A model study of telluric fields in two-dimensional structures

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Summary. In recent years telluric sounding has been replaced by MT (magnetotellurics). However, several new purely telluric parameters, besides the traditional Jacobian, have been shown to be efficient geophysical indicators of lateral conductivity variations. A set of typical two-dimensional structures is analysed to demonstrate the resolving power of the new indicators. For such telluric studies, a wide frequency band is a great asset, especially because the indicators are best displayed as pseudo-sections in the frequency domain. However, a wide frequency range is easily achieved when only the telluric field needs to be measured. In MT the magnetic sensing coils often severely reduce the available bandwidth.

Key words: modelling, telluric field, 2-D structures

1 Introduction

The telluric method was introduced and patented as an autonomous method of geophysical investigation in the 1930s by Schlumberger (1936). Later it was widely applied in the field of oil prospecting as an initial method of investigation in sedimentary basins. At the same time, various methods were developed for analysing telluric signals, mainly aimed at determining a single parameter for each measurement site. In most cases, this was the Jacobian $J$, derived over suitable and relatively wide frequency bands. In 1977 Yungul published a full review of these methods, all of them analogic, and stressed their fields of application and validity (the so-called 'S' band of the apparent resistivity curves in magnetotellurics). A semi-quantitative interpretation of the $J$ results obtained in these frequency bands (an insulating substratum being hypothesized as a constraint) may be carried out with two-dimensional (2-D) models, whose response is obtained by the method of Kunetz & Chastenet de Géry (1956), and has more recently been applied to various 2-D structures by other authors (e.g., Roy 1973; Kumar 1980).
The advent of magnetotellurics (MT) (Cagniard 1953) has relegated the telluric method to a back seat, because MT is capable of providing the resistivity itself and information on its lateral and depth variations by making use simultaneously of the magnetic components of the natural electromagnetic (em) variation field.

During the last 10 years, the telluric method has regained favour, rather as an auxiliary method of MT itself (Hermance & Thayer 1975; Stodt, Hohmann & Ting 1981). This is because if the horizontal magnetic field does not vary laterally each telluric station may become a full MT sounding, with evident savings in time and equipment costs.

In the last few years we have reconsidered the telluric method in its own right as a technique of geophysical exploration (Iliceto & Santarato 1981). We have investigated the suitability of new parameters, apparently more useful in the definition of buried structures than the admittedly fairly effective J method mentioned above. We have also worked on the development of new methods which are more efficient than the traditional methods of data acquisition and analysis.

2 Theoretical considerations

2.1 SIGNAL ANALYSIS IN THE DC FIELD APPROXIMATION

It is well known that the classic analysis of telluric fields is based on linear equations:

\[ E_u = aE_x + bE_y \]
\[ E_v = cE_x + dE_y \]

where subscripts x, y refer to the reference Cartesian axes of the base station and u, v to those of the satellite station. Implicit in equations (1) is the hypothesis of a uniform primary or inducing field.

In the 'S' frequency range (Berdichevskii 1960) the telluric field may be considered as a DC field, i.e. a, b, c and d are real constants which depend only on the position of the satellite station.

Equations (1) define the transfer tensor T,

\[ T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \]

and the Jacobian J,

\[ J = \det(T). \]

For 1-D situations, equations (1) and (2) reduce to:

\[ E_u = E_x \]
\[ E_v = E_y \]

provided u∥x and v∥y.

In 2-D situations, and when the directions of measurement coincide with the principal structural axis, equations (1) and (2) become:

\[ E_u = aE_x \] i.e. \[ T = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \]
\[ E_v = dE_y \]

and

\[ J = ad = (E_u/E_x) \cdot (E_v/E_y) \]

where in general \( a \neq d \).
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When the measurement axes do not coincide with the principal directions, $b$ and $c$ are different from zero, but $b = c$, that is $T$ is symmetrical, for every $\gamma$ between the measurement axes and the principal directions.

The meaning of equations (1) is that a circularly polarized electric field at the base station is transformed into an elliptically polarized field at the satellite station. Berdichevskii (1960) showed that it is possible to determine the angle $\gamma_1$ between the major axis of the polarization ellipse described by the tip of the electric vector and the $u$-axis:

$$\tan \gamma_1 = \frac{ac + bd}{a^2 + b^2 - g_1} \tag{5}$$

where

$$g_1 = \frac{L^2}{2} - \sqrt{\frac{L^4}{4} - L^2} \quad (L^2 = a^2 + b^2 + c^2 + d^2).$$

For a symmetric tensor $T$ (2-D situations), the principal axes of the tensor (the strike direction and the direction orthogonal to it) coincide with the axes of the polarization ellipse.

2.2 Analysis of the Telluric Field as a Function of Frequency

Let us now consider the behaviour of the telluric field as a function of frequency beyond the frequency band of the ‘S’ range. First of all, with the same hypothesis of a uniform primary magnetic field, the form of equations (1)–(4) still holds at all frequencies in the validity range of the classic MT approximation of Maxwell’s equations. However, coefficients $a$, $b$, $c$ and $d$ now depend not only on the relative position of the satellite and base stations, but also on frequency, and they therefore become complex functions. These functions may be derived from the auto- and cross-powers of $E_x$, $E_y$, $E_u$ and $E_v$, obtained via FFT from the digitized time series, e.g. by means of the formulae given by Sims, Bostick & Smith (1971) for the MT impedance tensor elements. We obtain a series of frequency-dependent parameters such as the ratio and phase between parallel components of the telluric field at the satellite and base stations, the Jacobian itself and the direction of geological strike.

At every frequency the strike can be determined in the following way: assuming a nearly 2-D situation, an angle $\gamma$ exists such that $|b(\gamma)| = |c(\gamma)| = 0$. We therefore transform the $T$ tensor by way of a mathematical rotation:

$$T' = \Gamma TT\Gamma^t$$

where $\Gamma$ is the rotation matrix and $\gamma$ the rotation angle (positive clockwise) of the axes:

$$\Gamma = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix}$$

and try to satisfy a minimum condition:

$$|b(\gamma)|^2 + |c(\gamma)|^2 = \text{min}. \tag{7}$$

or some equivalent condition. Condition (7) does not resolve the $\pm 90^\circ$ ambiguity in identifying the strike, as also occurs with the equivalent maximum condition applied to the antidiagonal elements of tensor $Z$ (Swift 1967) in four-component MT. In the frequency limit of the ‘S’ band ($a$, $b$, $c$ and $d$ real) and in purely 2-D situations ($b = c$ for every $\gamma$),
condition (7) gives an equation identical to equation (5) (i.e. \( \gamma_1 = \gamma \)), provided a suitable choice of sign is done.

However, the telluric method allows resolution of the \( \pm 90^\circ \) ambiguity, by examining the behaviour of terms \( a \) and \( d \) of tensor \( T \) in the new directions on varying the frequency (Iliceto & Santarato 1981). In fact, if we take into account the characteristic different behaviour of the field components in \( E \)- and \( H \)-polarizations (to be examined in detail later), the strike axis is identified as the one whose respective diagonal element of tensor \( T \) more closely approaches 1 in wide frequency bands (e.g. if this element is \( a \), the strike axis is \( x \)).

3 Analysis of telluric parameters versus frequency

It is important now to investigate whether the telluric fields are capable of giving meaningful information on buried structures in frequency bands other than the ‘S’ band. At the same time, it is important to verify if useful information can be obtained in structural situations other than the conductive overburden-resistive substratum typical of sedimentary basins. This might indeed demonstrate the usefulness of the telluric method as an efficient and cheap one for geophysical prospecting. With this in mind, we shall look at a number of characteristic structures of general interest.

In recent years, several numerical methods have become available to compute the MT response of 2-D structures and we will restrict ourselves here to such structures. In general, these computer programs derive not only the apparent resistivities and MT phases, but also the behaviour versus frequency of the telluric fields and associated parameters. If the computed telluric responses correlate well with the original structure – specifically, if strongly structured pseudo-sections derived in the frequency are seen to be closely linked with the original structure – it will be possible to envisage using the telluric method in the opposite direction, i.e. as a prospective tool for deriving fairly constrained structural information from experimental pseudo-sections, provided the vertical resistivity distribution of one station is known.

When computing the MT and telluric responses of a 2-D model, the calculated parameters always refer to the principal directions, i.e. the strike (\( H \)-polarization in the telluric case) and the orthogonal direction (\( E \)-polarization). This does not compromise the interpretation of experimental data obtained in arbitrary measuring directions, since it is always possible, in true 2-D situations, to restore the experimental parameters to the principal directions.

Let us assume that the \( x \)-axis coincides with the strike. Among all the parameters which relate to the telluric fields, we have chosen the following:

1. modulus \( R_{1\omega} \) of the ratio \( E_\omega/\bar{E}_\omega \) (\( H \)-polarization);
2. phase \( \Phi_{1\omega} \) of ratio 1;
3. modulus \( R_{2\omega} \) of the ratio \( E_\omega/\bar{E}_\omega \) (\( E \)-polarization);
4. phase \( \Phi_{2\omega} \) of ratio 3;
5. Jacobian \( J(\omega) \), defined according to the last of equations (4), as:

\[
J(\omega) = R_{1\omega} R_{2\omega}^{-1} \]

6. horizontal gradient \( G_{1\omega} \) of \( R_{1\omega} \).

The importance of parameter (6) will become clear later, when we discuss the normalization of the telluric parameters to the base position.

It seems opportune to mention here a ‘skewness’ parameter \( \Sigma \), defined as:

\[
\Sigma = \frac{|b + c|}{|a + d|} \]
in analogy with the same MT parameter: although it is nil in the main directions of a true
2-D situation, this parameter allows immediate ascertainment of the effective existence of
a 2-D or 3-D situation while experimental data are processed.

4 Computer program

Parameters (1)--(6) were calculated on a frequency range of six decades, between $10^{-4}$ and
$10^2$ Hz. Such a wide frequency range provided a homogeneous view of all structures
examined, characterized by variable sizes, resistivities ($1-10^3$ Ohm) and resistivity ratios
($1:100$ and $100:1$). For useful comparison with more familiar data, we also computed the
MT resistivities and phases for both field polarizations. The results are plotted as pseudo-
sections, with horizontal distances as abscissa and the logarithm of frequency (in Hz) as
ordinate.

The computer program used is the last published update of that of Jones & Pascoe (i.e.
Jones & Thomson 1974).

This program computes the components of the natural em field at the ground surface by
means of the finite differences method. The fields must satisfy the following differential
equations:

\begin{align}
(E-pol) \quad &\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = i\mu_0 \omega E_x \\
(H-pol) \quad &\frac{\partial}{\partial y} \left( \frac{1}{\sigma} \frac{\partial H_x}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\sigma} \frac{\partial H_x}{\partial z} \right) = i\omega \mu_0 H_x
\end{align}

where $x$ is the reference axis parallel to the strike (SI units). $E_y$ is obtained from:

\begin{equation}
E_y = \frac{1}{\sigma} \frac{\partial H_x}{\partial z}.
\end{equation}

5 Computation of the response of a set of 2-D models

Among the infinity of possible 2-D models, a limited set of characteristic ones was selected
and analysed. These standard models are sketched in Fig. 1 and comprise the following
structures: (1) the ‘horst’, (2) the ‘graben’, (3), (4) and (5) dipping planes with various dip
angles.

For each structure we studied the response when the resistivity contrast between the
cover and the substratum was inverted. We also checked the effects of resistivity variations
when their ratio was left unaltered. As was to be expected, such variations do not affect the
shape but only the vertical position of features in the pseudo-sections, according to the well-
known principle of similarity:

\begin{equation}
\mu_0 \omega L^2 = \mu' \omega' \sigma' L'^2
\end{equation}

with obvious meanings of the symbols.

In both modelling and processing telluric data, the parameters must be normalized to
those of the base position. As we shall see, the model results are fairly linear and meaningful
when the normalization base is put very far from the 2-D structure (theoretically at infinity,
in practice at one edge of the grid). In this situation the telluric fields at the base are equiva-
 lent to those of a 1-D structure. However, this situation is not common in practice, as it is
almost never possible to achieve in field measurements. It is of course possible to change the
base, i.e. to refer the telluric parameters to a 'new' and more suitable base. In main
directions this change can easily be operated: for example, on ratio $R_1 = E'_n/E'_o$ (and
analogously on $R_2$) by multiplying it by the ratio $E'_n/E'_o$ of 'new' and 'old' base stations. We
therefore traced the pseudo-sections of the same parameters, putting the base both at
infinity and in the neighbourhood of the buried structure.

6 Analysis of results
In order to evaluate the usefulness of the parameters defined in Section 3, we examine the
pseudo-sections obtained from the 'horst' model, with the base put at infinity to the left.
We present as examples of the influence of the base position the pseudo-sections of $R_1$ and
$R_2$, with the base between the centre and the right edge of the structure. We also present the
pseudo-sections of the MT apparent resistivities and phases, for the purpose of comparison.
Finally, we show the most significant pseudo-sections of the other models, emphasizing and
discussing their diagnostic properties with regard to the buried structures.

6.1 'HORST' MODEL
A 2-D 'horst model' was devised, composed of an overburden and a substratum (see Fig.
10), where the horst has the same resistivity as the substratum and intrudes into the over-
burden.

The geometrical size of the horst is $3 \times 3$ km; its top lies at 0.5 km below the surface.
Two resistivity combinations were chosen for the overburden and the substratum, i.e. 1:100
and 1000:10 $\Omega$m respectively.

We begin with the 1–100 $\Omega$m combination, looking at the MT apparent resistivity $\rho_a$ and
phase $\varphi$ pseudo-sections (Fig. 2).

The $H$-polarization MT parameters show the expected sharply accentuated response of
the buried structure while, as was to be expected, the $E$-polarization response is much
weaker and is smoothed out over large distances (cf. Fischer et al. 1986). Since we only
consider structures composed of two simply connected formations with different conductivities, we avoid the problems which may arise when a highly resistive intermediate layer can lead to very large $H$-polarization adjustment lengths (cf. e.g. Ranganayaki & Madden 1980).

Let us now examine the various telluric parameters. $R_1(\omega)$ (Fig. 3a, b) with the base at left infinity gives results comparable with those of $\rho_0$ in $H$-polarization MT, while with the
Figure 3. Distance–frequency pseudo-sections of electric field ratios $R(\omega)$ with reference base at left infinity (a) or located over the structure at b (b), $R(\omega)$ (c) with reference base at left infinity and $G(\omega)$ (d) with reference base located at b: horst-like structure (1:100 resistivity ratio).

near base we obtain a less accentuated sketch, still fairly well linked to the actual shape of the structure however. In fact, the isolines of unity closely correspond to the borders of the buried structure and the isoline values decrease when going away from the structure. While the same trend holds for $R(\omega)$, its pseudo-section (Fig. 3c) is exceedingly smooth. From
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Field data obtained under these conditions, it will be difficult to infer much about the causative structure.

Phase $\Phi_1(\omega)$ gives the pseudo-sections shown in Fig. 4. These are affected by the position of the base, but a clear anomaly may be observed, correlating strongly with the lateral shift.

Figure 4. Distance–frequency pseudo-sections of the phase $\Phi_1(\omega)$ of the horst model (1:100 resistivity ratio) with reference base at left infinity (a) and located at b (b); (c) and (d) report pseudo-sections of $R_1(\omega)$ with the reference base at left infinity and located at b respectively for the horst model with 1000–10 resistivity ratio.
position and vertical extent of the horst. The Jacobian $J(\omega)$, calculated according to equation (8), is not reported here, as it gives a pseudo-section which is very similar to that of $R(\omega)$; this is a consequence of the smoothness of $R(\omega)$. The pseudo-section of $\Phi(\omega)$ too is not reported here, because of its smoothness and very limited range of variations (not more than $5^\circ$).

The parameter whose pseudo-section is the least affected by the base position is the horizontal gradient $G(\omega)$ as shown in Fig. 3(d). In both base positions it underscores the lateral dimensions of the horst. This was to be expected since the gradient is defined as the ratio between the difference of two successive values and their separation in space, and is thus rather insensitive to level variations resulting from a different normalization.

As regards the horst model with inverted resistivities (1000 Ohm for the cover and 10 Ohm for the substratum and the horst), the pseudo-sections of the telluric parameters normalized to infinity continue to delineate not only the shape of the structure but also the different resistivity contrasts satisfactorily and they compare favourably with those of the MT functions. In effect, the Fig. 4(c, d) networks of isolines of $R(\omega)$ are something like mirror images of those of Fig. 3(a, b). Again in analogy with the preceding case, the horizontal gradient $G$ is the one which is the least affected when the base position is changed.

In the horst example just studied, all the telluric parameters give pseudo-sections which are indicators of the buried structure, but the most telling diagnostic features seem to be contained in the pseudo-sections of $R(\omega)$, of its horizontal gradient $G(\omega)$ and of the phase $\Phi(\omega)$.

The pseudo-sections of $R(\omega)$ and $\Phi(\omega)$ also give quite an accurate picture of the structure, marking its vertical extent even more precisely than $R(\omega)$ and $\Phi(\omega)$. However, they yield much smaller parameter variations and in practice will be very difficult to interpret, particularly if the data are scattered and the base is near the structure.

Of the other models shown in Fig. 1, only the pseudo-sections of $R$ will be examined here.

### 6.2 Graben Model

The pseudo-section of $R$ obtained with the graben model (Fig. 1 @) with the base at infinity (Fig. 5a) again offers an accurate picture of the underground structure.

The same may be said for $R$ with the base placed near the centre of the structure (Fig. 5b), whose pseudo-section still gives a pattern of isolines which are unquestionably different from those corresponding to the horst model with either ratio of resistivity (cf. figs 3a, b and 4c, d).

### 6.3 Tilted-Plane Models

The responses of a series of structures of the type shown in Fig. 1 @, @ and @ with resistivity ratios between cover and substratum of 1:100 and 100:1 were studied.

Figs 5(c, d) and 6 show the pseudo-sections of $R$ obtained for the two combinations of resistivity. The most obvious observation is that the pseudo-sections of identical geometrical arrangement but with inverted resistivities are completely different. On the other hand, when the resistivity ratio is the same, the pseudo-sections show very little variation with varying geometry, i.e. with the angle of tilt. This is particularly true for the ratio of 10:1000 between cover and substratum, respectively. For the opposite ratio (1000:10) somewhat greater differences are observed.
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Figure 5. Distance–frequency pseudo-sections of $R_l(\omega)$ for the graben model (1:100 resistivity ratio) with reference base at left infinity (a) and located at b (b); (c) and (d) report pseudo-sections of the Step model (4) with resistivity ratios 10–1000 and 1000–10 respectively.

The $R_l(\omega)$ parameter thus seems to depend much more on the lateral changes of electrical properties than on the exact geometry of the structure. A variation of the angle from 150° (pinch-out structure), through 90° (step) towards 30° (sloping plane) and with a 1:100 resistivity ratio yields pseudo-sections with remarkable but relatively similar charac-
Figure 6. Distance–frequency pseudo-sections of $R_l(\omega)$ with reference base at left infinity of model (3) with resistivity ratio $10^{-1}$–$10^{3}$ (a) and $10^{3}$–$10^{10}$ (b), and of model (5) with resistivity ratio $10^{-1}$–$10^{3}$ (c) and $10^{3}$–$10^{10}$ (d).

teristics. These consist in a sort of verticalization of the isolines at the upper angle of the tilting plane (Figs 5d and 6a, c).

Although these common features remain with the 100:1 resistivity ratio (cf. Figs 5d and 6b, d), the right portion of the pseudo-sections contains more structure. Whereas
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Fig. 6(b) is fairly representative of a structure with gradually thickening overburden, it will be more difficult to distinguish field data arising from a step (Fig. 5d) from similar data caused by a pinch-out structure (Fig. 6d).

7 Conclusions

A detailed analysis of the telluric fields over a wide frequency range suggests that the telluric method is not only highly flexible, but one that can give very significant information, especially as regards lateral variations of buried structures. To achieve this aim it is necessary, however, to secure data over a rather wide frequency band and to extend the study to several new parameters beside the traditional Jacobian.

Our analysis of the responses of these parameters in a series of 2-D models has made it possible to evaluate the resolving power of the telluric method, both in terms of geometry and distribution of the electrical properties of the buried structures. The results we have obtained clearly demonstrate that the telluric method may be applied effectively as an autonomous method of geophysical prospecting in the investigation of arbitrary 2-D structures, as well as a preliminary survey to MT soundings, based on the fact that the weak influence of the model structure on E-polarization is an advantage for MT studies of the Earth beneath the shallower 2-D structures.

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References


