

## CFD Simulation of Thermodynamic Effect Using a Homogeneous Cavitation Model Based on Method of Moments

<sup>1</sup>Shin-ichi Tsuda\*; Satoshi Watanabe

<sup>1</sup>Department of Mechanical Engineering, Kyushu University, Fukuoka, Japan

### Abstract

Recently, we have constructed a new homogeneous cavitation model, which includes various elementary processes in cavitation bubbles on the basis of a method of moments. We already confirmed that CFD simulation with this model can predict lift coefficient of a hydrofoil in cavitating water flow better than some conventional methods while the temperature decrease by thermodynamic effect in liquid nitrogen as a cryogenic fluid is underestimated. In this paper, we re-applied the cavitation model to a CFD simulation around the same target, cavitating liquid nitrogen flow around NACA16-012 hydrofoil. As a result, the temperature decrease by the thermodynamic effect is fairly well reproduced compared with the previous result when we arranged the model parameter of the growth/shrinkage while a limitation of homogeneous model and that of turbulence model are still indicated.

**Keywords:** cavitation; thermodynamic effect; NACA16-012 hydrofoil

### Introduction

Cryogenic fluids such as liquid hydrogen or liquid oxygen have been used as a liquid rocket propellant, and it is well known that the suction performance of turbo-pump inducer is better in cryogenic fluids than in cold water due to thermodynamic effect. Thermodynamic effect is caused by temperature decrease inside a cavity region that arises from the latent heat transfer across the interface of cavity, and it brings about depression of bubble growth. To better understand the suction performance of cavitating cryogenic inducers, we should take account of the temperature changes due to thermodynamic effect and Computational Fluid Dynamics (CFD) is a powerful tool for this purpose. However, accurate evaluation of the temperature depression affecting the cavitating flow is even now one of the difficult tasks although some CFD studies have been carried out [1-3].

Recently, we have constructed a new homogeneous cavitation model calling “Multi-process cavitation model”, which includes various elementary processes in cavitation bubbles based on a method of moments. We already confirmed that CFD simulation with this model can predict lift coefficient of a hydrofoil in cavitating water flow better than some conventional methods [4] while we could not successfully reproduce the thermodynamic effect in cryogenic fluids. In this paper, we retried the application of the present model for a cryogenic fluid of liquid nitrogen around NACA16-012 hydrofoil (angle of attack is 8 deg.) whose experiment was conducted by Niiyama et al [5].

### Multi-process cavitation model

The multi-process model was constructed based on the moment method. In this method, “*i*th moment  $M_i$ ” is defined using the size distribution function  $f(R, t)$  as follows.

$$M_i = \int R^i f(R, t) dR, \quad i \in \{0, 1, 2, 3\}, \quad (1)$$

where  $R$  is the bubble radius and  $t$  is the time. Taking the time derivative for the quantity  $M_i$  based on a simple approximation [4], we can finally derive the relationship below,

$$\frac{DM_i}{Dt} = iM_{i-1}G(\bar{R}) + S_{p,i}. \quad (2)$$

Here,  $\bar{R}$  is the mean bubble radius defined as  $M_1/M_0$ ,  $G(\bar{R})$  is the bubble growth/shrinkage rate, and  $S_{p,i}$  reflects the change of the number of bubbles, which occurs with inception/collapse and coalescence/break-up. For each  $i$  ( $i=0, 1, 2, 3$ ), the equation set is shown in Fig. 1 [4].

\*Corresponding Author, Shin-ichi Tsuda: tsudashin@mech.kyushu-u.ac.jp

Expansion/Shrinkage    Inception/Collapse    Coalescence/Break-up

$$\textcircled{1} \frac{DM_0}{Dt} = \downarrow + J_s + S_b - S_c$$

$$\textcircled{2} \frac{DM_1}{Dt} = G(\bar{R})M_0 + R_c J_s + \bar{R}(S_b - S_c)$$

$$\textcircled{3} \frac{DM_2}{Dt} = 2G(\bar{R})M_1 + R_c^2 J_s + \bar{R}^2(S_b - S_c)$$

$$\textcircled{4} \frac{DM_3}{Dt} = 3G(\bar{R})M_2 + R_c^3 J_s + \bar{R}^3(S_b - S_c)$$

$$\textcircled{5} \frac{D(\rho_g M_3)}{Dt} \approx \dot{m}_v M_2 M_3 + \rho_g \frac{DM_3}{Dt}$$

Evaporation/Condensation    ↑  
from Eq.(4)

Main Symbols

$\bar{R}$  ; mean radius of bubbles  
 $G(\bar{R})$  ; expansion/shrinkage speed  
 $R_c$ ; critical size  
 $J_s$ ; inception/collapse rate  
 $S_b$ ; break-up rate  
 $S_c$ ; coalescence rate  
 $\dot{m}_v$ ; evaporation/condensation flux

Void Fraction

$$\phi = \frac{(4\pi/3)M_3}{1 + (4\pi/3)M_3}$$

Quality

$$Y = \frac{(4\pi/3)\rho_g M_3}{\rho_l + (4\pi/3)\rho_g M_3}$$

Fig. 1 Equation set of multi-process cavitation model

As shown in Fig. 1, main elementary processes such as growth/shrinkage, inception/collapse, and coalescence/breakup are reflected in each source term. The fifth equation in Fig. 1 is an extended equation of Eq. (2) reflecting the evaporation/condensation, which is the source of thermodynamic effect. The specific expression of each elementary process is described in detail in Ref. [4].

### Numerical method and validation target

“Multi-process cavitation model” was implemented to a C-CUP (Cubic interpolation propagation - Combined Unified Procedure) flow solver [1] where compressibility of liquid-vapor two-phase flow can be incorporated with Baldwin-Lomax turbulence model as a zero-equation model. Using the solver, we simulated cavitating flow of liquid nitrogen around the hydrofoil of NACA16-012. The experimental set up by Niiyama et al. [5] in which five thermo-sensor to detect the temperature depression by thermodynamic effect and the present computational grid are shown in Fig. 2. In computation, each total cell number is around 30,000, and the flow velocity was given at the inlet boundary while the pressure was given at the outlet in the computational domain.

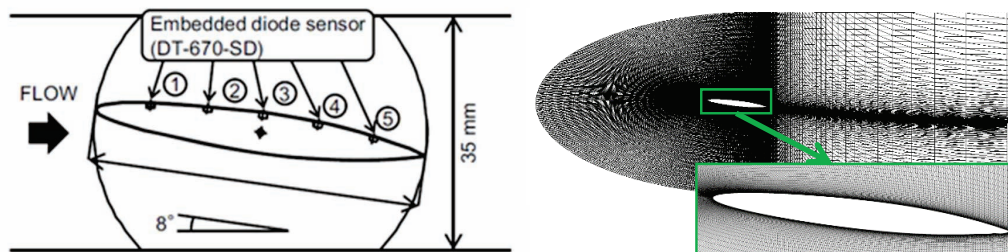


Fig. 2 Experimental set of thermos-sensor [5] and the computational grid for NACA16-012

### Results and discussion

In advance, we investigated the influence of the model parameters of Multi-process cavitation model, and confirmed that the most sensitive model parameter is the coefficient of Growth/Shrinkage (GS) process,  $C_g$ , and the second one is that of Evaporation/Condensation (EC) process,  $C_{ec}$ . Each model for the two elementary processes is as follows, in which the expression for GS process is slightly different from that in Ref. [4].

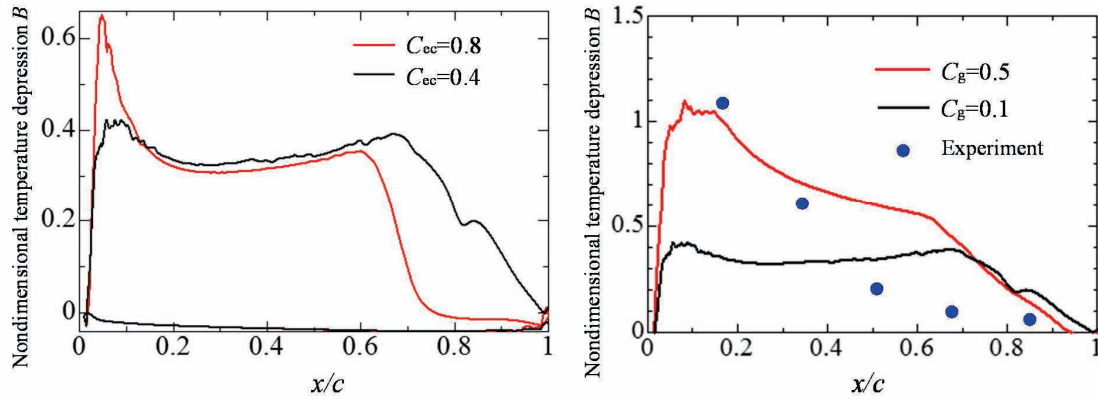


Fig. 2 Influence of the model parameters at  $\sigma = 0.37$  (left: influence of  $C_{cc}$ , right: influence of  $C_g$ )

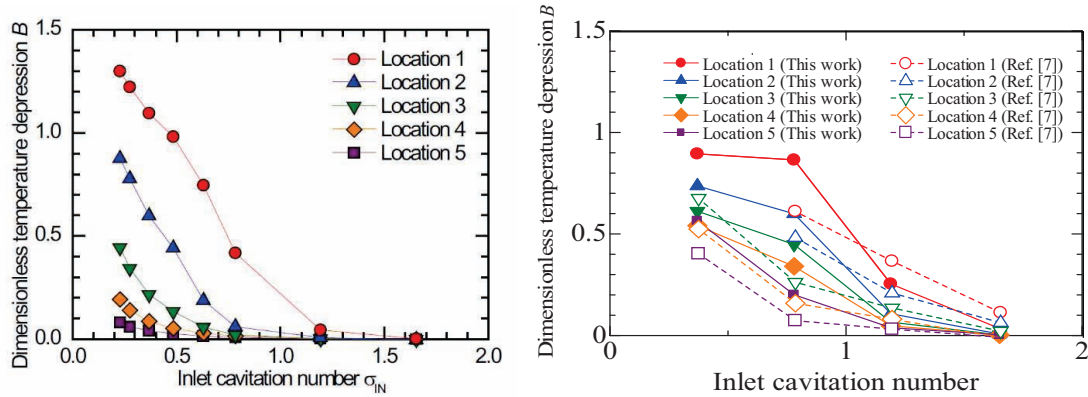


Fig. 3 Dimensionless temperature depression against cavitation number (left: experiment, right: present calculation)

$$G(\bar{R}) = C_g \sqrt{\frac{2 p_v - p_l - 2\sigma_s / \bar{R}}{3 \rho_l}} \text{ (for GS process), } \dot{m}_v = \frac{2C_{cc}}{2 - C_{cc}} \frac{p_{s,in} - p_v}{\sqrt{2\pi R_v T}} \text{ (for EC process)} \quad (3)$$

The former is a well-employed simplified Rayleigh-Plesset model [3], and the latter is a classical evaporation/condensation model by Schrage [6]. Here,  $p_v, p_l, \rho_l, \sigma_s, R_v, T$  are saturation vapor pressure, liquid pressure (given in CFD simulation), liquid density, surface tension, gas constant, and temperature, respectively.  $p_{s,in}$  is the pressure inside bubble, which is given by Kelvin equation [6].

Figure 2 is a result showing an example of the influence of the two parameters (cavitation number  $\sigma$  is 0.37), and the experimental result by Niiyama et al. [5] is plotted in the right figure. Here, the vertical dimensionless temperature decrease  $B$  is defined as the following form.

$$B_N = \frac{T_{in} - T_N}{T^*} \quad (N = 1, 2, 3, 4, 5), \text{ where } T^* = \frac{\rho_v(T_{in})L(T_{in})}{\rho_l(T_{in})C_{p,L}(T_{in})}. \quad (4)$$

Here,  $T_{in}$  is the inlet temperature,  $L$  is latent heat,  $C_{p,L}$  is liquid specific heat at constant pressure,  $\rho$  is density (the subscript  $l$  and  $v$  mean liquid and vapor), respectively. The index  $N$  of  $B$  corresponds to each thermos-sensor in Fig. 1. As shown, the previous model parameter set ( $C_g = 0.1, C_{cc} = 0.4$ ) [4] clearly underestimates the temperature

depression at the leading edge side while just modification of  $C_{ec}$  does not reproduce the temperature distribution at the trailing edge side. In this work, we set the value  $C_g = 0.5$  without changing  $C_{ec}$  ( $C_{ec}$  is kept to be 0.4).

Figure 3 compares the temperature depression at the five points along the chord direction in some cavitation numbers between experiment and this work. Also, the computational result by Takeda et al. [7] who captures the liquid-vapor interface with a method coupling VOF and Level-Set is plotted in the left side in Fig. 3. Note that the legend 'Location 1-5' in Fig. 3 corresponds to each temperature sensor 1-5 in Fig. 1. As shown, the present results using the new parameter set qualitatively reproduce the temperature depression in the experiment, which is different from the simulation using the old parameter set [4]. It suggests that the model parameter of  $C_g$  is very important for the evaluation of thermodynamic effect. In addition, our results apparently show closer result against the experiment compared with the previous CFD result by Takeda et al. [7]. However, we still have two deviations from the measurement. One is that, at the higher cavitation number, the present work overestimates the temperature depression because our simulation based on a homogeneous cavitating flow precedes the cavitation inception. In the homogeneous cavitating flow, bubble nuclei whose quantity is given according to water quality [4] are also present on the every wall surface imposed non-slip condition, and in this case, the inception starts at the suction peak of negative pressure surface. This overestimation will be common to homogeneous medium models. On the other hand, at the lower cavitation number, the present simulation even underestimates the temperature depression near the leading edge while it overestimates the depression near the trailing edge side. The reason would be in inaccurate evaluation of turbulent flow rather than the cavitation model because the present simulation employs a simple turbulence model among zero-equation models. Therefore, the revision of the flow field itself is still necessary.

## Conclusion

We have applied "Multi-process cavitation model", in which the basic elementary processes in cavitation are taken into account based on the moment method, to the cryogenic cavitation around NACA16-012. As a result, the temperature decrease by the thermodynamic effect is evaluated when the model parameter of the growth/shrinkage rather than that of the evaporation/condensation is arranged. However, a limitation of homogeneous model and that of turbulence model are still present, absolutely indicating the improvement of the two factors.

## Acknowledgment

This study was partially supported by Ebara Co. Ltd., paying special thanks to Dr. Motohiko Nohmi (Ebara Co. Ltd.).

## References

- [1] Tani, N. and Nagashima, T., 2002, Numerical Analysis of Cryogenic Cavitating Flow on Hydrofoil –Comparison between Water and Cryogenic Fluids–, Proceedings of 4th International Symposium on Launcher Technology, #27 (CD-ROM)
- [2] Hosangadi, A. and Ahuja, V., 2005, Numerical Study of Cavitation in Cryogenic Fluids, Trans. ASME J. Fluids Eng., Vol. 127, pp.267-281
- [3] Tsuda, S., Tani, N., Yamanishi, N., Development and Validation of a Reduced Critical Radius Model for Cryogenic Cavitation, 2012, Trans. ASME J. Fluids Eng., Vol. 134, 051301
- [4] Tsuda, S. and Watanabe, S., Application of Multi-Process Cavitation Model for Cavitating Flow in Cold Water and in Liquid Nitrogen around a Hydrofoil, Proc. ASME/JSME/KSME 2015 Joint Fluids Engineering Conf., Paper No. AJKFluids2015-05532
- [5] Niiyama, K., Yoshihida, Y., Hasegawa, S., Watanabe, M., and Oike, M., 2012, Experimental Investigation of thermodynamic effect on cavitation in liquid Nitrogen, Proc. 8th International Symposium on Cavitation, No. 231
- [6] Fujikawa, S., Yano, T., Watanabe, M., 2011, Vapor-Liquid Interfaces, Bubbles and Droplets, Springer
- [7] Takeda, N., Himeno, T., Umemura, Y., Watanabe, T., 2014, Numerical Simulation of Cavitation Considering Thermodynamic Effect by Direct Interface Capturing Approach, Proc. 17th Symposium on Cavitation (Japan), Paper No. 0004