Studying of flow model and bed load transport in a coarse bed river: case study – Aland River, Iran

Kiyoumars Roushangar, Yousef Hassanzadeh, Mohammad Ali Keynejad, Mohammad Tagi Alami, Vahid Nourani and Dominique Mouaze

ABSTRACT

This paper describes a mathematical model which solves the 1D unsteady flow over a mobile bed. The model is based on the Richtmyer second-order explicit scheme. Comparison of the model results with the experimental flume data for alluvial steady flow (aggradation due to overloading) and unsteady flow shows that, by using the two-step method of Richtmyer, one can solve the equations, governing the phenomenon, in a coupled method with the desired accuracy. Firstly, the Badalan reach located at the Aland River is considered. Variations of flow rate, water level and bed level profiles due to flood hydrographs are assessed. Secondly, bed load discharge data were collected from the Aland River and a variety of bed load discharge formulae were compared with measured data. Results show that, by using the grain size of the bed surface layer to predict the bed load discharge, a larger relative error will occur compared to the other two cases and a proper choice of grain size has the main role in reduction of the relative error of bed load discharge estimation in gravel bed rivers. The applicability of formulae varies depending on flow rate, and should be split into low and high flow transport formulae.

Key words | bed load, Richtmyer, Saint-Venant–Exner equations, simulation, unsteady flow

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INTRODUCTION

Nowadays, due to the limitations of the measured hydraulic data on the one hand and the development of numerical methods on the other, mathematical simulation of the flow behavior, sediment discharge, aggradation and degradation processes is necessary and unavoidable. Over the last three decades, refined numerical modeling of alluvial rivers has received considerable attention in the field of river engineering for the purposes of hydropower generation, flood control and disaster alleviation, water supply and navigation improvement as well as environmental enhancement. A large number of numerical river models for the water–sediment–morphology fluvial system have been developed. The models have been used as one of the primary tools in river hydraulics research and engineering practice. Regarding the bed load evolution, many attempts have been accomplished in recent decades to understand and analyze particle motion and to evaluate the bed load. These proposed methods are based on statistical correlations, a combination of the theoretical models, logical assumptions and the experimental information. However, because of the actual restrictions in understanding the mechanism and complex motion of particles, a general analyzing procedure has not yet been established.

Flow model over a mobile bed

The one-dimensional modeling of unsteady sediment transport and bed evolution in alluvial channels is most often performed

doi: 10.2166/hydro.2010.010

by supplementing the St. Venant equations (De Saint Venant 1871), describing fluid continuity and flow momentum conservation, with the Exner equation (Exner 1925; Graf 1971) describing sediment continuity. Various researchers have focused their attempts on the one-dimensional simulation of the flow with movable beds in the alluvial channels and natural rivers. Among this research one can refer to a few analytical models for channel aggradation and degradation (Soni *et al.* 1980; Gill 1983a, b, 1987; Ribberink & Van der Sande 1985; Zhang & Kahawita 1987, 1990; Begin 1988; Lenau & Hjelmfelt 1992). While this type of analytical model provides an easy-to-use approach to predict the response of river channels to change simple water and sediment hydrograph or base lowering, these models are based heavily on assumptions. First, the flow is assumed to be quasi-steady, leading to the elimination of $\partial h/\partial t$ and $\partial u/\partial t$ in the water–sediment mixture continuity and momentum equations. Second, in the momentum equation the nonlinear convective acceleration term $U\partial U/\partial x$ is ignored, yielding a diffusion model for the bed elevation evolution (Soni *et al.* 1980; Gill 1983a, b; Begin 1988; Lenau & Hjelmfelt 1992). A slightly modified kind of model, namely a hyperbolic model, has been developed by including the nonlinear convective effect to some extent using a perturbation technique (Ribberink & Van der Sande 1985; Gill 1987: Zhang & Kahawita 1987, 1990). Finally, in the sediment continuity equation the sediment storage term is almost exclusively not taken into account in order to make the analytical solution tractable. One of the major challenges in using these analytical models is the determination of the model coefficients involved. Additionally, it appears not to be encouraging to use these analytical models with highly variable hydrographs (complicated boundary conditions). More comments on these analytical models can be found in Zanre & Needham (1996).

Currently, finite difference techniques in the simulation of unsteady flow with movable bed in open channels have become a predictive mathematical tool. In recent years, a large variety of them have been applied and a comparison among them is extremely difficult to make. These one-dimensional (1D) numerical river models have been applied with two fundamental aspects. The first aspect is associated with simplifications in the governing equations. Alluvial flows over erodible beds are distinguished from those over fixed beds in that the flow may entrain sediment from the bed or, in contrast, cause the sediment carried by the flow to be deposited on the bed, which usually causes river bed degradation or aggradation. This is referred to as the bottom mobile boundary problem. At the same time, the water–sediment mixture may have properties different from clear water. In spite of these apparently known features of erodible-bed flows, it is often assumed that the rate of bed morphological evolution is of a lower order of magnitude than flow changes with adequately low sediment concentration. Accordingly, the water–sediment mixture continuity equation is almost exclusively assumed to be identical to that for a fixed-bed clear-water flow without considering the river bed mobility, i.e. $\partial z/\partial t$ in Equation (1) is neglected (e.g. Cunge *et al.* 1980; Jaramillo & Jain 1984; Holly & Rahuel 1990; Needham 1990; Morse *et al.* 1991; Needham & Hey 1991; Wormleaton & Ghumman 1994; Cui *et al.* 1996; Zanre & Needham 1996; Sieben 1999). The effect of this treatment appears to have been quantitatively addressed only by Correia *et al.* (1992) and discussed by Rahuel (1993). Stevens (1988) claimed that bed mobility is important for complete coupling of water and sediment in discussing Lyn's (1987) analysis. The second aspect to be considered is the numerical solution procedure of the coupled system of governing equations. The water–sediment–morphology fluvial system is strongly coupled, as clearly demonstrated in the formulated partial differential Equations (1)–(3) and with the auxiliary closure relations (4) and (5). In existing numerical models, these equations are mostly solved in an asynchronous procedure. Specifically, in a given time step, the mixture continuity and momentum equations are solved, assuming negligible bed change rate (or fixed morphology). Then the sediment continuity equation is solved, using the newly obtained flow variable. Models involving the asynchronous solution are usually referred to as decoupled. There have been semi-coupled models in which the flow and bed equations are solved iteratively in a given time step (Park & Jain 1986, 1987; Kassem & Chaudhry 1998). There is an ongoing debate about which approach is the most suitable for morphological river modeling. Decoupled models have been criticized as being mathematically ill-posed and numerically unstable; incapable of handling rapidly changing boundary conditions or supercritical flow (Lyn 1987, Correia *et al.* 1992; Cao *et al.* 2002). However, Kassem & Chaudhry (1998) claimed that decoupled models are not unstable, as usually referenced in the literature.

Bed load transport

Bed load transport is a basic factor in determining the morphologic development of alluvial river reaches. Few bed load discharge equations were available at the beginning of the systematic regulation of rivers, but since the middle of the 20th century a variety of bed load discharge formulae have been developed. They are the result of laboratory investigations with controlled boundary conditions, equilibrium transport and bed level stability. At the end of the 1970s numerical sediment transport models allowed the calculation of river bed level changes over lengthy periods of time (Subcommittee on Sedimentation 1988). Existing bed load discharge formulae have been classified by Graf (1971) into Du Boys-type equations (Du Boys 1879) that have a shear stress relationship, Schokitsch-type equations (Schoklitsch 1934) that have a discharge relationship and Einstein-type equations (Einstein 1950) that are based upon statistical consideration of lift forces. Additionally, Gomez & Church (1989) distinguished stream power equations (see, e.g., Bagnold 1980).

Numerous bed load discharge equations have been derived hitherto, but only a limited number of field studies are available for validation or for the further development of formulae. Major reviews of bed load discharge formulas (see, e.g., Johnson 1939; Vanoni *et al.* 1961; Shulits & Hill 1968; White *et al.* 1973; Mahmood 1980; Carson & Griffith 1987; Zanke 1987; Gomez & Church 1989; Chang 1994; Reid *et al.* 1996, Lopes *et al.* 2001) were performed using either laboratory data or field data. The use of prototype data allows a more realistic evaluation of bed load formulae. In many of the evaluations, the formulae, the fit or appropriateness of the formulae self-evidently gave good results. Most evaluations concluded with a recommendation or representative formula, but no universal relationship between bed load discharge and hydraulic conditions was established. A recent review (Wilcock 2001) highlights the reasons why one cannot expect highly predictive power under selected prototype conditions. Generally, there is a lack of field data with which to test and to verify formulae, or to deal with sediment transport complexities related to a deficit of bed load. Most of the formulae rely on a limited database, untested model assumptions and a general lack of field data. Consequently, the application of many formulae is limited to special cases of their development; only a few are generally accepted for practical use, for the formulae were based on their applicability to gravel bed rivers. One can refer to Schoklitsch (1934); Meyer-Peter & Müller (1948), Einstein (1950), Yalin (1963) and Parker *et al.* (1982) for equilibrium conditions and Zanke (1987), Sun & Donahue (2000) and Wilcock & Crow (2003) for both non-equilibrium conditions and equilibrium conditions. In this research, one-dimensional, unsteady flow equations (Saint-Venant equations) and sediment continuity equation are solved numerically by the second-order accurate, explicit finite difference two-step scheme developed by Richtmyer & Morton (1967). All three governing equations are solved simultaneously during any step so that the water flow equations and the sediment continuity equation are coupled. The computed results are compared with the experimental data obtained in a laboratory flume for steady flow and unsteady flow over a mobile bed. Secondly, the bed load transport process in the Badalan interval located on the Aland River (a coarse bed river) is considered. Bed load discharges (field data under uncontrolled bed equilibrium conditions) were collected from the Aland River and a variety of bed load discharge formulae were compared with measured data. Then, the role of bed layers and bed material load grain size on the error of the bed load discharge estimation and evaluation of bed load formulae were investigated.

Therefore, the aims of this paper are:

- To report the results of an investigation on the performance of a mathematical model based on the Richtmyer scheme in the river under study.
- To compare bed load transport formulae with prototype data and analyze advantages and disadvantages of the formulae in the river under study.
- To describe the influence of the proper choice of input parameters and grain size of bed layers and bed material load for the predicting of bed load rate.

GOVERNING EQUATIONS

Consider flow in an erodible alluvial channel with an idealized rectangular section of constant width. The partial differential equations governing the model, which encompasses the unsteady flow with a movable bed, are of nonlinear hyperbolic type. These equations can be expressed as

$$(h)_t + (hu)_x = 0 \quad \text{Continuity (for water)} \tag{1}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x - gh(S_\circ - S_f) = 0 \quad \text{Momentum (for water)} \tag{2}$$

$$(z)_t + \left(\frac{1}{1-p}\right)(q_s)_x = 0 \quad \text{Continuity (for sediment)} \tag{3}$$

where t = time; x = streamwise coordinate; h = flow depth; S_o = bed slope; $q = uh$ = water discharge for unit width; q_s = bed load discharge for unit width; u = cross-section-averaged streamwise velocity; z = bed elevation; g = gravitational acceleration; S_f = friction slope and p = porosity of bed layer.

To close the governing Equations (1)–(3), the flow resistance and sediment discharge need to be specified. In this study, the friction slope is determined via the Manning relation:

$$S_f = \frac{m^2u^2}{{R_h}^{4/3}} \tag{4}$$

where m is the Manning roughness coefficient and R_h is the hydraulic radius.

The following relationship for unit sediment discharge is used here:

$$q_s = f(u, h, \delta, \ldots) \tag{5}$$

where δ = coefficients depending on sediment characteristics.

Equations (1)–(3) are a set of nonlinear hyperbolic equations, and closed-form solutions are available only for idealized cases. Therefore, they are solved by numerical schemes. In this model, a finite difference scheme developed by Richtmyer (Richtmyer & Morton 1967) is used. The modified Lax–Wendroff scheme by Richtmyer & Morton (1967) is a combination of the Lax–Friedrichs scheme and a midpoint leapfrog scheme, with each of the two steps being applied at half the time interval, consecutively. The Richtmyer scheme is an explicit scheme, is simple to implement and does not require inversion of large matrices. It is also easy to incorporate general empirical equations for roughness and sediment discharge. The scheme is second-order accurate in both space and time. The solution grid for two steps and boundary conditions are presented in Figure 1.

The system used here is the conservative system, applying the above form to that system with source terms included:

Step 1

At grid point i

$$h_{i+1/2}^{n+1/2} = \frac{1}{2}(h_{i+1}^n + h_i^n) - \frac{\Delta t}{2\Delta x}((hu)_{i+1}^n - (hu)_i^n) \tag{6}$$

$$(hu)_{i+1/2}^{n+1/2} = ((hu)_{i+1}^n + (hu)_i^n) - \frac{\Delta t}{2\Delta x}\left[\left(hu^2 + \frac{1}{2}gh^2\right)_{i+1}^n - \left(hu^2 + \frac{1}{2}gh^2\right)_i^n\right] + \frac{\Delta t}{2}\frac{(\varphi_{i+1}^n + \varphi_i^n)}{2} \tag{7}$$

$$z_{i+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2}(z_{i+1}^n + z_i^n) - \frac{\Delta t}{2(1-p)\Delta x}\left(q_{s_{i+1}}^n - q_{s_i}^n\right). \tag{8}$$

Now, at grid point $i-1$:

$$h_{i-1/2}^{n+1/2} = \frac{1}{2}(h_{i-1}^n + h_i^n) - \frac{\Delta t}{2\Delta x}((hu)_i^n - (hu)_{i-1}^n) \tag{9}$$

$$(hu)_{i-1/2}^{n+1/2} = \frac{1}{2}((hu)_{i-1}^n + (hu)_i^n) - \frac{\Delta t}{2\Delta x}\left[\left(hu^2 + \frac{1}{2}gh^2\right)_i^n - \left(hu^2 + \frac{1}{2}gh^2\right)_{i-1}^n\right] + \frac{\Delta t}{2}\frac{(\varphi_i^n + \varphi_{i-1}^n)}{2} \tag{10}$$

$$z_{i-\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2}(z_{i-1}^n + z_i^n) - \frac{\Delta t}{2(1-p)\Delta x}\left(q_{s_i}^n - q_{s_{i-1}}^n\right) \tag{11}$$

Step 2:

$$(hu)_i^{n+1} = (hu)_i^n - \frac{\Delta t}{\Delta x}\left[\left(hu^2 + \frac{1}{2}gh^2\right)_{i+1/2}^{n+1/2} - \left(hu^2 + \frac{1}{2}gh^2\right)_{i-1/2}^{n+1/2}\right] + \Delta t\frac{(\varphi_{i+1/2}^{n+1/2} + \varphi_{i-1/2}^{n+1/2})}{2} \tag{12}$$

$$z_i^{n+1} = z_i^n - \frac{\Delta t}{\Delta x}\left(q_{s_{i+\frac{1}{2}}}^{n+\frac{1}{2}} - q_{s_{i-\frac{1}{2}}}^{n+\frac{1}{2}}\right) \tag{13}$$

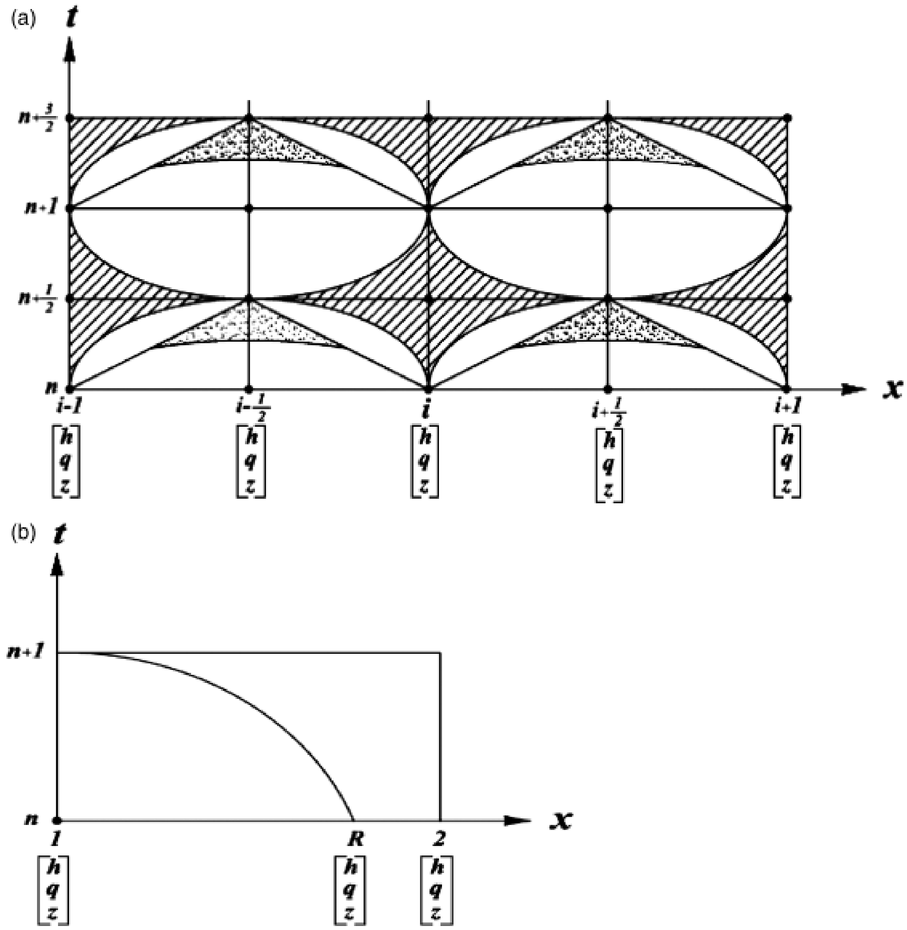

Figure 1 | (a) Schematic of Richtmyer two-step scheme. (b) Characteristics method for boundary conditions.

where φ is the source term defined by

$$\varphi^n_{i\pm1} = gh^n_{i\pm1}(S_0 - S_t)^n_{i\pm1} \text{ and}$$

$$\varphi^n_{i\pm1/2} = gh^n_{i+1/2}(S_0 - S_f)^n_{i+1/2} \tag{14}$$

$$(S_f)_{i\pm1} = \frac{m^2|u_{i\pm1}|u_{i\pm1}}{R^{4/3}_{hi\pm1}}; \quad (S_f)_{i\pm1/2} = \frac{m^2|u_{i\pm1/2}|u_{i\pm1/2}}{R^{4/3}_{hi\pm1/2}} \tag{15}$$

Higher-order schemes like the Richtmyer scheme usually produce numerical oscillations near the steep wave fronts. These oscillations, caused by the dispersive errors associated with the odd leading term in the truncation error, may be dampened by introducing artificial viscosity. For this purpose, a procedure developed by Jameson *et al.* (1981) may be used.

First the variables Ψ and Γ are defined as

$$\Psi = \begin{cases} \dfrac{|h_i - h_{i-1}|}{|h_i| + |h_{i-1}|} & \text{for } i = k+1 \\ \dfrac{|h_{i+1} - 2h_i + h_{i-1}|}{|h_{i+1}| + 2|h_i| + |h_{i-1}|} & \text{for } i = 2-k \\ \dfrac{|h_{i+1} - h_i|}{|h_{i+1}| + |h_i|} & \text{for } i = 1 \end{cases} \tag{16}$$

$$\Gamma^+ = 0.5Max(\Psi_{i+1}, \Psi_i), \quad \Gamma^- = 0.5Max(\Psi_i, \Psi_{i-1}) \tag{17}$$

where the final values are corrected as

$$\begin{bmatrix} h \\ hu \\ z \end{bmatrix}^{n+1}_i = \begin{bmatrix} h \\ hu \\ z \end{bmatrix}^{n+1}_i + 2\Gamma^+ \begin{bmatrix} h_{i+1} - h_i \\ (hu)_{i+1} - (hu)_i \\ z_{i+1} - z_i \end{bmatrix}^{n+1} - 2\Gamma^- \begin{bmatrix} h_i - h_{i-1} \\ (hu)_i + (hu)_{i-1} \\ z_i - z_{i-1} \end{bmatrix}^{n+1}, \tag{18}$$

BOUNDARY CONDITIONS

In order to calculate the values of h, u and z at nodes 1 and $k+1$, for the subcritical flow conditions according to the characteristics method (Vardy 1977), two upstream boundary conditions and one downstream boundary condition should be known.

The upstream boundary condition is the inlet hydrograph of the flood. Using the characteristics method, other dependent variables are calculated as follows. At the upstream, first, the values of ${h_1}^{n+1}$ and ${u_1}^{n+1}$ are obtained by defining the values of variables at point R and using the inlet hydrograph. Then the value of ${x_R}^n$ is corrected. Using the corrected values, the operations are iterated until the desired convergent solution is achieved. In this way, the values of the unknown variables at the upstream nodes is obtained:

$$x_R^n = (c-u)_1^n.\Delta t \tag{19}$$

$$\begin{bmatrix} uh \\ h \\ z \end{bmatrix}_R^n = \begin{bmatrix} uh \\ h \\ z \end{bmatrix}_2^n - \left(\frac{x_2^n - x_R^n}{\Delta x}\right) \begin{bmatrix} (uh)_2 - (uh)_1 \\ h_2 - h_1 \\ z_2 - z_1 \end{bmatrix}^n \tag{20}$$

$$h_1^{n+1} = \frac{((uh)_1^{n+1} - (uh)_R^n) - \Delta t(\xi_R^n)}{(u+c)_R^n} + y_R^n \tag{21}$$

$$x_R^n = \frac{1}{2}\Delta t((c-u)_1^{n+1} + (c-u)_R^n) \tag{22}$$

$$\begin{bmatrix} uh \\ h \\ z \end{bmatrix}_R^n = \begin{bmatrix} uh \\ h \\ z \end{bmatrix}_2^n - \left(\frac{x_2^n - x_R^n}{\Delta x}\right) \begin{bmatrix} (uh)_2 - (uh)_1 \\ h_2 - h_1 \\ z_2 - z_1 \end{bmatrix}^n \tag{23}$$

$$h_1^{n+1} = \frac{2((uh)_1^{n+1} - (uh)_R^n) - \Delta t(\zeta_R^n + \zeta_1^{n+1})}{(u+c)_R^n + (u+c)_1^{n+1}} + h_R^n \tag{24}$$

where $c = \sqrt{gh}$ and $\xi = gh.(S_0 - S_f)$.

The downstream boundary condition is the discharge–depth diagram. Therefore, the same procedure as for the upstream boundary nodes is used, considering the positive slope of the characteristics curve (Vardy 1977).

Stability. For stability, the Richtmyer scheme must satisfy the Courant–Friedrichs–Lewy (CFL) condition (Courant *et al.* 1928). It is given for a rectangular section by the following formula:

$$C_n = \left(\frac{q}{h} + \sqrt{gh}\right)\frac{\Delta t}{\Delta x} \leq 1 \tag{25}$$

in which C_n is the Courant number and Equation (25) must be satisfied at every grid point for the scheme to be stable.

COMPUTATIONAL PROCEDURE

To start the computations, the initial conditions, i.e. the values of ${h_i}^n$, ${q_i}^n = (uh)_i{}^n$ and ${z_i}^n$ at $t = 0$ are known at all grid points. The values at the end of the time interval Δt are computed as follows.

The values of $h_{i\pm\frac{1}{2}}^{n+\frac{1}{2}}, q_{i\pm\frac{1}{2}}^{n+\frac{1}{2}}$ and $z_{i\pm\frac{1}{2}}^{n+\frac{1}{2}}$ at the interior nodes ($i = 2,\ldots,k$) are computed by using Equations (6)–(11), and their values at the boundaries ($i = 1$ and $i = k+1$) are computed by using the boundary conditions. Then, h, q and z at the end of time interval Δt, *i.e.*, ${h_i}^{n+1}$, ${q_i}^{n+1}$, and ${z_i}^{n+1}$ are determined by using Equations (12) and (13). The values determined in step 2 are modified to dampen higher-order oscillations by using the procedure presented before. ${h_i}^1$, ${q_i}^n$, and ${z_i}^n$ for the next time interval are set equal to ${h_i}^{n+1}$, ${q_i}^{n+1}$, and ${z_i}^{n+1}$, respectively, and the time interval Δt for the next step is determined from Equation (25). The procedure is repeated until the required time is reached.

The coupling of flow equations and the sediment continuity equation are achieved in this method because it uses a kind of two-level predictor–corrector approach. Strictly speaking, there is no coupling during the first step. However, the predicted values of h and q are both used to determine q_s and evaluate the spatial derivative term in Equation (3). Then in the second step, the ultimate computation of each dependent variable at the end of the time step takes into account the changes in all the other variables. On the other hand, coupling is not achieved if Equation (3) is solved after completely solving Equations (1) and (2).

TEST OF MODEL

Aggradation due to sediment overloading

The model presented in the previous sections was used to simulate the aggradation process observed by Soni *et al.* (1980) in a laboratory channel. They considered a rectangular alluvial channel carrying a constant unit discharge q_0 at a uniform flow depth of h_0. The equilibrium between the water flow and the sediment flow was disturbed by increasing the sediment inflow at the upstream end from the equilibrium value of q_{s0} to $q_{s0}+\Delta q_s$.

The experiments were conducted in a channel 0.2 m wide and 30 m long. The sand forming the bed and the injected material had a mean diameter of 0.32 mm. The values of the empirical constants a and b in the sediment transport formula, $q_s = a(\frac{q}{h})^b$, found from the uniform flow experiments were 1.45×10^{-3} and 5.0, respectively. The Manning coefficient was approximately equal to 0.022 and the porosity p of the sediment bed layer was equal to 0.4. The values of the initial uniform water discharge $q_0 = 0.02$ m^2/s, the uniform flow depth $h_0 = 0.05$ m, the initial bed slope $S_0 = 3.56\times10^{-3}$, the equilibrium sediment discharge q_{s0}, and the increment in the sediment discharge at the upstream end Δq_s are considered equal to $\frac{\Delta q_s}{q_{s0}} = 4$ in this test.

In the mathematical model, uniform unit discharge, uniform flow depth and the initial bed elevations, as calculated from S_0, were specified at every finite difference node as the initial conditions. The transient state was initiated by increasing the sediment discharge at the upstream end by Δq_s. As mentioned earlier, one boundary condition at the downstream end and two at the upstream end were imposed for the satisfactory application of the model. The upstream constant discharge boundary could easily be implemented by specifying $q(0,t) = q_0$ for all $t \geq 0$ However, the inclusion of the second boundary $q_s(0,t) = q_{s0} = \Delta q_s$ was not as straightforward as the former, and it needed to be translated into an equation by which the bed elevation at the upstream end could be calculated. This was achieved by assuming a fictitious node upstream from node 1 and specifying the sediment discharge at that node equal to $q_{s0}+\Delta q_s$. Using the sediment continuity equation and applying the backward finite difference (Anderson et al. 2006) on the spatial differential term, one obtains the left-hand side of Equation (26) and therefore z at the unknown time level $k+1$ could be calculated:

$$\left[(1-p)z + \frac{q_s h}{q}\right]_1^{k+1} = \left[(1-p)z + \frac{q_s h}{q}\right]_1^{k} + \frac{\Delta t}{\Delta x}\left[(q_{s0}+\Delta q_s) - (q_s)_1^k\right]. \quad (26)$$

The downstream boundary condition was the constant depth, which was specified by $h(k\Delta x,t) = h_0$ for all $t \geq 0$. This was based on the assumption that the channel was long and the bed transients would not reach the downstream end within the period for which conditions were computed and that the variation in flow depth would be negligible.

Figure 2 shows the comparison between the computed transient bed and water–surface profiles with the measured values at $t = 40$ min for test case 1. The "measured" points here are not actual data points. They are the points of average transient profile as reported by Soni et al. (1980).

Unsteady flow over a movable bed

The model ability to simulate the process of aggradation and degradation due to change in flow rate was tested using data from flume measurements performed at the Hydraulics Laboratory of the Environment Canada at Burlington, Ontario, Canada (Krishnappan 1983). The sediment transport flume has 23 m length and 2 m width. In this test the initial bed slope is 0.2%, the grain size is uniform with $D_{50} = 1.2$ mm, the Manning coefficient is $m = 0.017$ and the sediment supply at the flume entrance is held constant at 0.06 kg/s. The Ackers & White (1973) method is used for determination of sediment transport rate. Figure 3 shows the water supply rate at the flume entrance.

Due to this hydrograph, both aggradation and degradation occurs. The initial conditions are specified using the measured water and bed level profiles at $t = 0$.

The flow rate and sediment input at the upstream and measured depth at downstream is imposed as boundary conditions.

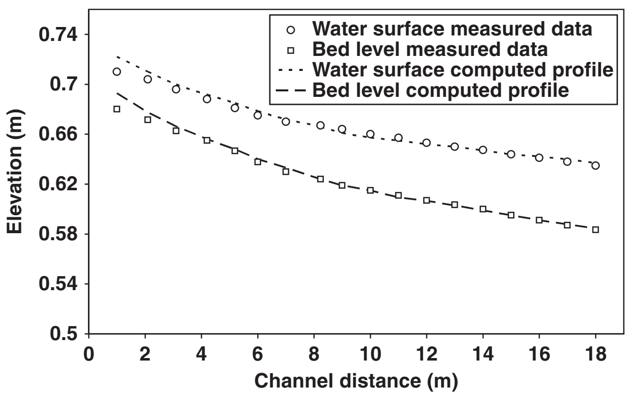

Figure 2 | Transient water and bed profiles at $t = 40$ min due to aggradation.

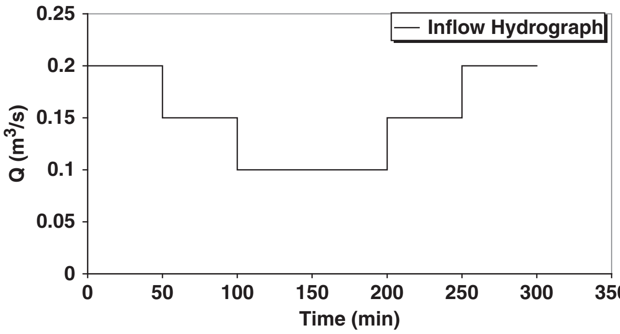

Figure 3 | Inflow hydrograph in laboratory flume.

VERIFICATION OF MODEL

Figure 2 compares the computed transient bed and water surface profiles with the measured values at $t = 40$ min for test 1. It is clear from this figure that the model simulates the aggradation of the channel bed and transient water surface profile satisfactorily. The difference between observed and computed values can be partially attributed to:

(a) Varying bed roughness during the experiment due to the presence of bed forms (roughness coefficient is considered constant),
(b) the uncertainty in the sediment transport equation,
(c) the small fraction of injected sediment deposited upstream of the first cross section (ignored by the numerical model).

For test 2, according to Figure 4, for bed level predicted data, due to the flume bed being covered by dunes during the experiments and bed measured data are not mean values and the model is not capable of predicting the dune shapes and their movement, some differences and errors are observed.

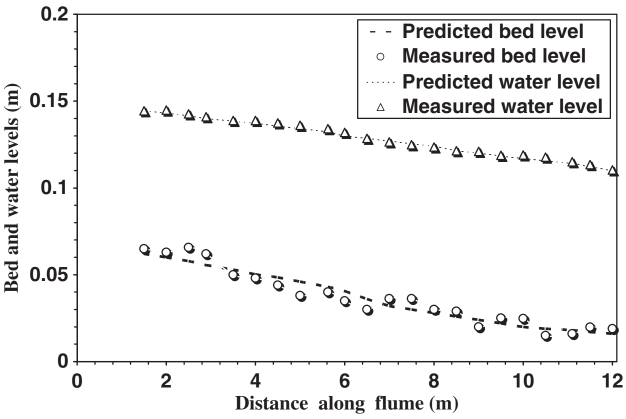

Figure 4 | Comparison between measured and predicted water level and bed level.

CASE STUDY: ALAND RIVER, IRAN

The Aland River emerges from the mountains on the Iran–Turkey border and, after a long distance, discharges into the Qotoor River in the south of Khoy city (see Figure 5). The Badalan reach located between the downstream of the Badalan hydrometric station and upstream of the diversion dam of Aland has been chosen and the mathematical model for the bed load transport evaluation and the hydraulic simulation of the flood flow in the mentioned interval has been studied by employing the proposed method. The features of the river within the considered reach are 400 m in length, 20 m in width and its bed slope is 0.67%.

For the flow resistance calculation, the surface layer of the bed is considered as the criterion. The Manning roughness coefficient in the considered reach has been calculated based on the following three methods.

Empirical methods

These formulae are in the form of $m = a(D_c)^b$ in which D_c is the particle grain size for which c% of all particles are smaller than that and a and b are constants. Some researchers

Figure 5 | Location map of Aland River and Badalan reach.

(Meyer-Peter & Müller 1948; Lane & Carlson 1953; Hederson 1966; Raudkivi 1976; Garde & Ranga Raju 1978; Bray 1979; Subramanya 1982) used different material sizes (D_{50}, D_{65}, D_{75} and D_{90}) and coefficients in their empirical formulae. The results are according to Table 1.

Qualitative methods

In these methods, the m value is chosen from a similar river with the same characteristics (morphological, hydraulic and geometric). Photographic methods or tables are used for this purpose (French 1986). In this method $0.034 \leq m \leq 0.039$ is obtained.

Direct measurement method

In this method m is estimated directly from discharge, water surface slope and hydraulic geometry of the channel in a representative reach. The length of the reach should be greater than or equal to 75 times the mean depth of flow and the fall of the water surface should be equal to or greater than 0.15 m. By applying the slope-area method, m can be estimated directly from the following formula (Bathurst 1986; French 1986):

$$m = \frac{1}{Q}\left[\frac{(Y+h_v)_1 - (Y+h_v)_N - \sum_{i=2}^{N}(\beta\Delta_v)_{i-1,i}}{\sum_{i=2}^{N}\frac{L_{i-1,i}}{(AR_h^{\frac{2}{3}})_{i-1}\cdot(AR_h^{\frac{2}{3}})_1}}\right]^{\frac{1}{2}} \quad (27)$$

in which N = number of cross sections along the reach, L = distance between two successive sections, Y = elevation of the water surface at the section with respect to a datum common to all sections, A = flow cross sections, Rh = hydraulic radius, h_v = velocity head at a section, $h_v = \alpha v^2/2g$, v = mean velocity, Q = measured discharge, α = velocity head correction factor and β = coefficient accounting for the non-uniformity of the channel ($\beta = 0$ for uniform and 0.5 for non-uniform reach). In this method $0.030 \leq m \leq 0.032$ is obtained. By applying the measured hydraulic parameters (A, Rh, S_f) and empirical roughness coefficients for discharge calculation (by the Manning formula) and comparison between calculated and measured discharge, and by considering the results of all three methods, the empirical formula of Bray (1979) (based on studies of 67 coarse bed rivers) gives the best estimate of m ($m = 0.031$).

Bed load data-measurement

Bed load discharge field data (under uncontrolled bed equilibrium conditions) were collected from two Aland River sites (i.e. upstream site, Badalan hydrometric station and downstream site, Aland diversion dam) during April–September 2007 by using the Helley–Smith sampling technique. The Helley-Smith sampler has an intake opening of 152 mm × 152 mm, with a mesh bag of 4000 cm^2 and mesh size of 0.2 mm, which made it useful for this study as it can catch large sized particles. The bed load measurements were carried out mostly in the afternoon as water discharge started to rise at this time and gain its peak value (because of the snowmelt regime). These measurements were continued till sunset and in some occasions after sunset. The duration of sampling depended on the bed load discharge to ensure that only 30% of the basket was filled (Hubbell *et al.* 1986). In principle, the Helley-Smith sampler was employed in the manner prescribed by Emmett (1980). The sampling lasted

Table 1 | Calculated roughness coefficients with empirical correlations

Methods / Coefficients	Meyer-Peter & Müller (1948)	Lane & Carlson (1953)	Hederson (1966)	Raudkivi (1976)	Garde & Ranga Raju (1978)	Bray (1979)	Subramanya (1982)
a (m)	0.038	0.0317	0.0415	0.0041	0.0476	0.056	*0.47*
b (m)	0.166	0.166	0.166	0.166	0.166	0.176	*0.166*
c (%)	90	75	50	75	50	65	*50*
m	*0.025*	*0.03*	*0.024*	*0.025*	*0.028*	*0.031*	*0.027*

30 s; during very low transport rates it was 60 s. There was a time interval of about 5–7 min between individual samples. At each vertical, three individual samples were obtained to remove short term fluctuations of the bed load (see, e.g., Ergenzinger 1992). Averaging of the three values allows one to obtain the mean transport rate per vertical. To check and maintain the quality of the collected data, the standards adopted by Bathurst *et al.* (1985) were followed during a bed load data collection study. Bed load data were collected according to the following conditions:

- The water and bed load discharge were steady during the period of measurement, i.e. when the discharge varied less than 5% throughout the sampling period, normally about 12 h.
- The bed load discharge varied with the water discharge; and
- The maximum possible bed load flux occurred, i.e. bed load transport was at capacity.

In addition to bed load sampling, hydraulic and sedimentological data were obtained. These included the following parameters: water discharge, flow width and depth, bed and water surface slopes, and grain size distributions of bed load, subsurface and surface bed material (see Figure 14).

MODEL PERFORMANCE AND RESULTS

Due to the direct path, the absence of sharp variations in the longitudinal profile of the bed, fixed width walls in the high flow rates and finally a large ratio of width/depth, the simulation of river flow is considered as one-dimensional. According to the inlet hydrograph in Figure 6, the flow rate increases from $Q_1 = 27$ m^3/s to $Q_2 = 80$ m^3/s in the period of $t_1 = 2400$ s and then decreases during the period of $t_2 - t_1 = 6600$ s.

To investigate the proposed mathematical model (Zanke 1987) the bed load transport formula (see the results of bed load evaluation) is applied to the sediment continuity equation. The interval is divided into eight spatial intervals of 50 m to be followed by the appropriate time steps to ensure the stability of the model. The three nonlinear partial differential equations along with the inlet hydrograph as an upstream boundary condition and depth rating curve as a downstream boundary condition have been solved. The hydraulic characteristics variations have been illustrated in Figures 7–13 with respect to the various parameters.

The comparison of the model results with the experimental data show that, by using the two-step method of Richtmyer, one can solve the equations governing the phenomenon in a coupled method with the desired accuracy. The delay between y_{max} and Q_{max} in the closed loop of the unsteady flow depicted in Figure 8 and the maximum rate difference between the inlet and outlet cross sections of Figure 7 represent the ability of the model to solve the Saint-Venant equations. According to Figure 10, the Froude number increases at $t = 40$ min and decreases at $t = 150$ min along the whole reach. This behavior is due to the time delay, the flow volume existing between sections and flood wave extension. Figures 7 and 9 show the effect of path length dissipation on the hydrograph variations of a flood and the hydraulic parameters like flow depth. Figure 11 shows that

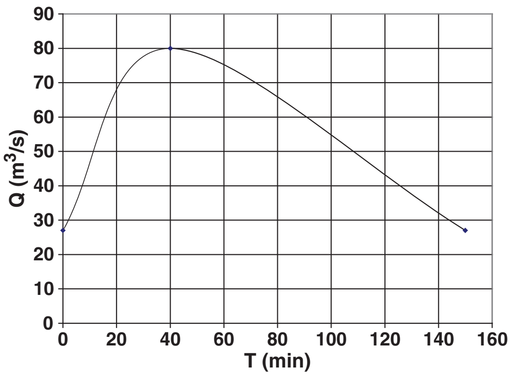

Figure 6 | Upstream inlet flow hydrograph in river.

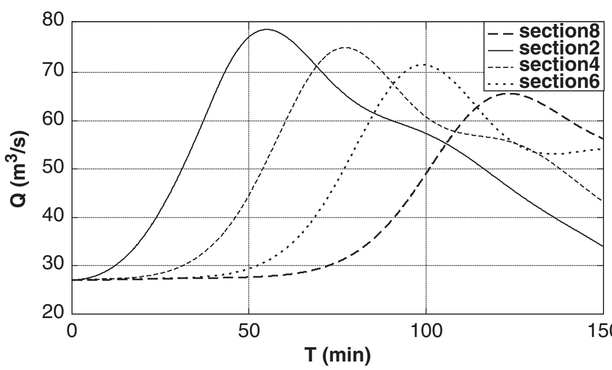

Figure 7 | Variations of flow discharge with time in different sections.

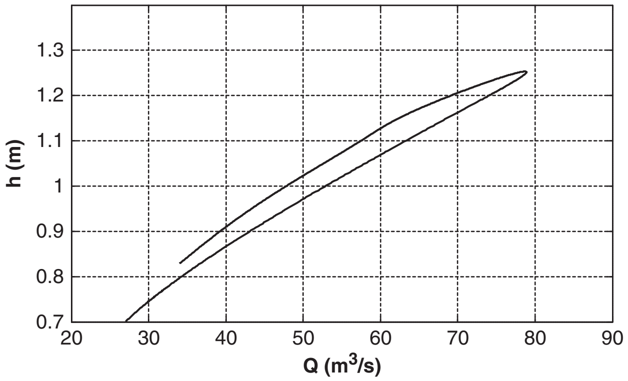

Figure 8 | Variations of flow discharge with water depth in section 2.

the variations of the Manning coefficient can produce around 10% error for estimating maximum flow discharge in the final section hydrograph. Therefore, due to the effect of the roughness coefficient variation on the hydraulic parameters such as velocity and depth or on sediment discharge formulae, it would be necessary and desirable to revise the amount of the roughness coefficient. It leads to a reduction of the expenses and an increase of the accuracy of calculations for problems in river engineering and designing of flood control structures.

Evaluation of bed load transport formulae

The collection of good quality bed load transport data is expensive and time-consuming. Therefore it is unavoidable to rely on predicted bed load transport rates determined from existing equations. The predictive abilities of the bed load transport formula for the Aland River were unknown due to lack of field measurements for testing. Therefore, eight formulae based on their applicability to gravel and coarse bed rivers (by considering the range of validity of formulae with measured data) were applied for the prediction of bed load discharge in the Aland River. These formulas are divided into:

(a) Shear-stress-based formula (Schoklitsch 1934; Meyer-Peter & Müller 1948; Yalin 1963). Formulae based on thresholds are sensitive to the value of the initiation of motion, depending on the grain size. For practical purposes the reach-averaged, non-dimensional Shields stress (Graf 1971):

$$\tau_i^* = \frac{\tau}{\rho(s-1)gd_i} \tag{28}$$

was calculated, where τ bottom shear stress; $(s-1)=$ submerged specific weight of the bed load; $\rho=$ density of water; $g=$ acceleration due to gravity and $d_i=$ grain size of the ith fraction. The gravel limit for Shields stress is 0.056. Zanke (1987) related this to a probability of 10% for particles to move. Several formulae are based on a comparison between actual and critical shear stress, e.g. the Meyer-Peter & Müller (1948) formula uses a critical Shields stress of 0.047.

(b) Stochastic probabilities-based formula (Einstein 1950; Zanke 1987; Sun & Donahue 2000; Wilcock & Crow 2003). The initial entrainment and motion of sediment particles is generally believed to constitute a stochastic process. The instantaneous shear stress, drag and lift forces induced by the temporal fluctuations of turbulent flow appear to be the main contributors to the stochastic

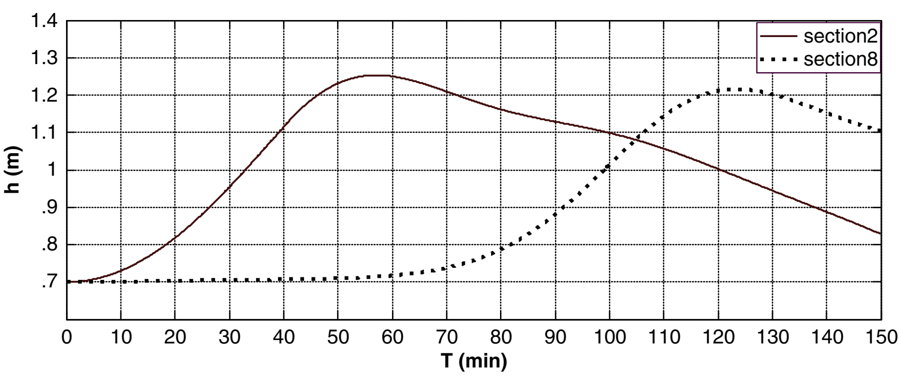

Figure 9 | Variations of water depth with time.

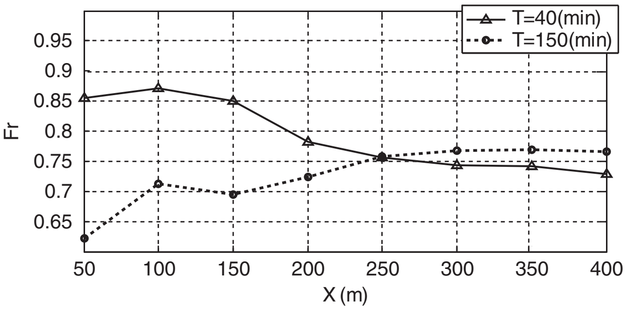

Figure 10 | Variations of Froude number along the channel.

nature of the sediment entrainment problem. For instance, Einstein (1950) used the pickup probability to derive the bed load function.

(c) Fractional bed load discharge formulas (Einstein 1950; Sun & Donahue 2000; Wilcock & Crow 2003). These methods aim to compute sediment transport rates for the various size fractions forming non-uniform mixtures. In one approach of these methods the bed material load is calculated after the computation of transport capacities corresponding to each size group by the summation of fractional sediment transport rates. In other approach, the formulae for sediment transport, developed for uniform sediments, are extended to non-uniform mixtures by using shear stress correction for individual size groups. In order to accomplish this, the shear stresses acting on each size fraction are accounted for by the use of a correction factor.

(d) Formulae based on stream power equations (Meyer-Peter & Müller 1948; Yalin 1963). In this study the methodology to evaluate these formulae is to compare measured data and calculated data with different water discharges. Different grain size distributions between the surface, subsurface and bed material load are used for calculating

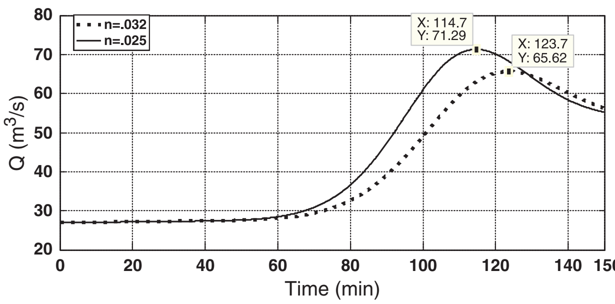

Figure 11 | Effect of the roughness coefficient variations on the outlet hydrograph and Q_{max}.

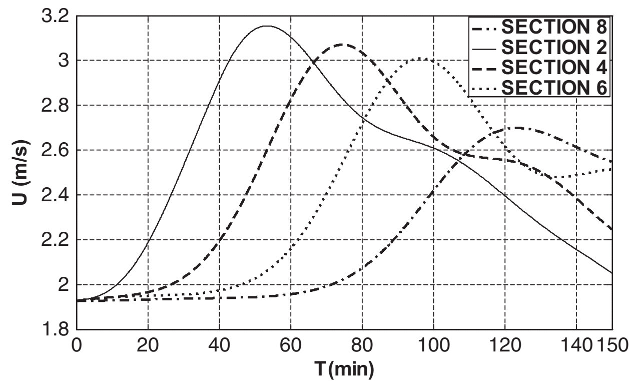

Figure 12 | Variations of velocity with time in different sections.

bed load discharge to describe the influence of the proper choice of input parameters. Relative average error

$$RAE = \left(\frac{\sum_{i=1}^{n} |(q_b measured - q_b calculated)|}{\sum_{i=1}^{n} q_b measured} \right)$$

and the root mean square of error

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^{n} (q_{bmeasured} - q_{bcalculated})^2 \right]^{0.5}$$

(Mendenhall & Sincich 1994) are used for comparison between calculated data by different bed load transport formulae and measured data. Results are shown in Figures 15(a)–(h) and Table 2.

All Figures 15(a)–(h) and Table 2 show that the grain size is a key variable in the calculation of the bed load transport rate. According to these figures, methods like Schocklitch (1934), Meyer-Peter & Müller (1948) and Einstein (1950) substantially underestimate the bed load discharge and, in some cases, predict zero transport. These formulae, based on the threshold of motion, are sensitive to the value of initiation of motion and give reasonable results for low bed load discharges ($Q \leq 20$ m^3/s). Other formulae like Zanke (1987), Sun & Donahue (2000) and Wilcock & Crow (2003) tend to over-predict the bed load rate and perform well (especially for $Q > 20$ m^3/s) using a subsurface layer or bed material load grain size. Therefore, according to Figures 15(a)–(h) and Table 2, the following results are obtained:

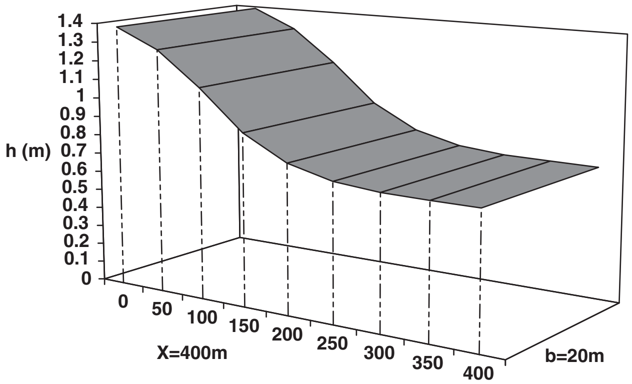

Figure 13 | Longitudinal section of river with flood wave at $t = 40$ min.

(1) Proper choice of grain size has the main role in the reduction of relative error of bed load discharge estimation in gravel bed rivers. Figures 15(a)–(h) and Table 2 show that, by using the grain size of the bed surface layer to evaluate the bed load discharge, a larger relative error will occur compared to the other two cases. Therefore, the estimation of the bed load rate using the physical properties of the bed subsurface layer and bed material load give rise to better results. Table 2 shows that using bed material grain size leads to smaller errors in comparing with subsurface layer grain size.

(2) According to Figures 15(d), 15(e) and Table 2, due to large errors, the Yalin (1963) and Parker *et al*. (1982) equations fail to predict this reach.

(3) Shear-stress-based formulae using single values for the threshold of motion, like Schoklitsch (1934) and Meyer-Peter & Müller (1948), are not suitable methods for predicting the bed load at high flow rates in this reach (see Figures 15(a) and (b)). In this work, samplings were obtained in uncontrolled conditions. It means that, in high flow rates, non-equilibrium conditions like aggradation and degradation occur and these formulae were not developed for non-equilibrium conditions.

(4) Equilibrium and non-equilibrium conditions coexist in natural rivers. Therefore, formulae like Zanke (1987),

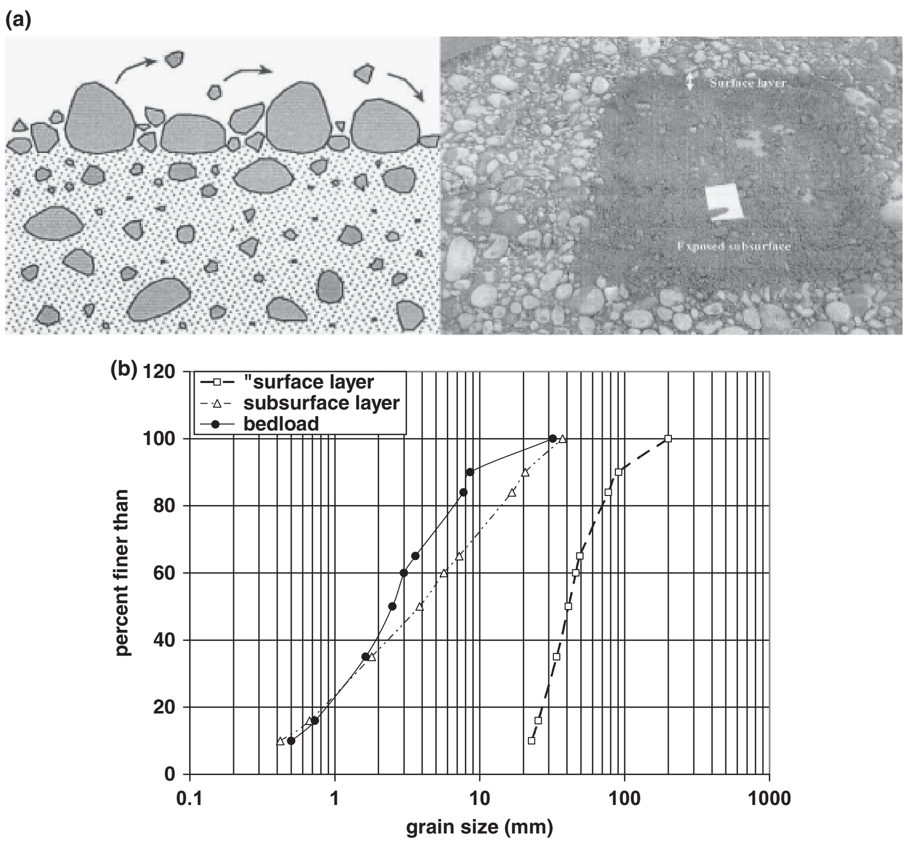

Figure 14 | (a) Bed surface layer and bed subsurface layer. (b) Grain size distributions of bed load, surface and subsurface layer.

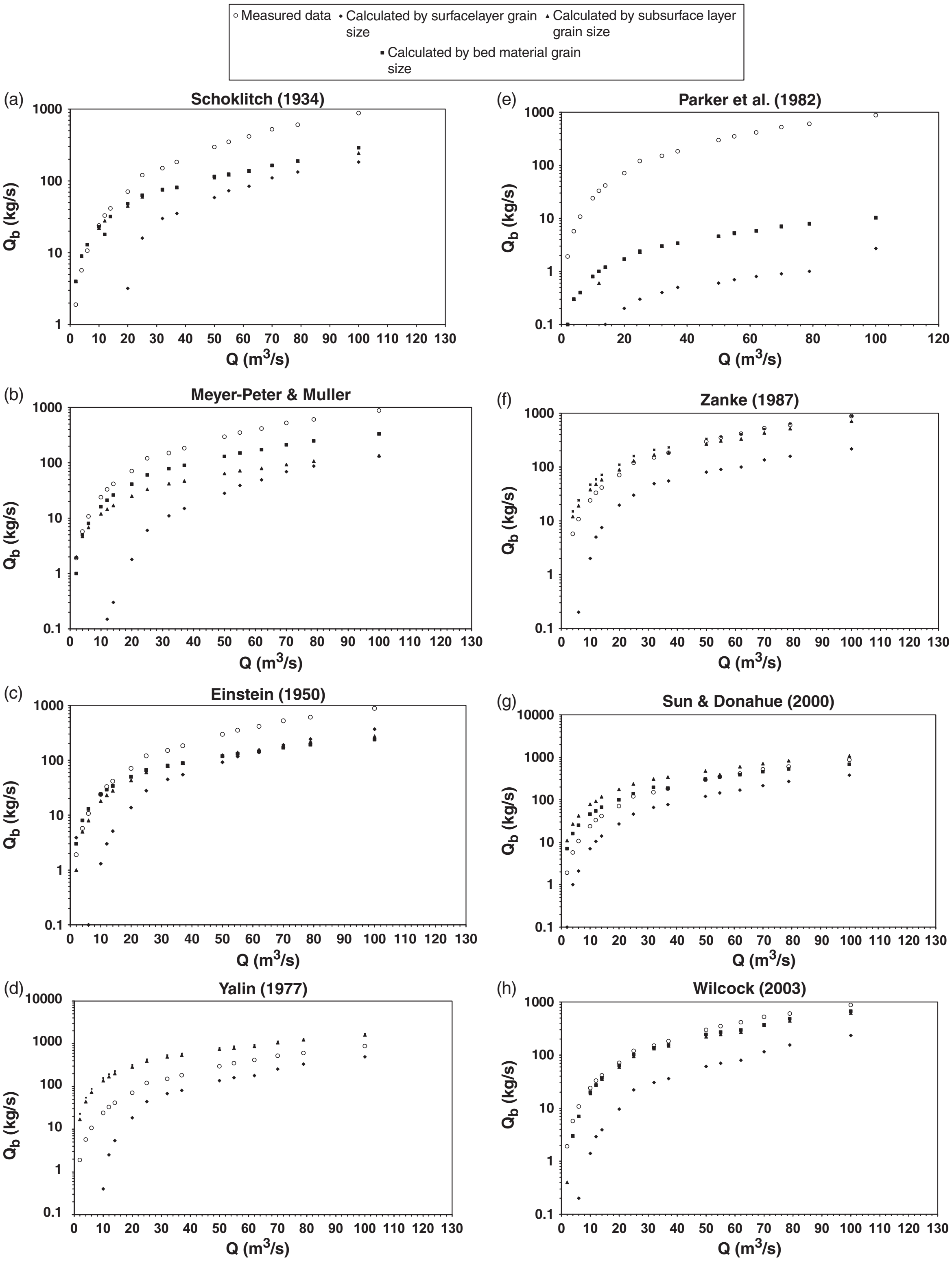

Figure 15 | (a)–(h) Comparison of the results of bed load discharge formulas with measured data by using different grain size of bed layers.

Table 2 | Evaluation of error of bed load discharge formulae by using different bed layers grain size

		Surface layer		Subsurface layer		Bed material	
	Bed layer Method	RAE	RMSE	RAE	RMSE	RAE	RMSE
For all Q (m^3/s)	Schoklitsch	81%	270	63%	235	62%	227
	Meyer-Peter	80%	297	79%	281	57%	204
	Einstein	65%	248	60%	222	62%	233
	Yalin	52%	538	134%	376	153%	428
	Parker	99%	337	98%	336	98%	335
	Zanke	74%	254	11%	48	10%	31
	Sun & Donhaue	58%	195	47%	134	4%	57
	Wilcock	82%	259	28%	99	21%	81
For $Q \leq 20$ m^3/s	Schoklitsch	98%	34	18%	10	21%	11
	Meyer-Peter	99%	34	56%	21	36%	14
	Einstein	97%	23	48%	8	16%	4
For $Q > 20$ m^3/s	Zanke	74%	334	11%	77	6%	34
	Sun & Donhaue	57%	260	42%	170	5%	75
	Wilcock	77%	345	25%	132	20%	108

Sun & Donahue (2000) and Wilcock & Crow (2003) that allow the calculation of bed load discharge for both conditions perform better for high flow rates (see Table 2).

(5) According to Figure 15 and Table 2, the formulae applicability varies depending on water flow rate: they should be split into two categories, transport rates that occurred during low water flow rate and transport rates that occurred during high water flow rate.

(6) For weak bed load discharge, the threshold shear stress formula perform comparatively well in this reach.

(7) Due to non-uniformity of the subsurface layer and bed material load (Figure 14(b)), good results are obtained by stochastic and fractional bed load discharge formulae like Einstein (1950), Sun & Donahue (2000) and Zanke (1987).

(8) For this reach, the Zanke method has the most reliable results by considering bed material load grain size.

It is apparent that care must be taken in applying any of the existing bed load discharge formulae to a particular river reach and, even then, the results are dubious unless augmented by some bed load measurements. Unfortunately, this indicates that, at present, one cannot predict bed load transport discharge with any degree of reliability without an adequate number of observations to guide us in choosing the appropriate formula.

CONCLUSION

This paper has described a mathematical model which solves the 1D unsteady flow over a mobile bed; it is based on the Richtmyer second-order explicit scheme. The proposed model has the following capabilities:

- The possibility of applying any boundary conditions.
- The possibility of employing any kind of sediment transport equation.
- It is unnecessary to solve the nonlinear equations system.
- The possibility of water surface and bed variations profile extraction at any temporal and spatial interval.
- The possibility of assessment of the flood hydrograph and its dissipation from upstream to downstream, at any point in space.

Secondly, the bed load transport rate was considered in a natural gravel bed river. For this purpose, comparison between measured data and calculated bed load discharge in the Aland River indicates that, by using the grain size of bed surface layer to predict the bed load discharge, a larger relative error will occur compared to the other two cases.

Formulae applicability varies depending on water flow rate: they should be split in to two categories, bed load transport rates that occur during low water flow rate and bed load transport rates that occur during high water flow rate. In the river under study, shear-stress-based formulae using single values for the threshold of motion like Schoklitsch (1934) and Meyer-Peter & Müller (1948) are not reliable methods for predicting the bed load at high flow rates. In contrast, stochastic and fractional bed load discharge formulae like Einstein (1950), Zanke (1987), Sun & Donahue (2000) and Wilcock & Crow (2003) perform well at high flow rates, and for this reach the Zanke method has the most reliable results by considering bed material load grain size.

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First received 4 January 2010; accepted in revised form 21 July 2010. Available online 17 December 2010