VISVE, a Vorticity Based Model Applied to Cavitating Flow around a 2-D Hydrofoil

[1]Lu Xing*; [1]Chunlin Wu; [1]Spyros A. Kinnas

[1] *Ocean Engineering Group, The University of Texas at Austin, Austin, TX, USA*

Abstract

In this study, a method solving the viscous vorticity equation (VISVE) and the mixture model, which introduces an additional transport equation for the vapor volume fraction, are coupled to predict the partial cavitating flow around a two-dimensional hydrofoil. The VISVE method is designed to be both spatially compact and numerically efficient in comparison with the commonly used RANS models. The cavitation is modeled by the Schnerr-Saucer or the Zwart-Gerber-Belamri models. The predicted vorticity, velocity, pressures and the vapor volume fraction are compared with those from a commercial RANS solver.

Keywords: vortex method; mixture model; cavitation; hydrofoil

Introduction

Cavitation is a complex physical phenomenon of rapid phase change between liquid and vapor, resulting in very large and steep variation of density and viscosity. In many engineering applications – particularly hydrofoils, liquid pumps and marine propellers – cavitation, as a subject of research has attracted much attention mainly due to the detrimental effects (e.g. structural failure, corrosion, and noise radiation) of collapsing bubbles [4]. In particular, a type of cavitation that is very common on marine propellers is sheet cavitation, in which a distinct thin vapor bubble is attached to the blade surface. Other types of cavitation which also occur include cloud and bubble cavitation. However, this work addresses the computational analysis of sheet hydrofoil cavitation, though the method by nature is eligible to predict cloud and bubble cavitation.

VISVE method has been proposed and implemented in our previous work by Tian and Kinnas [1]. The VISVE method is designed to be both spatially compact and numerically efficient in comparison with the commonly used RANS models. Especially, after parallelization the efficiency can be improved significantly. The method was recently extended in the case of 2-D and 3-D hydrofoil in backing conditions by Wu et al [5] and cylinder in alternating flow by Li and Kinnas [6]. Reliable results were obtained in both cases. Mixture Model is a homogeneous fluid approach based on solving the transport equation for vapor volume fraction, where mass transfer rate due to cavitation is modeled by a mass transfer model. The mass transfer between two phases is controlled by including appropriate source terms in the volume fraction equation. There are several models [2][3][7] to formulate the simple rational expression of these source terms. In our work, we applied the model proposed by Schnerr and Sauer [2] and the model by Zwart-Gerber- Belamri [3].

The goal of this study is to incorporate in VISVE the mixture model to predict the cavitating flow around a hydrofoil. To verify the coupled scheme, the results are compared with those from the commercial RANS solver *ANSYS/Fluent*.

Numerical Methods and Models

1. Governing system

The cavitation mixture model assumes that the bubble-liquid flow under investigation is a single fluid with the homogeneous mixture of two phases (liquid and vapor). Therefore, only one set of equations is needed to simulate the cavitating flows:

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla} \cdot (\rho_m \vec{u}) = 0 \tag{1}$$

$$\frac{\partial \omega}{\partial t} + \vec{\nabla} \cdot (\omega \vec{q}) = \nabla^2(\nu\omega) + \vec{\nabla} \cdot \left[\nu \frac{\overrightarrow{\nabla\rho}}{\rho}\omega\right] \tag{2}$$

Phases are considered to be incompressible and to share the same velocity field [3]. Equation (1) represents the continuity equation and Equation (2) is the simplified vorticity equation under the assumption that the two-dimensional hydrofoil is placed along the x-axis, with inflow also along the x-axis, such that $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial z}$, especially within the narrow region close to the hydrofoil and along the wake where $\omega \neq 0$. The complete formulation of vorticity

*Corresponding Author, Lu Xing: xinglu@utexas.edu

equation (2) can be found in Kinnas [9]. The above equations are closed with the constitutive relations for the density and dynamic viscosity:

$$\rho_m = \alpha\rho_v + (1-\alpha)\rho_l;\ \mu_m = \alpha\mu_v + (1-\alpha)\mu_l \tag{3}$$

Where, α represents the volume vapor fraction within the mixture. Detailed implementation schemes will be presented in Xing [8].

2. Pressure Calculation

Pressure is not a direct solution out of vorticity equation. It can then be calculated through post-processing. The momentum equation in the direction normal to the hydrofoil surface is given by the following equation [9]:

$$\frac{\partial \boldsymbol{q_n}}{\partial t} + \frac{\partial}{\partial n}\left(\frac{q^2}{2}\right) = -\frac{\partial P}{\partial n}\cdot\frac{1}{\rho} + \boldsymbol{q}\times\omega - \upsilon\nabla\times\omega - \frac{(\boldsymbol{\nabla}\cdot\boldsymbol{q})}{\rho}\cdot\frac{\partial\mu}{\partial n} \tag{4}$$

Where t is time, P is pressure, ρ is density, υ is the kinematic viscosity, $\boldsymbol{q}$ is the total velocity, ω is vorticity and μ is dynamic viscosity. Under the assumption that the normal velocity $\boldsymbol{q_n}$ near the hydrofoil surface can be neglected, equation (4) then be simplified as [9]:

$$\frac{\partial P}{\partial n}\cdot\frac{1}{\rho} = -\frac{\partial}{\partial n}\left(\frac{q^2}{2}\right) - \upsilon\boldsymbol{\nabla}\times\boldsymbol{\omega} - \upsilon\boldsymbol{\nabla}\times\boldsymbol{\omega} - \frac{(\nabla\cdot\boldsymbol{q})}{\rho}\cdot\frac{\partial\mu}{\partial n} \tag{5}$$

Since in VISVE the grid is generated normal to the hydrofoil, a line integral can be conducted along the direction normal to the hydrofoil from the outer boundary to any point (X) within the domain to calculate the pressure at that point. Therefore, as long as the pressure on the outer boundary of the domain is given, the pressure within the whole domain can be obtained.

$$P_X = \int_{outer\ boundary}^{X} \frac{\partial P}{\partial n}\cdot d\boldsymbol{n} \tag{6}$$

3. Cavitation model

With Equation (3), the continuity equation (1) can then be split into the following system:

$$\frac{\partial(\alpha\rho_v)}{\partial t} + \boldsymbol{\nabla}\cdot(\alpha\rho_v\boldsymbol{q}) = R \tag{7}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{q} = R\left(\frac{1}{\rho_v} - \frac{1}{\rho_l}\right) \tag{8}$$

Where, R is the phase change rate: $R = R_c$ or $R = -R_e$, with R_c, R_e representing condensation rate and evaporation rate, respectively, which can be obtained by the following two models. Equation (7) is the vapor transport equation, while Equation (8) models the "compressibility" of the mixture due to the generation and collapse of the cavity bubbles.

3.1 Schnerr and Sauer's model

In this model, the vapor volume fraction is defined as the ratio of the volume of vapor over the cell volume and then reformulated [2]:

$$\alpha = \frac{V_{vapor}}{V_{cell}} = \frac{N_{bubbles}\cdot\frac{4}{3}\pi\Re_B{}^3}{V_{vapor}+V_{liquid}} = \frac{n_0 V_{liquid}\cdot\frac{4}{3}\pi\Re_B{}^3}{n_0 V_{liquid}\cdot\frac{4}{3}\pi\Re_B{}^3 + V_{liquid}} = \frac{n_0\cdot\frac{4}{3}\pi\Re_B{}^3}{1+n_0\cdot\frac{4}{3}\pi\Re_B{}^3} \tag{9}$$

Where V_{vapor}, V_{liquid}, V_{cell} represent the volume of vapor, liquid and cell respectively; $\Re_B$ is the radius of the vapor bubble; n_0 is the bubble concentration per unit volume of pure liquid and is set to be $10^{13}/m^3$. The evaporation and condensation phase change rate is then modeled from Rayleigh-Plesset equation as follows:

$$R_e = \frac{\rho_l\rho_v}{\rho_m}\alpha(1-\alpha)\frac{3}{\Re_B}\sqrt{\frac{2}{3}\frac{(P_V-P)}{\rho_l}} \tag{10}$$

$$R_C = \frac{\rho_l\rho_v}{\rho_m}\alpha(1-\alpha)\frac{3}{\Re_B}\sqrt{\frac{2}{3}\frac{(P-P_V)}{\rho_l}} \tag{11}$$

$$\Re_B = \left(\frac{\alpha}{1-\alpha}\frac{3}{4\pi}\frac{1}{n_0}\right)^{\frac{1}{3}} \tag{12}$$

Here, P_V is the vapor pressure.

3.2 Zwart- Gerber- Belamri's model

This model assumes constant bubble radius and simulate the bubble growth and collapse as [3]:

$$R_e = F_{evap} \frac{3\alpha_{nuc}(1-\alpha)\rho_v}{\Re_B} \sqrt{\frac{2}{3}\frac{(P_V - P)}{\rho_l}} \tag{13}$$

$$R_C = F_{cond} \frac{3\alpha\rho_v}{\Re_B} \sqrt{\frac{2}{3}\frac{(P - P_V)}{\rho_l}} \tag{14}$$

In the above equations, α_{nuc} is the nucleation site volume fraction, F_{evap} and F_{cond} are two empirical calibration coefficients for the evaporation and condensation process, respectively. In VISVE, the above parameters are set as follows: $\alpha_{nuc} = 5.0 \times 10^{-4}, \Re_B = 1.0 \times 10^{-6} m$, $F_{evap} = 50$, $F_{cond} = 0.01$.

4. Boundary treatment

The boundary treatment scheme is shown in Figure 1. After solving the above coupled system, we could obtain the vorticity field. However, velocities calculated from vorticity field using Biot-Savart law will not satisfy the non-slip boundary conditions on the wall. A vorticity creation scheme based on the Boundary Element Method (BEM) is designed to eliminate tangential and normal velocity, denoted as $\boldsymbol{q^n}, \boldsymbol{q^s}$ on the wall. After assigning the newly created vorticity into cells in the first layer, the non-slip boundary condition will then be satisfied.

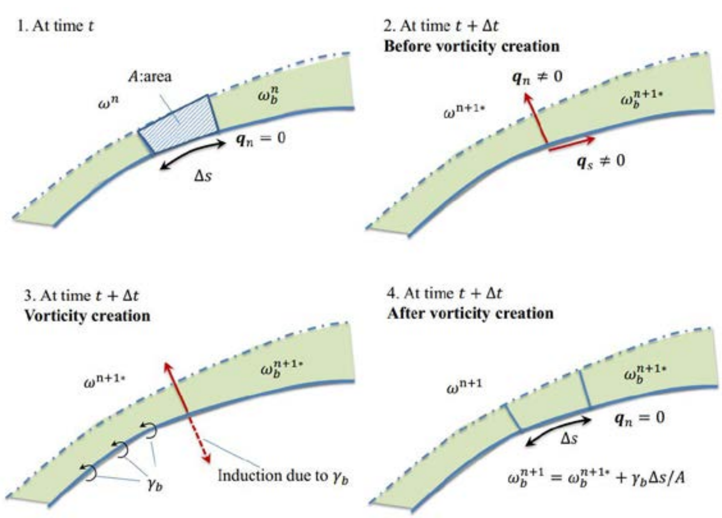

Figure 1: Schematic figure for the vorticity creation algorithm [1]

Results and comparison with RANS

To compare the computation time, VISVE and RANS are run on the same Stampede2 node of Texas Advanced Computing Center using 16 CPUs, and the parameters are shown in Table 1. VISVE is proven to be compact and efficient. Time step is set as $\Delta t = 0.0001\ s$ in both VISVE and RANS, and they are run for 10000 steps, so we get results for the first second. CPU time is calculated simply by taking the average of several cases.

1. Grid Configurations and numerical setup

The computation domains were generated around a NACA66 hydrofoil with thickness to chord ratio 4% ($\frac{t}{L} = 4\%$), camber to chord ratio1% ($\frac{c}{L} = 1\%$). The grid configurations in VISVE and RANS[1] are shown in Figure 2 and Figure 3, respectively. It can be noticed that VISVE enjoys a much more compact domain than RANS model. The total number of cells for the two models are shown in Table 1.

[1] All the RANS simulations shown in this paper have been performed by using *ANSYS/Fluent*.

Table 1: CPU time comparison

	Total Cell Number	CPU Time
VISVE	20,000	55min
RANS	50,000	4.5 h

In the RANS model, the cavitation is modeled as the steady flow. Second order discretization is used for density, momentum and turbulent kinetic energy. PRESTO!, QUICK and Least-square-cell-based schemes are applied to pressure, vapor phase transport equation and gradient calculation respectively. The simulations were run at Reynolds number Re=4,800, Angle of Attack $(AOA = 4°)$and the laminar model was adopted in VISVE and RANS. The boundary condition in RANS simulation is shown in Figure 3. The computation domain in VISVE is also open.

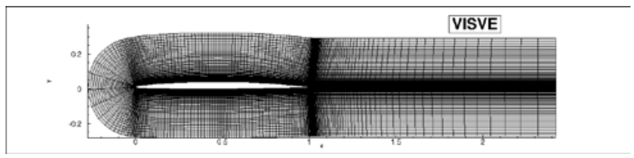

Figure 2: VISVE computation domain and grid configuration

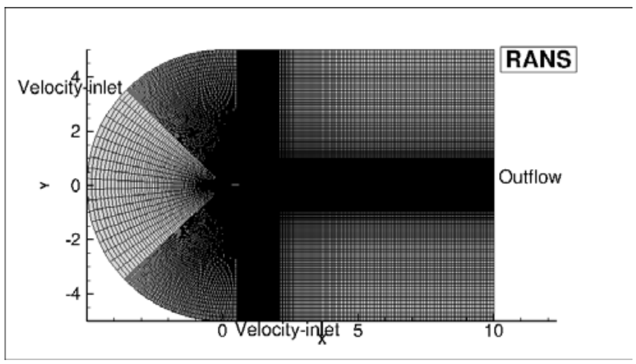

Figure 3: RANS computation domain and grid configuration

For the computational results, we use the following non-dimensional parameters:

$$\text{Cavitation Number } \sigma = \frac{P-P_v}{\frac{1}{2}\rho_l u_\infty{}^2} \quad (12)$$

$$\text{Reynolds Number } Re = \frac{u_\infty L}{v_l} \quad (13)$$

With chord length L= 1m, the free stream velocity u_∞ is set to be 2.44m/s. The results of a wetted case $\sigma = 1.6$ and cavitation cases of $\sigma = 1.2$, $\sigma = 1.0$ and $\sigma = 0.8$ are presented next.

2. Wetted case ($\sigma > 1.6$)

Before investigating the cavitating flow, the study of velocity and vorticity field was first carried out in fully wetted case. The pressure coefficient on the foil as well as the profiles of velocity and vorticity predicted by VISVE and RANS are compared in Figure 4 and Figure 5, respectively. The two models yield very close profiles to each other. The good correspondence verifies the reliability of VISVE model in fully wetted cases.

3. Steady Cavitation

The cavitating results shown below was simulated using the Zwart- Gerber- Belamri's model. However, with the runtime option given, users can easily choose their preferable model. More results are shown in Xing [8]. The pressure coefficients and the cavity shape at $\sigma = 1.2$, $\sigma = 1.0$ and $\sigma = 0.8$ are shown in Figure 6-11. It can be seen from Figures 6-9 that when the cavity is moderate ($\sigma = 1.2$, $\sigma = 1.0$), the pressure coefficient and cavity shape predicted by VISVE show good agreement to those simulated by RANS. However, for smaller cavity numbers ($\sigma = 0.8$), as shown in Figures 10 and 11, VISVE yields thicker and more diffused cavity shape, compared to that from RANS.

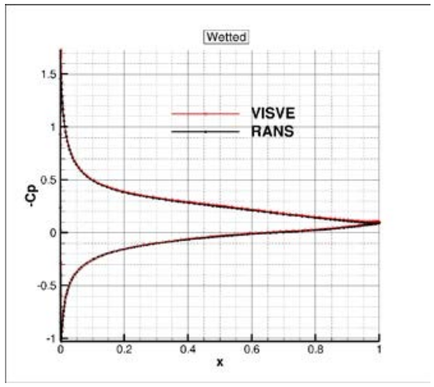

Figure 4: Pressure coefficient comparison between VISVE and RANS in wetted case.

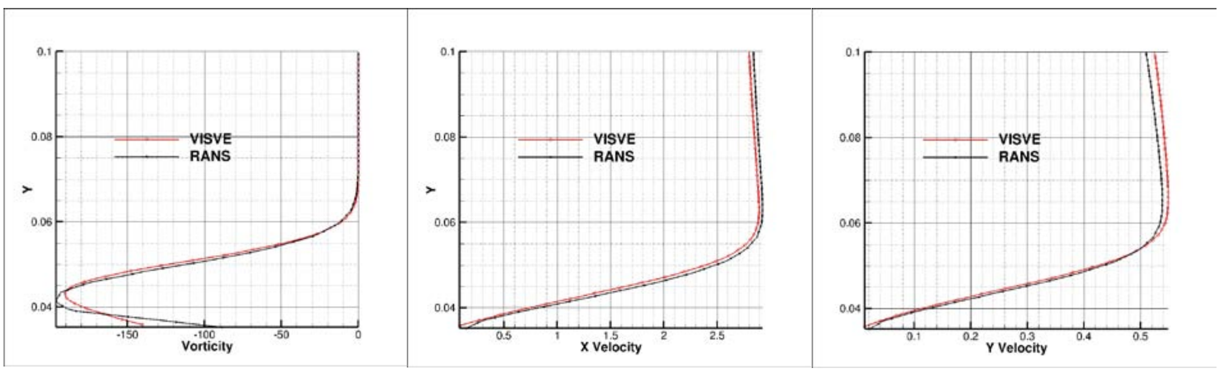

Figure 5 Vorticity, x-velocity, y-velocity profile comparison at 10% chord at Re= 4,800

Conclusions

A new numerical model which couples VIScous Vorticity Equation (VISVE) model with mixture model to simulate cavitating flow is proposed and the predicted velocity, vorticity, pressure and cavity patterns are compared with those from Reynold Averaged Navier-Stokes (RANS) simulations. In fully wetted conditions, the VISVE model predicts vorticity and velocity profiles, and pressure distributions which are very close to those predicted from RANS. In the cases of moderate cavitation number ($\sigma = 1.2$, $\sigma = 1.0$), VISVE predicts pressures and cavity shapes which are comparable to those from RANS. With lower cavitation number, $\sigma = 0.8$, VISVE predicts larger cavity than that from RANS. Further efforts will be made in the near future to resolve the differences in the cavity size predicted by the present method and by RANS.

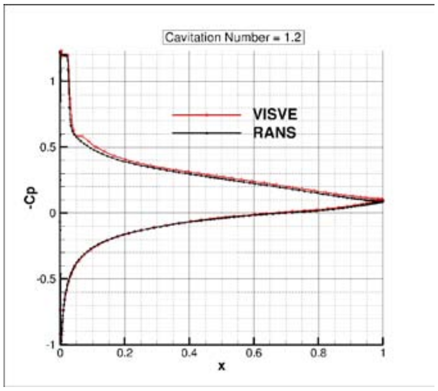

Figure 6: Pressure coefficient comparison between VISVE and RANS at Re= 4,800 $\sigma = 1.2$

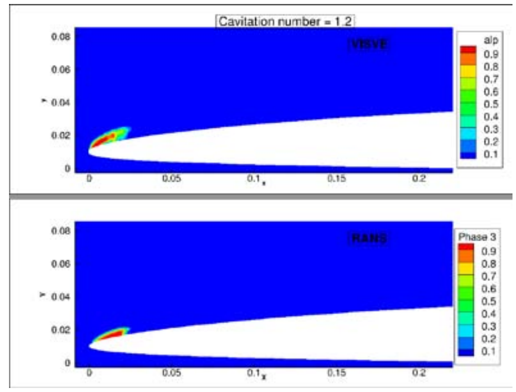

Figure 7: Vapor fraction contour comparison between VISVE and RANS at Re= 4,800 $\sigma = 1.2$

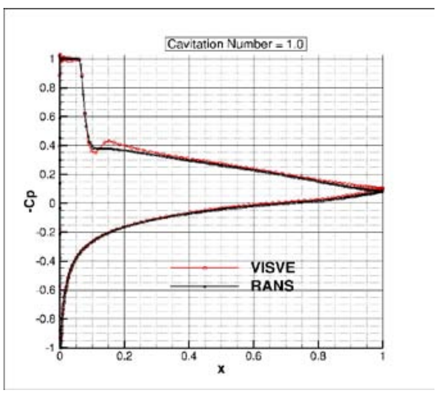

Figure 8: Pressure coefficient comparison between VISVE and RANS at Re= 4,800 $\sigma = 1.0$

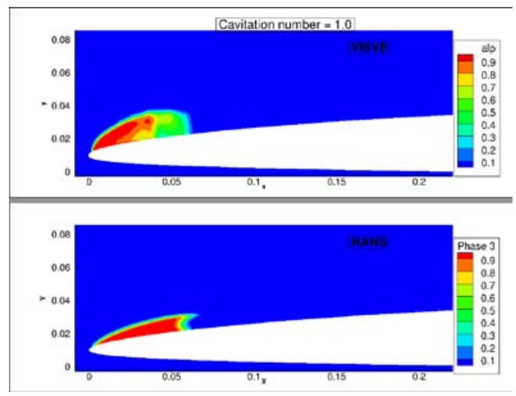

Figure 9: Vapor fraction contour comparison between VISVE and RANS at Re= 4,800 $\sigma = 1.0$

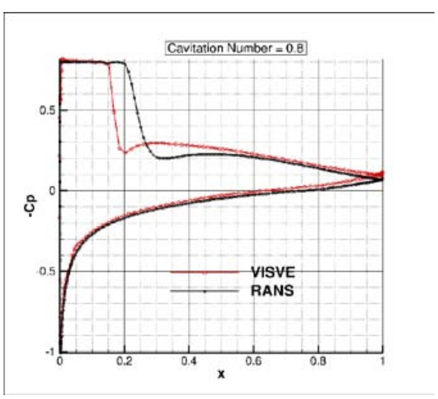

Figure 10: Pressure coefficient comparison between VISVE and RANS at Re= 4,800 $\sigma = 0.8$

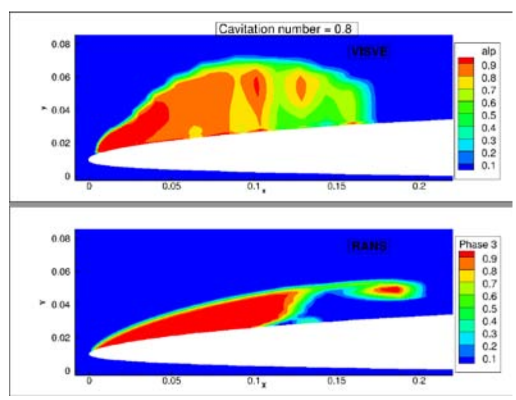

Figure 11: Vapor fraction contour comparison between VISVE and RANS at Re= 4,800 $\sigma = 0.8$

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