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or letting $t \to \infty$

$$T - T_0 = \frac{\sqrt{2\lambda}}{4\pi^{3/2}} \frac{q}{k} e^{-\lambda v\xi} \int_0^\infty \frac{d\tau}{\tau^2} e^{-\frac{\lambda R^2}{2\tau^2} - \frac{\lambda\tau^2}{2}} \ldots\ldots [C]$$

The infinite integral appearing in Equation [C] can be solved by means of the Bessel function $K_{1/2}(x)$. To this end put a new variable α, such that

$$\tau = \alpha R\sqrt{2\lambda} \ldots\ldots\ldots\ldots\ldots\ldots [D]$$

The substitution of Equation [D] in Equation [C] gives

$$T - T_0 = \frac{q\sqrt{2\lambda}}{4\pi^{3/2}\,k} e^{-\lambda v\xi} \times \frac{1}{R\sqrt{2\lambda}} \int_0^\infty e^{-(\lambda vR)^2\alpha^2 - 1/4\alpha^2} \alpha^{-2} d\alpha$$

But according to the theory of Bessel functions (10)

$$\int_0^\infty e^{-(\lambda vR)^2\alpha^2 - 1/4\alpha^2} \alpha^{-2} d\alpha = \sqrt{2\lambda R}\; K_{1/2}(\lambda vR)$$

and furthermore

$$K_{1/2}(\lambda vR) = \frac{\sqrt{\pi}}{\sqrt{2\lambda vR}} e^{-\lambda vR},$$

hence finally

$$T - T_0 = \frac{q}{4\pi k} e^{-\lambda v\xi} \frac{e^{-\lambda vR}}{R}$$

This expression has been derived previously in Part 1 in a more simple way for a three-dimensional case.

Using similar procedures for the line and plane instantaneous sources, Expressions [31] and [18] can be derived for the two-dimensional and linear heat flow, respectively.

Appendix 2

Expansion of Solution [45] in a Fourier Series

As shown by Carslaw,[11] if $f(y)$ is an even function of y which can be expanded, as also $f(y \pm 2na)$, in a Fourier series of cosines of multiples $\pi y/a$, then

$$\sum_{-\infty}^{\infty} f(y \pm 2na) = \frac{1}{a}\int_0^\infty f(y)dy + \frac{2}{a}\sum_1^\infty \cos\frac{\pi ny}{a} \int^\infty f(y')\cos\frac{\pi ny'}{a}\,dy$$

Let $f(y) = K_0(\lambda v\sqrt{\xi^2 + y^2})$, then

$$\sum_{-\infty}^{\infty} K_0\left(\lambda v\sqrt{\xi^2 + (y \pm 2na)^2}\right) = \frac{1}{a}\int_0^\infty K_0\left(\lambda v\sqrt{\xi^2 + y^2}\right)dy + \frac{2}{a}\sum_1^\infty \cos\frac{\pi ny}{a} \times \int_0^\infty K_0\left(\lambda v\sqrt{\xi^2 + y'^2}\right)\cos\frac{\pi ny'}{a}\,dy'$$

But, by virtue of the integral representation of Bessel function (10)

$$\int_0^\infty K_0\left(\lambda v\sqrt{\xi^2 + y^2}\right)dy = \frac{1}{\lambda v}\int_0^\infty e^{-(\lambda v\xi)^2t^2 - 1/4t^2}\,t^{-1}dt \times \int_0^\infty e^{-(\lambda vy)^2t^2}\,d(\lambda vy) = \frac{1}{\lambda v}\frac{\sqrt{\pi}}{2}\int_0^\infty e^{-(\lambda v\xi)^2t^2 - 1/4t^2}\,t^{-2}dt$$

$$= \frac{\pi}{2\lambda v} e^{-\lambda v|\xi|}$$

[11] Reference (9), p. 159.

Likewise

$$\int_0^\infty K_0\left(\lambda v\sqrt{\xi^2 + y'^2}\right)\cos\frac{\pi ny'}{a}\,dy' = \frac{1}{\lambda v}\int_0^\infty e^{-(\lambda v\xi)^2t^2 - 1/4t^2}\,t^{-1}dt \times \int_0^\infty e^{-(\lambda vy')^2t^2}\cos\left(\frac{\pi n}{\lambda va}\lambda vy'\right)d(\lambda vy')$$

$$= \frac{\sqrt{\pi}}{2\lambda v}\int_0^\infty e^{-(\lambda v\xi)^2 - \mu_n^2/4t^2}\,t^{-2}dt = \frac{\pi}{2\lambda v\mu_n} e^{-\lambda v\mu_n|\xi|}$$

where

$$\mu_n = \sqrt{1 + \left(\frac{\pi n}{\lambda va}\right)^2}$$

hence recalling that

$$2\lambda = \frac{c\rho}{k}$$

we obtain expression, Equation [48] or [49], according to whether ξ is positive or negative.

Discussion

R. H. Cameron.[12] This paper is of considerable interest to the writer as a mathematician, because it shows that data of important practical significance can be calculated by the use of higher mathematics and higher mathematical techniques. While undoubtedly many of the author's conclusions have been or will be checked directly by experiment, his methods make it possible quickly to calculate data and plot curves which could be obtained directly only by a very large number of time-consuming and costly experiments. His theory of quasi-stationary states simplifies many practical problems to the point where calculation is possible and provides a method of attack on other problems which have not yet been put on a mathematical basis.

The author's technique changes a problem involving moving heat sources into one involving only fixed sources or singularities, and thus extends the classical methods of treating fixed sources to the more difficult problems of moving sources. These methods apply to such apparently diverse problems as arc welding, the rate of extrusion of a continuous casting, continuous quenching, and the heating effects of the passage of a bullet in a gun barrel.

Finally, his methods can be used in connection with experimental work to suggest the design of experiments, the direction they should take, and the interpretation of the results. His method for determining the values of diffusivity and heat-dissipation ratio is an example in point.

M. W. Rubesin[13] and R. C. Martinelli.[14] The author has presented an extremly interesting and useful contribution to the field of heat conduction. It is interesting to note that, due to the quasi-stationary state existing about the moving source, the method of relaxation can be readily applied to the problem.

A heat balance is made on a lattice of sides δ and depth $\delta/2$ situated directly below the moving source and moving with the source at a velocity v. Consideration is taken of the heat generated by the source, the heat conducted into the lattice by conduction, and the heat carried into the lattice by the movement of material through the lattice. Heat balances are made on other lattices surrounding the one about the source and a set of relaxation patterns established. Calculations show that the results of this numerical method check reasonably with the analytical results of the author.

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Heat losses from the surface can also be accounted for by the method of relaxation.

Author's Closure

Professor Cameron's comments coming from a mathematician are very much appreciated. The technical man neither ignores nor belittles the value of the mathematics, but the thing he is primarily interested in is the final numerical value of the solution and not the way in which the latter has been obtained. Unfortunately, many a mathematician does not appreciate this point of view as fully as Professor Cameron does. They are inclined to consider the ultimate step of the analysis which consists of putting the solution in a form suitable for numerical computation as some sort of "puka" mathematics to be left to the computing machines. While the latter certainly are tremendous time savers, they hardly afford as complete a view of the problem as a mathematical analysis does, especially when it comes to the consideration of singularities. Some of the problems of heat flow require the use of the highest mathematical techniques, and it is only through the co-operation of mathematicians who are skilled in those techniques that a substantial progress in the theory of heat flow can be made.

The emphasis on the mathematical analysis does not preclude the use of numerical methods for particular applications. From this point of view the remarks made by Mr. Rubesin and Professor Martinelli are most welcome. The author is not enough familiar with the relaxation method to see the advantages that it offers in the treatment of a general problem of heat flow, like the one developed in the present paper, but he is aware of the possibilities which it affords in solving particular problems with specific boundary conditions. For example, he would be very much interested to see the discussers treat the following two problems which are of importance in arc welding:

1 Heat flow due to a moving plane source of a circular shape, and

2 Heat flow in a plate heated by a moving point source on one face and cooled by liquid medium on the other face.

The solution of the first problem may give a better insight into the phenomena of temperature distribution in the molten pool, and the second is of especial interest in ship-repair welding.

In closing the author wishes to thank all discussers for their kind interest in his paper.

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