

## Discussion: “Effect of Tip Clearance on the Thermal and Hydrodynamic Performance of a Shrouded Pin Fin Array” (Moore, K. A., and Joshi, Y. K., 2003, ASME J. Heat Transfer, 125, pp. 999–1006)

**P. Razelos**

Professor Emeritus  
College of Staten Island,  
CUNY, Staten Island, NY 10314  
and  
2 Kanigos St.,  
Athens 10677, Greece  
e-mail: razel@ath.forthnet.gr

[DOI: 10.1115/1.2227054]

In this paper the authors present the experimental results of the effects of tip clearance on the thermal and hydrodynamic performance of a shrouded pin fin array. The authors have employed the most modern experimental techniques to obtain their results. This is indeed an excellent experimental work. The authors refer to two cases of existing similar experimental work, where pin fins have been used for cooling purposes: (a) cooling of turbine’s blades in aircraft engines and (b) cooling of electronic equipment. A comprehensive discussion with regard to turbines’ experiments, it can be found in the recent book by Han et al. [1]. On the other hand, the authors’ work can be useful to manufacturers that produce ready made electronic coolers found in the market as CF-512, BF-02, etc.

Experimental endeavors are costly and time consuming, while analytical solutions of extended surface heat transfer requires only mathematical skills. Most of the analytical results have been obtained, assuming the surface heat transfer coefficient constant. In this respect the heat transfer community is indebted to experimentalists for providing valuable information that can be used to obtain more realistic mathematical results. However, experimentalists often, as in the present case, have to use analytically obtained results. For example, the authors employed mathematical results obtained previously by Kraus and Bar-Cohen, authors’ Ref. [20].<sup>1</sup> It is also apparent that rudimentary theoretical background of fin

analysis is a prerequisite for any experimental endeavor, for designing an experiment. Therefore, there is a blend between experiment and analysis.

In the following, discussion we offer some suggestions that might help the authors, and for the sake of the readers, to improve the representation of their results. These comments refer: to the authors’ mathematical expressions (Eqs. (9)–(12)), and to evaluation of the pin fins’ performance (fin efficiency).

It is of vital importance in any fin analysis to define the parameters that characterize the problem. For a single spine, there are three parameters, having definite physical significance, similar to those commonly used dimensionless numbers in other fields, e.g., Reynolds, Nusselt, Sherwood, etc. These dimensionless numbers are usually formed from the geometry and other given physical quantities. In the present case we have: the thermal conductivity  $k$  of the fin’s material, and heat transfer coefficient  $h$ . Note, that since the purpose of authors’ experiments is to determine experimentally  $h$ , one can use either  $\bar{h}$  or the authors’  $h_{100}$ , but for the moment we will simply call it  $h$ . The fins geometry is characterized by its height  $H$ , and the base and tip’s diameters  $D_{ew}$  and  $D_T$ . The first parameter introduced is the ratio  $\lambda$ , which is equal to the ratio of the two diameters  $\lambda = D_T/D_{ew} = r_T/r_{ew}$ . The other two are

$$u^2 = (H/r_{ew})^2 (hr_{ew}D_{ew}/k) = (H/r_{ew})\text{Bi} \quad (1)$$

$$\text{Bi} = hD_{ew}/k \quad (2)$$

The parameter  $u^2 = hH/(kr_{ew}/H)$  represents the ratio of convection/conduction energy, and the second is the well known Biot Number, that represents the ratio of the internal to the external thermal resistances. It has been shown [2–5] that in any pin fin analysis, the parameters  $u$  and  $\text{Bi}^{1/2}$  should represent the dimensional height and base semithickness of the pin fin, respectively. It is worth noticing that using  $u$  as dimensionless height of a fin is nothing new. For example, Gardner’s [6] efficiency graphs, appearing in all the textbooks and handbooks, are plotted versus  $u$ . Also, as shown in Ref. 2, the pin’s dimensionless heat dissipation it is also plotted versus  $u$ . Also, the Biot number, Bi, (actually it should have been  $\text{Bi}^{1/2}$ ), has been extensively used, either to derive the criteria, which will guarantee the fin will not introduce any insulating effect, Schneider [7,2], Kraus et al. [8], or to derive the criteria for the validity of the one-dimensional approach by Irey [9], (cylindrical pin fins), [2], and Lau and Tan [10]. Irey, Lau and Tan, and all the authors on fin analysis found in the heat transfer literature, instead of  $u$  are using the ratio  $H/D$ , or  $H/w$ , where  $w$  is the base thickness.

For some technical reasons the authors use trapezoidal instead of cylindrical profile pins, which introduces the additional parameter  $\lambda$ . In fact, the authors pin fins are nearly cylindrical, because the values of  $\lambda$  are:  $\lambda = 0.9175, 0.8750, 0.8325$ , and their average values are:  $\lambda_{\text{ave}} = 0.9583, 0.9375, \text{ and } 0.9183$ . I believe that with these values  $\lambda$  their calculated results would have not been any different, if the authors had employed instead the pertinent equations for the cylindrical spines, given below. In addition, on the author’s page 1003, there is no reference for the definition of

<sup>1</sup>In the following I will refer to the author’s Ref. [20] as K-B.

Contributed by the Heat Transfer Division of ASME for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received February 15, 2005; final manuscript received June 18, 2005. Review conducted by Vijay Dhir.

efficiency, which is different from the one defined by Gardner [6], while from their Eq. (10) one can see that by definition

$$\eta = \frac{\bar{\Theta}}{\Theta_{ew}} \quad (3)$$

where, in the foregoing equation, the numerator is the space average dimensionless temperature.

Let us now discuss Eqs. (11), (12a), (12b) that were derived by K-B. In spines and longitudinal fins analysis it is advantageous to locate the origin of the coordinate,  $x=0$ , at the tip of the fin. However, symbol  $b$ , in their nomenclature, that represents the distance between the tip and the base of the fin, is not correct. As is shown in the Fig. 3.10 of K-B,  $b$  symbolizes the distance between a fictitious tip, (determined by  $\lambda$ ), and the base of the fin, while the symbol  $a$ , (that does not appear in the author's nomenclature but only in the pertinent equations), represents the distance from the fictitious tip to the tip of pin. Therefore, the distance  $b$  depends on the parameter  $\lambda$ , while  $a=b-H$ . Thus, the extremely complicated expression for the dimensionless temperature, Eq. (3.73) in K-B, involves the distances the  $b$  and  $a$ , instead of the given parameters  $H$  and  $\lambda$ . All of this enormously complicated formulation by K-B could have been circumvented if one selects the real tip to be the origin of coordinates, ( $0 \leq x \leq H$ ). For this particular case, the pertinent equations, given below, are taken from Ref. 4.

The profile of the fin and the dimensionless temperature are as follows

$$z = \lambda + (1 - \lambda)\varphi \quad \phi = x/H \quad (4)$$

The dimensionless temperature, defined as  $\theta = T/\bar{T}_{fl}$ , where  $\bar{T}_{fl}$  is the average fluid temperature both measured above the ambient temperature, is equal to

$$\theta = \left(\frac{1}{z}\right)^{1/2} [C_1 I_1(2pz^{1/2}) + C_2 K_1(2pz^{1/2})] \quad p = u/(1 - \lambda) \quad (5)$$

The foregoing equations show that the dimensionless temperature  $\theta = \theta(\varphi, u, \lambda)$ , while for a given  $u$  is a function of  $\varphi$  and  $\lambda$ , with  $z(0) = \lambda$ , and  $z(1) = 1$ . Note that integrating Eq. (4) from 0 to 1, we obtain the average fin diameter,  $D_{ave} = D_{end} (1 + \lambda)/2$ , thus  $0.9163 \leq z_{ave} \leq 1$ . In order to obtain the dimensionless temperature,  $\Theta_x$ , the authors use Eqs. (11), (12a), (12b), (Eq. (3.73) in K-B). However, the K-B dimensionless temperature has been obtained using constant  $h$ , and initial conditions (I think they mean boundary conditions) of  $Q(x=b) = Q_{pf}$ , and  $\Theta(x=b) = \Theta_{ew}$ . However, the temperature is not specified at  $x=b$ . I believe that a more reasonable boundary condition would have been to consider an adiabatic tip,  $Q(x=b) = 0$ . For example, when the authors' parameter  $C=0$ , the top plate is adiabatic, while the authors also never considered the heat transfer from the tip, their Eq. (10). This new boundary condition would have simplified the equations considerably, especially the expression for the heat dissipation (see Ref. 5). Now, the boundary conditions that accompany (5) are  $Q(u) = Q_{pf}$  and  $(d\theta/dx)_{x=0} = 0$ . We may now observe that, because the values

of  $\lambda$ , are nearly one, which result in very large values of the parameter  $p = u/(1 - \lambda)$ , it would have been justified to use for their calculations the following much simpler expression for cylindrical spines

$$\theta = B \frac{\cosh(\xi)}{\cosh(u)}, \quad \xi = u \times (x/H) \quad (6)$$

In the foregoing equation the symbol  $B$  is equal to  $B = (k^2 Q_{pf} / \pi h \text{Bi}^{3/2} \theta_{en})$ .

Finally, it is necessary for those who are involved in any fin analysis or experiment endeavor, at the end they should always compare the heat transferred by the fin, to the heat that would have been transferred from the fin's base area in the absence of the fin. This ratio was introduced by Gardner [6], as fin effectiveness. The later could be calculated using the average heat transfer coefficient determined experimentally, and the results will be on the conservative side, because in the bare surface, where the heat transfer coefficient can be easily determined from known formulas, would be larger. It has been also shown in Refs. 2 and 3 that, for straight fins and spines the effectiveness is equal to  $1/\sqrt{\text{Bi}}$ , which poses restrictions on the values of Bi, that was neglected by the authors.

I may suggest further, that the authors extend their experimental work to the following two other important problems. The first one is the case where fins are used in heat exchangers, and the second the in arrays of fins. Notice that the boundary conditions in the first are constant temperature and insulated tip, and in the second the temperatures are specified. With regard of the second problem that authors should consult [11].

I hope these comments will help the authors to improve the presentation and to design their new experiments.

## References

- [1] Han, J.-C., Dutta, S., and Ekkand, S. V., 2000, *Gas Turbine and Cooling Technology*, Taylor and Francis, New York.
- [2] Razelos, P., 2003, "A Critical Review of Extended Surface Heat Transfer," *Heat Transfer Eng.*, **24**(6), pp. 11–28.
- [3] Razelos, P., and Georgiou, E., 1992, "Two-Dimensional Effects and Design Criteria for Convective Extended Surfaces," *Heat Transfer Eng.*, **38**(3), pp. 38–48.
- [4] Das, S., and Razelos, P., 1997, "Optimization of Convective Trapezoidal Profile Circular Pin Fins," *Int. Commun. Heat Mass Transfer*, **24**(4), pp. 533–541.
- [5] Razelos, P., 1983, "The Optimum Dimensions of Convective Pin Fins," *ASME J. Heat Transfer*, **105**, pp. 411–413.
- [6] Gardner, K. A., 1945, "Efficiency of Extended Heat Transfer," *Trans. ASME*, **67**, pp. 621–631.
- [7] Schneider, P. J., 1955, *Conduction Heat Transfer*, Addison-Wesley, New York, p. 69.
- [8] Kraus, A. D., Aziz, A., and Welty, J., 2000, *Extended Surface Heat Transfer*, Wiley, New York.
- [9] Irey, R. K., 1968, "Errors of the One-Dimensional Fin Solution," *ASME J. Heat Transfer*, **90**, pp. 175–176.
- [10] Law, W., and Tan, C. W., 1973, "Errors in One-Dimensional Heat Transfer Analysis in Straight and Annular Fins," *ASME J. Heat Transfer*, **95**, pp. 549–551.
- [11] Razelos, P., 1980, "An Efficient Algorithm for Evaluating Arrays of Extended Surfaces," *ASME J. Heat Transfer*, **102**, pp. 185–186.