

**Robust Adaptive Control for Fractional-Order
Systems with Disturbance and Saturation**

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Robust Adaptive Control for Fractional-Order Systems with Disturbance and Saturation

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To our families, for their love and support

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Preface

This book is devoted to an investigation of some issues of tracking control and synchronization control for fractional-order nonlinear systems in the presence of system uncertainty, external disturbance, and input saturation. On the basis of definitions of the fractional integral and fractional derivatives, lemmas of stability analysis for fractional-order systems, design techniques of disturbance observers, approximation methods of system uncertainty, and handling methods of input saturation, the main research motives of this book are given as follows:

- 1) In the modeling process, there exist a vast amount of uncertainties caused by modeling error, which may not only degrade the performance of the control system but also even lead to instability of the dynamics system. Therefore, the uncertainty should be considered in the control design to improve the closed-loop system performance of fractional-order systems. Furthermore, neural networks can approximate any continuous uncertain dynamics with an arbitrary accuracy. Many adaptive neural control schemes have been reported for uncertain integer-order nonlinear systems. However, the neural network approximation technique has rarely been considered in uncertain fractional-order nonlinear systems in past decades.
- 2) A practical system is often subjected to external unknown disturbances. The disturbance may lead to oscillations and even increase the instability of the system. In the field of traditional control, it is well known that feedforward control provides an effective disturbance compensation method which can achieve prompt disturbance attenuation. However, the disturbance has to be measured by sensors for the implementation of traditional feedforward control. Unfortunately, disturbances are usually difficult or even impossible to be measured physically using sensors. Since disturbance observers can estimate external disturbances from known information of the controlled plants, the output of disturbance observers can be used to design the control law. As a result, disturbance rejection is guaranteed to improve the performance and robustness of the closed-loop system. Therefore, disturbance estimation techniques could be used to alleviate the restriction faced by traditional feedforward control and reject the effect of external disturbances. However, disturbance observers have seldom been reported for uncertain fractional-order nonlinear systems subject to external disturbances in the existing literatures. In addition, neural-network-based fractional-order disturbance observers need to be further designed for uncertain fractional-order nonlinear systems.

- 3) Since the interactive design is rendered more difficult by incorporating the neural network and the disturbance observer, the neural network approximation technique and the disturbance observer have rarely been considered together for integer-order nonlinear systems, although disturbance observers have been widely developed for integer-order nonlinear systems. Conversely, neural-network-based fractional-order disturbance observers have not been designed for uncertain fractional-order nonlinear systems by the interactive design method in the existing literature. For fractional-order systems, the disturbance-observer-based adaptive neural control schemes need to be further developed for uncertain fractional-order nonlinear systems with unknown disturbances.
- 4) Saturation nonlinearity is a common problem for actuators in a wide range of practical systems. Input saturation can degrade system control performance and even lead to system instability if it is ignored in the control design. Furthermore, control design under consideration of input saturation is a challenging problem for any uncertain nonlinear system. So far, many control design schemes for integer-order nonlinear systems with input saturation have been studied. However, the issues of input saturation and disturbance have rarely been considered together in the control of fractional-order nonlinear systems, although a number of studies have considered input saturation. Therefore, new control schemes need to be further studied for fractional-order nonlinear systems in the presence of system uncertainty, external disturbance, and input saturation.

Based on these research motives, the main contributions of this book are contained in 12 chapters. Chapter 1 introduces some background knowledge. Chapter 2 provides definitions of the fractional integral and fractional derivatives and corresponding lemmas for the stability analysis of fractional-order systems, and introduces some typical fractional-order systems. Chapter 3 gives a fractional-order PID controller and a frequency-domain fractional-order disturbance observer. In Chapter 4, two fractional-order controllers are designed, for integer-order and fractional-order systems, respectively. Chapter 5 develops a disturbance-observer-based sliding-mode control scheme for fractional-order nonlinear systems with external disturbances. In Chapter 6, an adaptive neural control issue is investigated for a fractional-order rotational mechanical system subject to system uncertainties and external disturbance. Chapter 7 considers system uncertainties, external disturbance, and input saturation in the tracking control of fractional-order chaotic systems. In Chapter 8, a stabilization issue is studied for continuous-time fractional-order positive systems based on disturbance observers. Chapter 9 investigates an adaptive sliding-mode synchronization control of fractional-order chaotic systems with disturbances. In Chapter 10, the problem of anti-synchronization control is investigated for fractional-order nonlinear systems based on a disturbance observer and the neural network. In Chapter 11, the input saturation issue is considered for the synchronization of fractional-order systems, while Chapter 12 which considers the synchronization controller design for fractional-order chaotic systems with disturbance and input saturation.

This book intends to provide readers with a good understanding of how to achieve tracking control and synchronization control of fractional-order nonlinear systems with system uncertainties, external disturbance, and input saturation. The book can be used as a reference for the academic research on fractional-order nonlinear systems or used in Ph.D. study of control theory and engineering.

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Series Preface

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Symbols and Acronyms

D^α	Caputo fractional derivative
$D_{0,t}^{-\alpha}$	fractional-order integral
${}^{\text{GL}}D_{0,t}^\alpha$	Grünwald–Letnikov fractional derivative
${}^{\text{RL}}D_{0,t}^\alpha$	Riemann–Liouville fractional derivative
$E_{\alpha,\beta}(z)/\mathcal{E}_{\alpha,\beta}(z)$	Mittag–Leffler function
eig	eigenvalue
I	identity matrix
$\mathcal{L}(\cdot)$	Laplace transform
\mathfrak{R}	field of real numbers
R^r	r -dimensional real vector space
R^+	positive real numbers
sign	signum function
$\lambda_{\max}(\cdot)$	maximum eigenvalue
$\lambda_{\min}(\cdot)$	minimum eigenvalue
$ \cdot $	absolute value
$\ \cdot\ $	2-norm
$(\cdot)^\top$	transposition
\forall	for all
\in	belongs to
Σ	sum
FOCS	fractional-order chaotic system
FODO	fractional-order disturbance observer
FONS	fractional-order nonlinear system
FOPS	fractional-order positive system
GL	Grünwald–Letnikov definition
MIMO	multi-input and multi-output
ML	Mittag–Leffler
RL	Riemann–Liouville definition
SISO	single-input and single-output
SMDO	sliding-mode disturbance observer
SMFODO	sliding-mode fractional-order disturbance observer
T–S	Takagi–Sugeno

