NOTE: The following discussions are being reprinted from the September, 1969, issue of the JOURNAL OF BASIC ENGINEERING. The Author's Closure was not included with the paper due to unavoidable circumstances.

The Unsteady Flow and Wake Near an Oscillating Cylinder[1]

V. W. GOLDSCHMIDT.[2] The work presented adds great insight into the details of the described flow. It results from the author's careful and thorough experimental work. Two comments are in order. First, the change in the nature of the signal (as noted in Figs. 6–8) does not necessarily correspond to that due to separation. It could also be due to transition. Second, the noted asymmetry could be explained by the fact that what is being observed is a combination of a cooling effect due to u and v simultaneously. Another explanation is due to Kibens and Kovasznay[3] who suggested that the one-sidedness of the signal (when in an intermittent region) is due to the turbulent portions having a lower average velocity than the conventional mean flow velocity at the same location. They report measurements comparing these different averages for the particular case of a turbulent boundary layer.

Reference should be made to similar hot-wire anemometer measurements taken by Maekawa and Mizuno[4] in the mean wake of a (nonoscillating) circular cylinder. In addition, the author's earlier work recently published[5] substantiates the stated notions and conclusions.

G. H. KOOPMANN[6] AND C. W. VOTAW.[7] Professor Toebes should be complimented on his extensive and well-developed experimental study. However, attention should be directed to the author's somewhat questionable use of the word vorticity in interpreting a fluctuating velocity field as seen through his hot-wire probes. It is indeed a precarious jump to assert that one-dimensional velocity fluctuations extracted from an early wake flow field give a measure of vortex strength, but this seems to be what the author has done. One must presume that the correspondence between velocity variance and vortex strength is obtained by assuming a known model such as that developed by von Karman. This model is least applicable in the early wake where the author has made his measurements of the velocity variance.

In the Introduction, he references Koopmann's investigations to confirm the notion that cylinder oscillations directly influence the strength of the associated vortex wakes. Nowhere in Koopmann's paper is this conclusively born out and it would be interesting to ascertain from what basis the author draws this conclusion.

In the section titled "Unsteady Flow Structure," part (*b*), the author asserts that an increase in the velocity, u(rms), corresponds to an increase in vortex strength. This increase in the so-called vorticity is cited as being in accordance with Ferguson and Parkinson. In the Ferguson and Parkinson work it is assumed that the von Karman vortex model was again used to relate velocity fluctuations with discrete vortex strength. It would seem more reasonable that if the author is going to interpret his measurements within the framework of such a model flow pattern, a better place to do so would be at a point in the wake away from the cylinder where the wake has had time to form into discreet vortices.

Since the relationship between the velocity fluctuations in the early wake and the discrete vortex strength has not been definitely established or explained by the author, it is felt that conclusions 10 and 13 are not fully justified by the data given.

G. V. PARKINSON.[8] Although the importance of understanding the mechanisms of vortex-induced oscillation of bluff cylindrical bodies has been realized for many years, it is only in the past 5 years that an appreciable body of data on the actual fluid phenomena in the presence of cylinder oscillation has begun to accumulate, and the present paper adds to Professor Toebes' significant contributions to this data.

In the following comments, some of the findings of the present paper are compared with recent measurements in our laboratory additional to those of reference [12], commented on by Professor Toebes.

1 In the discussion of Fig. 3, it is suggested that a relative narrowing of the early wake occurs for the oscillating cylinder. This is in agreement with our measurements[9] at lower but comparable Reynolds numbers.

2 In the discussion of Fig. 4, it is mentioned that no evidence of slantwise vortex shedding was found. Recent measurements in our laboratory,[10] however, give strong evidence of such slantwise shedding from a circular cylinder, both at rest and in low-amplitude vortex-induced oscillation, at Reynolds numbers from 17,000–27,000.

3 Hot-wire correlation measurements in our laboratory [2] behind a circular cylinder in vortex-induced oscillation support the measurements of Fig. 5 both qualitatively and quantitatively.

4 Evidence from our laboratory supports those conclusions of the present paper which relate to phenomena we have investigated, specifically (1), (2), (6), (7), (9), (10), (13).

P. D. RICHARDSON.[11] Professor Toebes is to be congratulated for his new measurements that are particularly useful in confirming and advancing our understanding of the flow in the near wake of a circular cylinder in crossflow. There are six features of his measurements on which I would like to offer comments.

1 **Free Shear Layer: Coalescence of Vortices.** Professor Toebes describes his observations that, during convection downstream, the individual eddies that grow in the free shear layer after its separation from the cylinder seem to coalesce. This has been found also in some experiments performed at Brown University [28],[12] in which we also made some wave-analyzer measurements that showed that the frequency of the coalesced eddies was one half of the original frequency. This is direct evidence that the coalescence occurs between neighboring pairs of vortices in the free shear layer, as illustrated beautifully for a free shear layer developed in a different geometry by Becker and Massaro [29].

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[1] By G. H. Toebes, published in the Sept., 1969, issue of the JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 91, No. 3, p. 493.

[2] Associate Professor of Mechanical Engineering, Purdue University, Lafayette, Ind.

[3] Kibens, V., and Kovasznay, L. S. G., "Detailed Measurements in the Intermittent Zone of a Turbulent Boundary Layer," presented at the Twentieth Anniversary Meeting of the Division of Fluid Dynamics, American Physical Society at Lehigh University, November 20-22, 1967.

[4] Maekawa, T., and Mizuno, S., "Flow Around the Separation Point and in the Near-Wake of a Circular Cylinder," *The Physics of Fluids*, Vol. 10, No. 9, Supplement (*Proceedings of an International Symposium on Boundary Layers and Turbulence With Geophysical Applications*), Sept. 1967, pp. S184–S186.

[5] Protos, A., Goldschmidt, V. W., and Toebes, G. H., "Hydroelastic Forces on Bluff Cylinders," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 90, Sept. 1968, pp. 378–386.

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[8] Professor, Mechanical Engineering Department, University of British Columbia, Vancouver, B. C., Canada. Mem. ASME.

[9] Ferguson, N., "The Measurement of Wake and Surface Effects in the Subcritical Flow Past a Circular Cylinder at Rest and in Vortex-Excited Oscillation," MASc thesis, University of British Columbia, Sept. 1965.

[10] Feng, C. C., "The Measurement of Vortex-Induced Effects in Flow Past Stationary and Oscillating Circular and D-Section Cylinders," MASc thesis, University of British Columbia, Oct. 1968.

[11] Professor of Engineering, Brown University, Providence, R. I. Mem. ASME.

2 Free Shear Layer: Bloor Frequency. While Professor Toebes has quoted Mrs. Bloor's observations on velocity spikes, he has not mentioned comparison of his observations of the frequency that occurs first in the separated shear layer with Mrs. Bloor's finding that the "Bloor" frequency f_B is related to f_f', the Strouhal or shedding frequency, and Re approximately by

$$f_B/f_f' \simeq 0.12\ \mathrm{Re}^{1/2}$$

Our own measurements show that this is a good approximation, and it would be reassuring in assessing Professor Toebes' results to know that he found the same. Our wave-analyzer results showed that the Bloor frequency is not so sharp as the shedding frequency [28]; the Bloor frequency exhibits some bandwidth, so that an estimate gained by, say, counting peaks in line 2 of Fig. 9 would give only an approximation to the Bloor frequency.

3 Free Shear Layer: Sensitivity to Free-Stream Disturbances. It has been established by Gerrard [30] that the free shear layer is extremely sensitive to free-stream disturbances when these are at a frequency sufficiently close to the Bloor frequency. The expected Bloor frequency at a Reynolds number of 64,000—typical of Professor Toebes' results—is of the order of 200 c/s. In the energy spectrum of the free-stream disturbances of a wind tunnel, it is not unusual to see some contribution in the neighborhood of this frequency. It would be helpful in relating some features of Professor Toebes' results to other work if we could indicate the rms free-stream turbulence intensity of the tunnel he used, and the fluctuation energy spectrum, for a typical Reynolds number.

4 Effects on Separation. When the cylinder is oscillated mechanically, an unsteadiness is created in the flow upstream of separation. This unsteadiness is additional to that already present due to the formation of the wake vortices. As Professor Toebes notes, the formation of the wake vortices gives rise to velocity fluctuations at the shedding frequency ahead of the separation region, as we have noted with measurements of oscillating heat transfer and with hot wires [28]. The oscillation of a cylinder normal to its axis creates an unsteady boundary layer in addition to that due to the free stream. With oscillations at or near the shedding frequency, the oscillation boundary-layer thickness is about the same as the steady boundary-layer thickness. The streaming Reynolds number associated with oscillations of $\epsilon = 1$ in. and $f = 10$ c/s exceeds 10^4, so that one woud not expect strong outer streaming motion in the absence of the free-steam flow [31]; however, from the point of view of the time-average flow in the boundary layer at the front of the cylinder, the Reynolds stresses due to the forced oscillations exist within a significant thickness of the steady boundary layer and may well serve to alter the mean boundary-layer flow in the forced convection, just as they do in natural convection [32]. This may alter the mean rate of discharge of vorticity in the free shear layer.

In addition, the forced oscillations induce changes in the separation position of the boundary layer from the cylinder, as described in considerable detail by Professor Toebes. This produces some additional effects by modulating the rate of discharge of vorticity over and above the modulation induced by the wake vortex formation process itself. The results presented do not allow us to know to what extent the rate of discharge of vorticity is modulated; indeed, this would be a difficult measurement to make. However, it is useful to bear in mind the possibility because it has relevance to the wake formation process.

5 Wake Formation Process. The computations of Abernathy and Kronauer [33] suggested that the vortices of the Karman street are formed simply by inviscid interaction of two vortex sheets of mutually opposing sense. These calculations also illustrated how considerable vorticity crosses from one side of the wake to the other, where it is cancelled, so that the vorticity convected downstream in the developed Karman vortices is distinctly less than that discharged from the cylinder in the free shear layers. The computations of Gerrard [34], more directly pertinent to the wake flow behind a cylinder, even showed how calculations of this type could give a realistic value for the shedding frequency. Gerrard postulated that the shedding frequency is determined principally by the rate of discharge of vorticity in the free shear layers. This viewpoint is supported by our own measurements, Peterka and Richardson [28], in which we found that the shedding frequency at constant Reynolds number was hardly affected by considerable shortening of the wake formation region, this shortening being caused by forced enhancement of the free shear layer instability. The point to be made here is that, if Gerrard's postulate is correct, the range over which locking of the shedding frequency to the oscillation frequency (or with its harmonics) can occur is very limited. Modification may arise if the mean rate of discharge of vorticity is affected by the oscillation Reynolds stresses (as just noted) or if this discharge is modulated.

6 Properties of the Developed Karman Vortices. Professor Toebes draws some conclusions that involve properties of the developed Karman vortices which seem to be based on information not included in the body of the paper, and I should be very grateful if he could indicate some more details concerning their justification. For example, in Conclusions 4, 10, and 15, Professor Toebes refers specifically to the strength of the Karman vortices. The strength of a Karman vortex can be expressed in terms of an equivalent line vortex. It is certain that this quantity has a mean and standard deviation, but it does not seem so certain that the strengths of successive vortices are uncorrelated, or random, as suggested in Conclusion 4. Indeed, this somewhat contradicts the idea of the feedback process! In connection with Conclusion 11, perhaps Professor Toebes means by the term "vortex strength" the magnitude of the oscillations in the flow felt at the cylinder surface at the shedding frequency. This magnitude depends as much on the vortex formation length behind the cylinder as it does upon vortex strength. It is not clear from the results presented that Professor Toebes measured vortex strength per se, as was done by Bloor and Gerrard [35].

Professor Toebes mentioned that cylinder oscillation decreases the variance of the unsteady flow. This is comparable to our similar result with oscillations at the Bloor frequency [28].

References

28 Peterka, J. A., and Richardson, P. D., "Effects of Sound on Separated Flows," *Journal of Fluid Mechanics* (in press).

29 Becker, H. A., and Massaro, T. A., "Vortex Evolution in a Round Jet," *Journal of Fluid Mechanics*, Vol. 31, 1968, p. 435.

30 Gerrard, J. H., *Journal of Fluid Mechanics*, Vol. 22, 1965, p. 187.

31 Stuart, J. T., *Journal of Fluid Mechanics*, Vol. 24, 1966, p. 673.

32 DeVahl Davis, G., and Richardson, P. D., "Natural Convection From a Horizontal Cylinder in the Presence of a Sound Field Giving Large Streaming Reynolds Numbers," Brown University, Division of Engineering Report AF 1754/3, 1967.

33 Abernathy, F. H., and Kronauer, R. E., "The Formation of Vortex Streets," *Journal of Fluid Mechanics*, Vol. 13, 1962, p. 1.

34 Gerrard, J. H., *Philosophical Transactions of the Royal Society*, London, England, Series A, Vol. 261, 1967, p. 137.

35 Bloor, M. S., and Gerrard, J. H., *Proceedings of the Royal Society*, London, England, Series A, Vol. 294, 1966, p. 319.

L. V. SCHMIDT.[13] Professor Toebes is to be commended for the care exercised in measuring the motion dependence of flow about a circular cylinder. Although the measurements may be classified as indirect in nature in contrast to a gross property measurement such as unsteady sectional lift or drag, the results do indicate a remarkably strong trend indicating that motion improves the spatial correlation of the velocity perturbations in the potential flow field near the cylinder provided that the lateral motion is in accord with the dominant flow or aerodynamic Strouhal number. It is interesting to note that Jones[14] recently reported measurements in a higher Reynolds number range and similarly showed that the unsteady lateral load peaked when the forcing frequency was 99 percent of the aerodynamic frequency,

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[12] Numbers in brackets designate References at end of Discussion.

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and that this peaking was sharply dependent upon frequency. The results presented by Professor Toebes are in good accord from this standpoint.

Since the author stated that the data were recorded on magnetic tape, it should be pointed out that analog processing of the hot-wire data for zero-time lag cross correlation is readily performed by an alternate technique using an analog multiplier in conjunction with a low-pass filter and then recording the output as time averaged traces on an X-Y plotter. In addition to zero-time-lag cross correlations one can obtain, if digital conversion facilities were available, the family of R_{nm} (τ) versus τ curves for various $\Delta Z/D$ spacings. This presentation would demonstrate the existence of spanwise separation waves and the corresponding wave speeds. An interesting result would be the dependence of the spanwise wave speed upon motion, which appears to be within the capabilities of the information recorded during these tests.

The abstract stated that one of the goals of the test program was to create controlled wake characteristics by the use of vibrating cylinders. Because of the indeterminancy or random nature of the separation point on the circular cylinder, the question arises as to whether this fourth goal might not have been obtained by removing this indeterminancy, say, by placing trip plates on the cylinder. Some investigators have observed that small platelike projections on a circular cylinder produced an aeroelastic behavior similar to that associated with a rectangular section.

In conclusion, the mark of good work is that not only are answers obtained to interesting questions, but also that the work raises questions for future efforts, which ultimately provides man with a deeper insight into nature's mysteries.

Author's Closure

It is certainly gratifying to note that a fair number of discussers found the reported data sufficiently significant to share in their interpretation and their correlation with similar or related work. Therefore, it is doubly regrettable that the paper could present samples of only about a third of the extensive data that were collected. Simultaneous hot-wire measurements were made from locations at the leading edge up to 30-cylinder diameters downstream. Both the wake and the bounding potential flow were surveyed. Arrays in the longitudinal, the transverse, and the cylinder axis directions were used. Also, several mean velocity and dynamic pressure traverses were performed in the early wake region.

The data obtained by Dr. Parkinson and co-workers using fixed and oscillating cylinders are, in the author's opinion, among the finest to be found in the literature. Consequently, those comments by Dr. Parkinson that would confirm findings reported herein may well be assigned double weight. It is noted that the discusser could support most conclusions of the paper [(1, 2, 6, 7, 9, 10, 13)] that dealt with subjects also investigated by him. (Possible reservations in regard to conclusions 15 through 18 are discussed below.) The absence of evidence of slant-wise vortex formation in the present work is attributed to its higher Reynolds number. It is suggested, in keeping with conclusion 19, that, at Reynolds number above say 30,000, large scale turbulent flow disturbances are interfering with or imposed on the "slanting mechanism." It may be that detailed statistical analysis could uncover some tendency toward slanting that is not obvious from an inspection of data such as shown in Fig. 4.

Dr. Richardson's lucid discussion increases the value of reported results considerably. It is very interesting to note that there is confirmation of what was called vortex coalescence. As stated, Fig. 9 is but one of a collection of records that have not yet been carefully inspected to date. Some of these data seem to suggest that disturbances can propagate upstream. Cross-correlation functions should be determined if a more definite statement is to be made about this puzzling impression.

In regard to the so-called Bloor frequency the author felt constrained to introduce the concept. For one thing, the Reynolds numbers involved were rather different. Secondly, it was unclear what location to select. By counting peaks for wires 2 and 3 in records such as shown in Fig. 9 it is found that $f_2 \simeq 1.2$ to $1.8 f_B$ and $f_3 \simeq 0.6$ to $0.9\, f_B$. Now by rearranging wires as shown in Fig. 10, similar results are obtained by the wires 2 and 3. (Fig. 10 purposely presents a different Reynolds number.) The present data thus do bracket f_B nicely and suggest that f_B is found in a layer-like region. It is not believed that these results are affected by tunnel turbulence. The ambient turbulence level of the tunnel used is about 0.1 percent. Yet more convincing are the signals obtained from wire 1 as compared to those of wire 2 in both Figs. 9 and 10. Unfortunately, it is not practical to determine a variance distribution function from these graphical records. The author has no doubt that such a spectrum would be of the medium bandwidth noise type.

In his fourth and fifth comment Dr. Richardson addresses himself to the question of interlock between the frequencies of vortex shedding and cylinder oscillation. He quotes Gerrard for the notion that such lock-in can exist only over a very limited f_r'-range and opines that the results shown in Table 1 could well result from modification of the mean rate of discharge of vorticity and/or a modulation thereof. The author is in substantial agreement with these concepts. He feels, however, that on occasion the effect of vorticity discharge on the wake structure has been advanced to the near exclusion of the shape of the wake boundary as it is effected by cylinder oscillation and the momentum exchange due to the interaction of wake and outer flows. Actual measurement of unsteady vorticity discharge would certainly be important.

The question how one deduces statements regarding vortex strength from the type of data presented was raised by Richardson, Koopman, and Votaw. Indeed, no direct measurements of vorticity were made, and the term vorticity was used freely out of previous familiarity with both the flow field and the associated lift force records. Such familiarity does, of course, include the usual von Karman model notions as the discussers concluded. The author agrees that the present paper, taken separately, does not establish the direct correspondence between vortex strength and the variation in maximum amplitudes of the oscillatory velocities. The discussers are correct in pointing this out. However, taken together with the literature on force measurements, surface pressure measurements and flow visualization results, he sees nothing to disprove the wording used. It would be quite difficult to endow the term vortex strength with a much greater precision than its use in such manifest simplifications as the von Karman model which, in effect, never said anything about the early wake.

The conclusion 4 meant to convey that both the frequency and the amplitude of velocity fluctuations are a stochastic process. The generic term "random" does not mean that these parameters are uncorrelated in the fashion of white noise. On the contrary, they are of the limited band-noise type. The wording was selected to emphasize that the bandwidth is not as limited as is often thought. Most certainly successive parameters are correlated by feedback.

Dr. Goldschmidt's basic questions concerning the interpretation of the hot-wire signals are well taken. In the present context there appears sufficient circumstantial evidence to prefer "separation" over boundary layer "transition." It is well documented that transition to turbulence occurs for the boundary layer of circular cylinders in a cross flow at Reynolds numbers around 0.5×10^6. In the present work Reynolds numbers were around 0.5×10^5. Regarding u and v measurements: several measurements were made with cross wires. The rate of de-

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[14] Jones, G. W., Jr., Cincotta, J. J., and Walker, R. W., "Aerodynamic Forces on a Stationary and Oscillating Circular Cylinder at High Reynolds Numbers," NASA TR R-300, Feb. 1969.

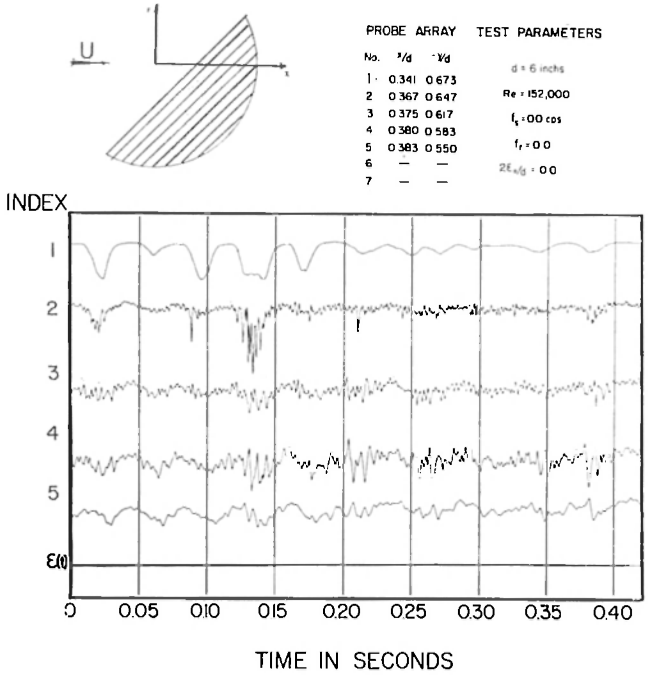

Fig. 10 A sample of 5 hot-wire signals for a stationary cylinder at Re = 152,000. All upward changes in u(t) correspond to flow accelerations.

crease in total velocity (assuming negligible velocity in the direction of the cylinder axis) was, on the average, greater than the rates of increase. It would have been better to state this incidental piece of information. Goldschmidt's reference 4 is identical to the author's reference [17].

Dr. Schmidt's comments on data reduction techniques point to his own fine contributions on the subject. The author agrees with all his recommendations. In fact, all of these were tried. The analog multiplier available proved to be unserviceable. All outputs on which the Figs. 5(*a*) and 5(*b*) are based, were indeed averaged from *X-Y* recorder traces of the correlator output. Digital conversion of the most important records and subsequent time series analysis were planned for extensively. The press for time prevented perusal. The author hopes to implement Dr. Schmidt's suggestion to determine several space-time correlation functions along the cylinder. The suggestion of a trip wire does not appear to be applicable. At 30-cylinder diameters downstream the gross distortion of the wake flow flattened the Strouhal peak in some samples almost beyond detection. This loss of correlation was not materially affected by cylinder vibration. A trip wire would appear to have little influence on features at the scale of the Strouhal oscillation. The indeterminacy of the separation line are held to be induced more by the downstream wake structure than random irregularities in the boundary layer upstream.

In conclusion, the author expresses once again his sincere appreciation for the contributions the discussers were willing to make.

ERRATA

The fifth line below Table 1 should read "at a ratio of $f_r = f_s/f_f$" rather than "$f_f = f_r/f_s$."