selection of the material to be used in a member subject to resonant vibrations, two other factors must be considered. The first of these concerns the length of the life of the member. The desired end is that the member last as long as possible, i.e., as many hours or minutes as possible. Even if the fatigue strengths, such as are listed for the various materials in Table 1, were based on the same number of cycles to failure, the life of members made of each of the materials would be different because the frequencies at which they vibrated would be different. The various members would have different stiffnesses and masses due to differences in their modulii and densities, and to differences in the amounts of material used in their construction, as required by other features of the design (static strength, etc.). Therefore they would have different natural frequencies and different lengths of life.

Another factor which must be considered in a comparison of materials for use in resonant members involves the rate at which the material absorbs energy due to internal friction. This energy appears as heat and unless the medium surrounding the member carries the heat away as fast as it appears, the member will heat up. As the member heats up the heat-transfer rate will increase until equilibrium is reached or until the member fails. Metals are not greatly affected by moderate increases in temperature, but plastics are much more sensitive. It is known that an increase of temperature of only 10 or 20 deg F above normal room temperature will have a detrimental effect upon the structural properties of a plastic.[7] Therefore any study of the relative resonant fatigue strengths of materials should include a review of the possibility of overheating.

Alex Yorgiadis.[8] The tendency has been to use the fatigue strength as the deciding factor in selecting materials for vibrating members, while damping capacity and dynamic modulus of elasticity were neglected. For resonant conditions, these last two properties are very significant, as ably explained by the authors.

The method of the authors, however, is not new. Using the same analysis as that of the authors, J. M. Robertson and the writer had derived an expression[9] for the resonant strength of a tension-compression member. This corresponds to the authors' Equation [14], which is for cantilever beams. These relationships can be rearranged as follows

$$\sigma_0 = \alpha C_1 E \sigma_m{}^2 = \frac{\alpha(\Delta W)_m}{\epsilon_m} \quad [1]$$

where

- σ_0 = maximum stress that would exist in member if exciting force P_0 were applied statically
- α = numerical constant depending upon method of loading
- α = 0.318 for tension-compression member
- α = 0.178 for cantilever beam (authors' example 1)
- C_1 = damping constant for material, as defined by the authors
- E = dynamic modulus of elasticity
- σ_m = fatigue strength of material
- $(\Delta W)_m$ = damping of material at stress σ_m (energy absorbed per cubic inch per cycle)
- ϵ_m = unit strain at stress σ_m

While the derivations and relationships of the authors are theoretically correct, their conclusions such as reached in the last columns of Table 1 of the paper, cannot be accepted in representing actual resonant strength values of the various materials. The principal discrepancy lies in the assumption that the damping constant and the dynamic modulus are stable values of the material which remain unchanged throughout its useful life. This is far from being the case, these properties changing appreciably due to repeated stresses high enough to cause fatigue failures. The values used by the authors were obtained on specimens with no previous stress history.

Owing to these changing properties, the exact condition of resonance (90 deg phase difference between sinusoidal exciting force and sinusoidal displacement) is very unstable and in practice cannot be maintained for more than a few hundreds of stress cycles at a time in highly stressed members. The authors' analysis does not take this into consideration and therefore is likely to yield inaccurate results. It is, however, a step in the proper direction.

Authors' Closure

The authors appreciate the interest shown by the discussers of this paper. The authors agree with most of the comments made, but consider that these comments deal with the quantitative aspects of the proposed fatigue strength-damping criterion. It was the object of the authors in presenting the paper to point out the general quantities which determine the resonant strengths of various structural members. Certain assumptions were made to simplify the analysis so that the final conclusions would not be obscured by more complicated exact relations. We agree with Mr. Plunkett that his method should lead to a more accurate result.

Camptograms for Beams in Compression[1]

H. Poritsky.[2] Rather than discuss the interesting graphical construction outlined by the authors, the writer would like to argue on what may be regarded as a constitutional right of authors, namely, the title of the paper, and in particular the spelling of the word "camptograms." This word, spelled with a "c," looks too much like a possible misspelling of "comptogram;" in fact, this is what it was assumed to be when the writer first saw the title of the paper.

Upon reading into it and finding out that the title is derived from the Greek word meaning "to bend," it was immediately recalled that there is no equivalent of the letter "c" in Greek, and the question arose why Greek kappa had been changed to a "c." Therefore the writer would like to suggest to the authors that the spelling "Kamptograms" would both make the title more distinctive and reveal its Greek origin. This suggestion agrees with common English practice, for instance in such words as "kinematics," "kinetics," etc.

M. C. Yovits.[3] The authors have given a lucid and extensive analysis of camptograms for compressed beam columns. The subject certainly merits further development.

A case of considerable practical importance which the authors have not included among their examples is the uniform compressed beam with both ends built-in and with a noncentral concentrated load, shown in Fig. 1 of this discussion. Since this

[7] "Relative Temperature Stability of Stressed Plastics," by J. A. Sauer, F. A. Schwertz, and D. L. Worf, *Modern Plastics*, vol. 22, March, 1945, pp. 153–156, 192, and 194.

[8] Design Engineer, Sonntag Scientific Corporation, Greenwich, Conn. Jun. ASME.

[9] See Reference 6, Equation [7], p. 76.

[1] By V. Rojansky and R. A. Beth, published in the September, 1947, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 69, p. A-202.

[2] Engineering General Division, General Electric Company, Schenectady, N. Y. Mem. ASME.

[3] Instructor in Physics, Union College, Schenectady, N. Y.

case involves a fundamental extension of their ideas, its solution is outlined herewith.

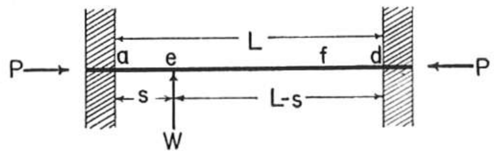

FIG. 1 BUILT-IN BEAM; NONCENTRAL CONCENTRATED LOAD

For the beam in Fig. 1 the static equilibrium conditions yield the equations

$$\frac{1}{2}(M_a - M_d) = -\frac{1}{2}\Phi N_a - \frac{1}{2}(W/\alpha)\phi_{ed} \ldots\ldots [1]$$

$$\frac{1}{2}(M_d - M_a) = \frac{1}{2}\Phi N_d - \frac{1}{2}(W/\alpha)\phi_{ae} \ldots\ldots\ldots [2]$$

These equations, however, are clearly not sufficient for the construction of the camptogram.

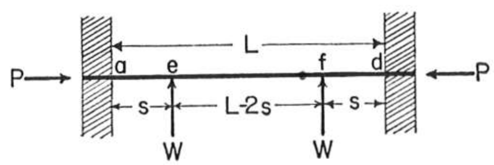

FIG. 2 BUILT-IN BEAM; TWO EQUAL SYMMETRICALLY PLACED LOADS

To obtain a third condition, we consider the built-in beam loaded as in Fig. 2. It can be shown analytically that

$$M_a = M_d = W/\alpha\left[\cos\left(\frac{1}{2}\Phi - \phi_{ae}\right) - \cos\frac{1}{2}\Phi\right]\Big/\sin\frac{1}{2}\Phi \,..[3]$$

For the beam of Fig. 1 we can show that

$$M_a = \frac{\sin\phi_{ae} + \sin\phi_{ed} - \sin\Phi + \phi_{ae} - \Phi\cos\phi_{ed} + \phi_{ed}\cos\Phi}{2(1 - \cos\Phi) - \Phi\sin\Phi} \ldots\ldots [4]$$

$$M_d = \frac{\sin\phi_{ae} + \sin\phi_{ed} - \sin\Phi + \phi_{ed} - \Phi\cos\phi_{ae} + \phi_{ae}\cos\Phi}{2(1 - \cos\Phi) - \Phi\sin\Phi} \ldots\ldots [5]$$

Hence

$$M_a + M_d = W/\alpha\left[\cos\left(\frac{1}{2}\Phi - \phi_{ae}\right) - \cos\frac{1}{2}\Phi\right]\Big/\sin\frac{1}{2}\Phi \,..[6]$$

This is precisely the expression for M_a in Fig. 2, given by Equation [3].[4]

Thus by drawing the camptogram for the beam in Fig. 2,[5] we obtain the horizontal line representing $\frac{1}{2}(M_a + M_d)$ for the beam of Fig. 1.

By combining this line with those representing Equations [1] and [2], we then obtain the lines representing M_a-versus-N_a and M_d-versus-N_d. These lines specify the boundary conditions, and the camptogram can now be drawn immediately.

Although the theory here presented may seem elaborate, the actual graphical construction of the camptogram is in fact simple.

In their concluding remarks the authors have suggested that camptograms can be used to deduce formulas for the bending moment and shear along compressed beam columns with various loadings. The writer has made such computations and, in particular, has derived from camptograms expressions for maximum moments and moments at built-in ends.

For this purpose camptograms can be drawn freehand. They offer an approach to the derivation of the formulas which is both simple and appealing.[6]

The writer has also used the general method of camptograms to solve the problem of the beam in tension. No convenient graphical solution previously has been proposed.

Whereas arcs of circles about $(0, \mu)$ as a center are solutions for the compressed beam column, arcs of hyperbolas about $(0, -\mu)$ as a center are solutions for the beam in tension. In the latter case, the angular length, αx, along the beam must be measured in hyperbolic radians rather than circular radians. The end points of these hyperbolic arcs may be found by solving certain equivalent circular problems, the gudermannian serving as the connecting link between the circular and hyperbolic angles.

A paper dealing with the graphical solutions for some of the more important cases of beams in tension is in course of preparation.

AUTHORS' CLOSURE

Dr. Poritsky's comments on the spelling of the word "camptogram" involve personal preferences, and are therefore difficult to discuss on strictly logical grounds. When the word was coined by Prof. Harrison C. Coffin, he suggested that either "k" or "c" could be used, and left the choice to the authors. One reason for adopting the "c" was the obscure claim by one of the authors that, when spelled with a "c," the word "*looked* better to his English-*speaking ear*." Another reason was the fact that in the Webster's New International Dictionary the following related words are listed: camptodrome, Camptolaemus, Camptosorus, camptotropal, and kamptulicon; and only one of them is spelled with a "k." Similarly, such words as "cacophony," "cathedral," "catholic," and so on, are usually spelled with a "c." Because of the softening effect of the "i," Dr. Poritsky's examples, namely, "kinematics" and "kinetics," involve a situation that does not arise in the word "camptogram;" but note the word "cinema."

The case of the compressed built-in beam with a noncentral concentrated load, considered by Mr. Yovits, is an instructive addition to the cases described by the authors. His extension of the camptographic method to beams in tension should be of interest also, especially if the introduction of the gudermannian will make the graphical procedure easily manageable.

For the benefit of readers who might be interested in extending the method to beams with more than two supports, it should be added that at the meeting of the ASME Applied Mechanics Division in June, 1947, Mr. David H. Ware of the General Electric Company, Schenectady, N. Y., reported progress in that direction.

A description of an application of the method to a problem in dynamics rather than statics was recently submitted by one of the authors (V. R.) to the *Journal of Applied Physics*, under the title "Gyrograms for Simple Harmonic Systems Subjected to External Forces."

[4] This result, incidentally, proves the superposition theorem for the present case.

[5] Because of symmetry, this camptogram can be drawn immediately by the authors' methods; only one half of it need in fact be drawn.

[6] The writer has found, for example, that the derivations by means of camptograms of Equations [3], [4], and [5] are particularly simple.