Stability of Quantized Chiral Soliton Coupled with the $\rho$ Meson

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Stability of the chiral soliton coupled with the $\rho$ meson which is quantized by taking breathing modes into account in addition to the spin-isospin rotational modes is examined on the basis of families of trial functions. It is shown that the parity-conserving states are not the ones with minimum energies and the solitons collapse through the spontaneous parity-violating states. The decay paths are represented by a parameter of trial profile functions.

§1. Introduction

It had been suggested a possibility that solitons of the simplest chiral Lagrangian which has only the kinetic term of chiral field may be stabilized if quantum effects of breathing mode are taken into account even when the Skyrme term is absent.\textsuperscript{1,2)} However, it was shown that there exists explicit paths of collapse of the chiral soliton in the absence of the Skyrme term.\textsuperscript{3)} The decaying paths are represented by a family of trial functions for the profile function $F(\rho; R(t))$ of the hedgehog ansatz for the matrix-valued chiral field $U(x) \in SU(2)$,

\begin{equation}
U(x) = A(t) \exp[i(\tau \cdot \hat{x})F(\rho; R(t))]A^\dagger(t),
\end{equation}

where $A(t) \in SU(2)$ is a dynamical variable of spin-isospin rotation and $R(t)$ is that of the breathing motion. By introducing a dimensionless variable $\rho = \tau/R(t)$ a family of trial functions is expressed as

\begin{equation}
F(\rho; C) = \frac{\pi}{N} \int_0^\infty \frac{dx}{(x^2+1)\sqrt{x^2+C}},
\end{equation}

where

\begin{equation}
N = \int_0^\infty \frac{dx}{(x^2+1)\sqrt{x^2+C}}.
\end{equation}

Explicit integrated expressions of $F(\rho; C)$ are given in Ref. 3). In the limit $C \rightarrow 0$, $F(\rho; C)$ tends to the collapsed one,

\begin{equation}
F(\rho; 0) = \begin{cases} 
\pi & \text{for } \rho = 0, \\
0 & \text{for } \rho \neq 0,
\end{cases}
\end{equation}

while in the limit $C \rightarrow \infty$ it tends to

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\[ F(\rho; \infty) = \pi \left( 1 - \frac{2}{\pi} \arctan \rho \right) \]  

which behaves as \( 2/\rho \) for large \( \rho \).

This family of trial functions is a generalization of the trial functions

\[ F(\rho; 1) = \pi \left( 1 - \frac{\rho}{\sqrt{\rho^2 + 1}} \right) \]

which is derived from the instanton solution by Atiyah and Manton.\(^4\) This form is the nearest trial function to the numerical solution among the family \( F(\rho; C) \). It gives static energy only 0.9% above the numerical results.\(^5\) The case with \( C=2 \) gives a trial function of arccosine form introduced by Igarashi, Otsu and two of the present authors.\(^7\)

By solving the Schrödinger equation for the wave function \( \phi(R) \) of the breathing motion it is shown in Ref. 3) that energies of chiral solitons without the Skyrme term tend to zero in both limits \( C \to 0 \) and \( C \to \infty \) for all spin-isospin states. As the parameter \( C \) goes to 0 corresponding to the collapsed profile function (3), both inertias of spin-isospin rotation and breathing motion tend to 0 as well as the coefficient of potential of breathing motion, so that energies of soliton-like systems go to zero. On the other hand in the limit \( C \to \infty \) both of the inertias of rotational and breathing motions tend to infinity, so that the rotational and the breathing motions are frozen and the system falls into the state which rests at the potential minimum point \( R(t) = 0 \), i.e., the collapsed one at the origin with zero energy.

The stability of chiral soliton coupled with the \( \rho \) meson has been discussed in classical level by many authors.\(^8\) It has been pointed out that though the kinetic term of \( \rho \) meson gives contribution to the static energy functional similar to that of the Skyrme term, the solutions are not stable because the spontaneous parity-violation (SPV) occurs if no additional stabilizer is included in the Lagrangian. Although it has been shown that the separate contributions from the kinetic term of the \( \rho \) meson and the quantized breathing motion are weak to prevent the collapse of solitons\(^3\),\(^7\) it is interesting to study the stability of solitons when both contributions coexist together. In this paper, we examine the stability of chiral soliton coupled with the \( \rho \) meson which is quantized by taking breathing modes into account in addition to the spin-isospin rotation.

As we will show in the following the chiral soliton coupled with the \( \rho \) meson is also instable even the quantized breathing motion is taking into account. Besides a family of trial functions \( F(\rho; C) \) for the pion field we use families of trial functions for the \( \rho \) meson profile functions which are related to \( F(\rho; C) \) in terms of two parameters, one is related to the breathing motion of \( \rho \) meson field and the other represents the amount of SPV contributions. The chiral solitons decay into the zero energy states through the SPV states in both limits \( C \to 0 \) and \( C \to \infty \). It is similar to the cases without the Skyrme term in Ref. 3).

In § 2 Lagrangian and the Schrödinger equation for the breathing mode of the chiral soliton coupled with the \( \rho \) meson are explicitly given. Based on the families of trial functions we solve the Schrödinger equation for the breathing mode and
examine the effects of rotational and breathing modes for the stability of the soliton in § 3. The final section is devoted to concluding remarks.

§ 2. Schrödinger equation of breathing mode

According to the formulation based on the hidden local symmetry in the nonlinear \( \sigma \) model on a manifold \( G/H(G=SU(2)_L \times SU(2)_R; H=SU(2)_V) \),\(^{10} \) we consider an effective Lagrangian for the pion and the \( \rho \) meson without any additional stabilizers,

\[
\mathcal{L} = \mathcal{L}_A + a \mathcal{L}_\nu + \mathcal{L}_\rho ,
\]

where

\[
\mathcal{L}_A = -\frac{1}{4} f_\pi^2 \text{Tr}(\partial_\mu \xi_L^* \xi_L^\dagger - \partial_\mu \xi_R^* \xi_R^\dagger)^2 ,
\]

\[
\mathcal{L}_\nu = -\frac{1}{4} f_\pi^2 \text{Tr}(D_\mu \xi_L^* \xi_L^\dagger + D_\mu \xi_R^* \xi_R^\dagger)^2 ,
\]

\[
\mathcal{L}_\rho = -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu}^2) ,
\]

We neglect the pion mass term. Matrix valued fields \( \xi_L(x), \xi_R(x) \) and \( U(x) = \xi_L^* \xi_R = \text{exp}[2i\pi(x)/f_\pi] \) are the elements of \( SU(2) \) group where \( \pi(x) = \pi^a(x) r^a/2 \) describes the pion fields. The gauge fields of hidden local symmetry \( V_\mu(x) = g V_\rho(x) r^a/2 \) are assigned to the \( \rho \) meson fields. In (6) \( f_\pi = 93 \text{ MeV} \) is the pion decay constant. A free parameter \( a \) in front of \( \mathcal{L}_\nu \) is fixed to \( a = 2 \) which realizes the \( \rho \) universality and the KSRF relation.\(^{11} \) Then the gauge coupling constant \( g = g_{\pi \pi} = 5.9 \) is determined by the KSRF relation \( 2g^2 f_\pi^2 = m_\rho^2 \).

We take the unitary gauge for \( \xi_L(x), \)

\[
\xi_L^*(x) = \xi_R^*(x) = \xi(x) ,
\]

\[
U(x) = [\xi(x)]^2 .
\]

For the pion field, we choose the same form as (1) for \( U(x) \) or for \( \xi(x) \)

\[
\xi(x) = A(t) \text{exp}[i(r \cdot \hat{x}) F(r; R(t))/2] A^*(t) .
\]
\( V_0(x) = A(t) \left[ \left[ -K \cdot \dot{\tau} + (\ddot{x} \cdot K)(\dddot{x} \cdot \tau) \right] G(\eta) + \left[ (\dddot{x} \times K) \cdot \tau \right] H(\eta) \right] \)

\[ + \left[ \frac{\dot{B}}{2B}(\dddot{x} \cdot \tau) \right] K(\eta) \right] A^t(t), \]

\( V_1(x) = A(t) \left[ \left[ (\dddot{x} \times \tau) \right] G(\eta)/2r + \left[ \tau_i - \dddot{x}_i (\dddot{x} \cdot \tau) \right] H(\eta)/2r \right] \)

\[ + \left[ \dddot{x}_i (\dddot{x} \cdot \tau) \right] K(\eta)/2r \right] A^t(t), \ (i=1, 2, 3) \quad (10) \]

where we define \( iK \cdot \tau = A^t A \) and \( \eta = \gamma/B(t) \), the dot being time derivative. Expressions for \( V_\mu(x) \) given by (10) are obtained from the sums of vector and axial vector currents of chiral field, \(-i\partial_\mu \xi^* (x) \xi(x)\), through the replacement of \( R(t) \), \( 1 - \cos F(\rho) \), \( \sin F(\rho) \) and \( \rho F'(\rho) \) by \( B(t) \), \( G(\eta) \), \( H(\eta) \) and \( K(\eta) \), respectively.\(^*\) By substitution of (9) and (10) into (6) we obtain the Lagrangian for the breathing and rotational motions,

\[ \mathcal{L} = \mathcal{L}_A + 2 \mathcal{L}_V + \mathcal{L}_p, \quad (11) \]

\[ \mathcal{L}_A = \frac{1}{2} f^2 \left[ r^2 F^2(\rho) \frac{\dot{B}^2}{B^2} - \left[ F^2(\rho) + 2 \sin^2 F(\rho)/r^2 \right] \right] \]

\[ + 2 \sin^2 F(\rho)(1 - \cos^2 \theta) \text{Tr}(\dot{A} \dot{A}^t) \right] \]

\[ \mathcal{L}_V = \frac{1}{2} f^2 \left[ K^2(\eta) \frac{\dot{B}^2}{B^2} - 2 [G(\eta) + \cos F(\rho)]^2 + 2 H^2(\eta) + K^2(\eta) \right]/r^2 \]

\[ + 2 [\overline{G}(\eta) + \cos F(\rho)]^2 + H^2(\eta)(1 - \cos^2 \theta) \text{Tr}(\dot{A} \dot{A}^t) \right] \]

\[ \mathcal{L}_p = -\frac{1}{g^2 r^2} \left[ [r \overline{G}^\prime(\eta) - H(\eta)K(\eta)]^2 + [r H^\prime(\eta) + \overline{G}(\eta)K(\eta)]^2 \right] \frac{\dot{B}^3}{B^3} \]

\[ - [\overline{G}(\eta) + H^2(\eta) - 1]^2/2r^2 + [\overline{G}(\eta) - H(\eta)K(\eta)/r]^2 \]

\[ + [H^\prime(\eta) + \overline{G}(\eta)K(\eta)/r]^2 \]

\[ + [r^2 [\overline{G}(\eta) - H(\eta)K(\eta)/r]^2 + r^2 [H^\prime(\eta) + \overline{G}(\eta)K(\eta)/r]^2 \]

\[ + [G^2(\eta) + H^2(\eta) - 1]^2](1 - \cos^2 \theta) \text{Tr}(\dot{A} \dot{A}^t) \right] \]

where \( \overline{G}(\eta) = G(\eta) - 1, \cos^2 \theta = (\dddot{x} \cdot K)^2/(K)^2 \) and the prime denotes differentiation with respect to \( r \).

The profile function \( K(\eta) \) is not an independent variable.\(^7,12\) The Euler-Lagrange equation for \( K(\eta) \) leads to

\[ K(\eta) = -\frac{\eta [G^\prime(\eta)H(\eta) - H^\prime(\eta)(G(\eta) - 1)]}{g^2 f^2 r^2/2 + [(G(\eta) - 1)^2 + H^2(\eta)]^2}, \quad (12) \]

\(^*\) For the most general expressions of the time component \( V_0(x) \), new functions \( G_0(\eta), H_0(\eta) \) and \( K_0(\eta) \) have to be used instead of those in (10). However, it is noted that the minimum energy eigenvalues are obtained by the same trial profile functions as \( G(\eta), H(\eta) \) and \( K(\eta) \) for the spatial components \( V_\mu(x) \).
where the prime denotes \( \eta \) differentiation.

Because of strong correlations between the chiral field and the \( \rho \) meson field which are imposed by the \( \mathcal{L}_\nu \) term, the breathing motions of both fields will strongly couple. Therefore we may assume that the main mode is a combined motion of these two breathing modes given by \( B(t) = aR(t) \) or \( \eta = \rho / \alpha \), where \( \alpha \) is a constant parameter and determined by minimizing the energy of soliton.

By taking constraints given by the terms \( \mathcal{L}_\nu \) and \( \mathcal{L}_\rho \) into account, we choose the following trial functions for the profile functions \( G(\eta) \) and \( H(\eta) \):

\[
G(\eta; C) = 1 - \cos F(\eta; C), \\
H(\eta; C, \beta) = \beta \sin F(\eta; C),
\]

where \( F(\eta; C) \) is given by replacing \( \rho \) by \( \eta \) in (2). By substitution of (13) into (12) we obtain the profile function \( K(\eta; C) \):

\[
K(\eta; C, \beta) = \beta \eta F'(\eta; C) q(\eta, \beta)
\]

with

\[
q(\eta, \beta) = \frac{1}{g^4 f^4 B^2 \eta^2 / 2 + [\cos^2 F(\eta; C) + \beta^2 \sin^2 F(\eta; C)]}.
\]

In (13) and (14) a free parameter \( \beta \) is introduced which will be determined by minimizing the energy of soliton. When \( \beta = 0 \), \( H(\eta; C) = K(\eta; C) = 0 \) corresponding to the parity conserved (PC) states. In this case the behavior of \( G(\eta; C) \) is almost the same with that of \( 1 - \cos F(\rho; C) \) and the term \( \mathcal{L}_\nu \) has very small contribution to energy. If \( \beta = 0 \) and \( \alpha = 1 \) (or \( \eta = \rho \)) the term \( \mathcal{L}_\nu \) vanishes and the \( \mathcal{L}_\rho \) term becomes the same with the Skyrme term. On the other hand, the case \( \beta \neq 0 \) corresponds to the parity violating states. If \( \beta = 1 \) and \( q(\eta, \beta) = 1 \) in (14), then \( K(\eta; C, \beta) = \eta F'(\eta; C) \).

Therefore the \( \rho \) meson field has the spherically symmetric pure gauge form \(^{(12)}\) which is given by \( V_\rho(x) = i\tau(x)\partial_x \chi(x)^{-1} \) with \( \chi(x) = A(t) \exp[i(\tau \cdot \vec{x})F(\eta; C)/2]A(t) \), so that the contributions of the \( \mathcal{L}_\rho \) term become zero.

From (11), by use of families of trial functions \( F(\rho; C), G(\eta; C) \) and \( H(\eta; C, \beta) \), we obtain the Lagrangian for the spin-isospin rotation and the combined breathing motion,

\[
L = a(R) \dot{R}^2 - V(R) + \lambda(R) \text{Tr}[\Delta \dot{A}],
\]

where \( a(R) \) and \( \lambda(R) \) are \( R \)-dependent inertia of the breathing motion and moment of inertia of spin-isospin rotation, respectively, and \( V(R) \) is potential of the breathing motion. Inertias \( a(R) \) and \( \lambda(R) \) are given by

\[
a(R) = a_1 R^2 + \frac{a_2}{R}, \\
\lambda(R) = \lambda_1 R^3 + \lambda_2 R,
\]

where
The potential $V(R)$ is given by

$$V(R) = b_1 R + \frac{b_2}{R}, \quad (18)$$

where

$$b_1 = 2\pi f^2 \int_0^\infty [\rho F'(\rho)^2 + 2\sin^2 F(\eta) + 4\cos F(\eta) - \cos F(\rho)]^2 \rho d\rho,$$

$$b_2 = \frac{4\pi\rho^2}{g^2} \int_0^\infty \left[ (1 - \beta^2 q)^2 \sin^2 F(\eta) + \beta^2 (1 - q)^2 \cos^2 F(\eta) \right] d\rho,$$

In (17) and (18) the primes of $F'(\rho)$ and $F'(\eta)$ denote the differentiation with respect to $\rho$ and $\eta$, respectively. In order to obtain (17) and (18), we have made an approximation for $q(\eta, \beta)$ in (15) where we use the mean value $\overline{B}$ instead of the breathing variable $B(\eta)$ in the first term of the denominator of $q(\eta, \beta)$. In the following calculation we fix the mean value $\overline{B}$ to 1 GeV$^{-1}$ which is reasonable as we discuss later when the parameter $C$ is not far from one.

The Lagrangian (16) with collective coordinates $R(t)$ and $A(t)$ has three parameters $C$, $a$ and $\beta$. The static classical solitons are the solutions which make $V(R)$ stationary. The minimum value of $V(R)$ is the classical mass of chiral soliton and a functional of $F(\rho)$, $G(\eta)$ and $H(\eta)$ and given by

$$M[F, G, H] = 2\sqrt{b_1 b_2} \quad (19)$$

However, now in terms of trial functions, the functional $M[F, G, H]$ is reduced to a function of three parameters $C$, $a$ and $\beta$.

The Schrödinger equation of wave function $\psi_f(R)$ for breathing mode with spin-isospin $I=J$ is

$$\left[ -\frac{1}{4\alpha(R)} \left\{ \frac{d^2}{dR^2} + h(R) \frac{d}{dR} \right\} + V(R) + \frac{J(J+1)}{2\lambda(R)} - E_f \right] \psi_f(R) = 0, \quad (20)$$

where

$$\alpha(R) = \frac{4\pi f^2}{g^2}, \quad h(R) = \frac{b_1}{R} + \frac{b_2}{R^2}, \quad V(R) = \frac{b_2}{R}.$$
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$$h(R) = \frac{3\lambda(R)}{2\lambda(R)} - \frac{a(R)'}{2a(R)}$$

and the prime implies differentiation with respect to $R$.

§ 3. SPV and PC solutions and instability

We solve numerically the Schrödinger equation (20) for $I=J=1/2$ and vary the parameters of the trial functions, $C$, $a$, and $\beta$.

As the first step, varying $C$ and $\beta$ and fixing the parameter $a$ to a value near 1.0, we solve Eq. (20) for $I=J=1/2$. It is found that when $C$ is near 1, the parameter $\beta$ which minimizes the energy eigenvalue $E_{1/2}$ is zero. This corresponds to the PC solution. On the other hand, in the regions where $C<1$ and $C \gg 1$, the energy eigenvalue $E_{1/2}$ is minimized when the parameter $\beta$ takes $\beta \approx \pm 1.0$ and $\beta \approx \pm 0.9$, respectively. Then we obtain the SPV solution in both small and large $C$ regions.

As the next step, based on the

Fig. 1. $a$ dependence of energy eigenvalue $E_{1/2}$ for several values of $C$ in the PC case and the SPV case where $\beta$ has been chosen to minimize the $E_{1/2}$. The PC case is shown by the broken line and the SPV cases are by solid lines.

Fig. 2. $\beta$ dependence of energy eigenvalue $E_{1/2}$ for several values of $C$ in the PC case and the SPV case where $a$ has been chosen to minimize the $E_{1/2}$. The PC case is shown by the broken line and the SPV cases are by solid lines.

Fig. 3. $C$ dependence of energy $E_J$ for the eigenstates with $I=J=0$ (broken line), 1/2 and 3/2 (solid lines). The PC states which are not the minimum energy states are shown by dotted lines.
results of the first step we fix values of parameter \( \beta \) as \( \beta = 0, \beta = 1.0 \) and \( \beta = 0.9 \) in the regions where \( C \) is close to 1, \( C < 1 \) and \( C > 1 \), respectively, and solve Eq. (20) for \( I = J = 1/2 \) varying \( C \) and \( \alpha \). Figure 1 shows that the \( \alpha \) dependence of energy \( E_{1/2} \) for typical values of \( C \) in both PC and SPV cases. Here we note that in the region where \( C \) is near to 1 the value of \( \alpha \) which gives smaller energy eigenvalue of the PC state lies around 1.3. On the other hand values of \( \alpha \) become much smaller than 1 in the region with \( C < 1 \) and decrease as \( C \to 0 \).

As the final step, we check again the \( \beta \) dependence of energy eigenvalue \( E_{1/2} \) by use of \( \alpha \) values which are chosen to make the energy minimum. The \( \beta \) dependence of energy \( E_{1/2} \) for typical values of \( C \) is shown in Fig. 2. As is shown in Fig. 2 the results obtained in the first step are confirmed, that is, when \( C \) is near 1, the minimal point is located at \( \beta = 0 \); while \( C \) is far from 1, the minimal point of energy is located near \( \beta = \pm 1.0 \).

In the same way with \( I = J = 1/2 \), we determine the values of free parameters \( \alpha \) and \( \beta \) for the states with \( I = J = 0 \) and 3/2 by minimizing the eigenvalue \( E_0 \) and \( E_{3/2} \) for varying \( C \). The calculated results of \( C \) dependence of energy \( E_i \) for \( I = J = 0, 1/2 \) and 3/2 are shown in Fig. 3. As shown in Fig. 3 the energies of the PC states in the region of \( C \) near one are rather large so that these PC states are not stable. As the parameter \( C \) decreases from 1 the energies of PC states increase after slight decrease reflecting the increasing contributions from the kinetic energies of the \( \rho \) meson and breathing mode. However there occur "phase transitions"\(^7,12\) from the PC states to the SPV states at around \( C \approx 10^{-3} \) and the energies of these SPV states decrease towards zero. The critical values of parameter \( C, C_s \), are 0.005, 0.002 and 0.0004, for \( I = J = 0, 1/2 \) and 3/2, respectively.

Fig. 4. Trial profile functions \( F(\rho; C) \), \( G(\rho; \alpha; C) \) and \( H(\rho; \alpha; C, \beta) \) for several values of \( C \) where \( \alpha \) and \( \beta \) have been chosen to minimize the energy \( E_{1/2} \). \( F(\rho; C) \) are shown by solid lines, \( G(\rho; \alpha; C) \) by broken lines, \( H(\rho; \alpha; C, \beta) \) by dotted lines.

Fig. 5. The \( C \) dependence of the parameters of Lagrangian. The pion decay constant and the gauge coupling constant are taken as \( f_\pi = 93 \) MeV and \( g = 5.9 \). The parameters \( \alpha \) and \( \beta \) are chosen to minimize the energy eigenvalue \( E_{1/2} \). \( \alpha_1 \) and \( \alpha_2 \) are shown by broken lines, \( \lambda_1 \) and \( \lambda_2 \) by dotted lines and \( b_1 \) and \( b_2 \) by solid lines.
As the parameter $C$ increases from 1 the energies of PC states decrease monotonically and tend to a limiting value $E_{1/2}^c=1.122\,\text{GeV}$ in the limit $C\to\infty$. However, “phase transitions” from the PC states to the SPV states also occur at the critical values of parameter $C$, i.e., at $C_0=200$, 500 and 2000, for $I=J=0$, 1/2 and 3/2, respectively. The energies of these SPV states in the region $C\gg 1$ rapidly decrease towards zero as $C$ increases. The trial functions $F(p; C)$, $G(p/\alpha; C)$ and $H(p/\alpha; C, \beta)$ for several values of $C$ are shown in Fig. 4 where the values of parameters $\alpha$ and $\beta$ are chosen to minimize the energy eigenvalue $E_{1/2}$.

The parameters of the Lagrangian $a_1$, $a_2$, $b_1$, $b_2$, $\lambda_1$ and $\lambda_2$ which are used to obtain minimum eigenvalue $E_{1/2}$ are shown in Fig. 5. The $C$ dependence of parameters of the Lagrangian which give minimum eigenvalues of the states $I=J=0$ and 3/2, $E_0$ and $E_{3/2}$, are almost the same with those of the $I=J=1/2$ state shown in Fig. 5. As shown in Fig. 5, all parameters $a_1$, $a_2$, $b_1$, $b_2$, $\lambda_1$ and $\lambda_2$ decrease as the parameter $C$ decrease in the SPV states of small $C$. On the other hand, except for $b_2$ all parameters of Lagrangian increase as $C$ increase in the SPV states of large $C$.

The results obtained so far are based on the approximation where we fix the dynamical variable $B(t)$ in $q(\eta, \beta)$ at the mean value $\bar{B}=1\,\text{GeV}^{-1}$. This approximation is valid in the region where the parameter $C$ lies near one. In the limit $C\to 0$, the mean value $\bar{B}$ becomes very large because the inertia of breathing motion $a(R)$ and the moment of inertia of rotational mode $\lambda(R)$ become very small. In this region the solitons are in the SPV states with $\beta=\pm 1$, so that $q(\eta, \beta)$ reduces to

$$q(\eta, \pm 1) = \frac{1}{g^2 f^2 \bar{B}^2 \eta^2/2 + 1}.$$  \hspace{1cm} (21)

Analytical calculation by use of this $q(\eta, \pm 1)$ shows that all the parameters of Lagrangian in the SPV states tend to zero as $1/\ln^2 C$ in the limit $C\to 0$. Then the energy of SPV state tends to zero. Therefore the SPV state represents the explicit path of collapse of chiral soliton coupled with the $\rho$ meson.

On the other hand, when the parameter $C$ goes to the limit $C\to\infty$, $\bar{B}$ becomes very small because $a(R)$ and $\lambda(R)$ become very large, the approximation for $q(\eta, \beta)$ is not applicable also in the region $C\to\infty$. In this region the solitons in the SPV states with $\beta=\pm 1$ have smaller energies. Then we can use $q(\eta, \pm 1)$ given by (21) and obtain analytical expression of $V(R)$ in the limit $C\to\infty$, which behaves as $26\pi^2 f^2 R + \sqrt{2} \times \pi^2 g f^2 R^2$ in the small $R$ region, so that $V(R)$ is zero at $R=0$. In the limit $C\to\infty$ both inertias $a(R)$ and $\lambda(R)$ go to infinity and the rotational and breathing motions are frozen \cite{3,6} and the systems fall into the state which rest at the potential minimum point $R(t)=0$, i.e., the collapsed one at the origin with zero energy. Therefore the SPV state represents the explicit decaying path of the chiral soliton coupled with the $\rho$ meson in the other way of $C\to\infty$.

If we take $a=1$ in the case of PC solutions, the contribution of $\mathcal{L}_V$ term to the energy completely disappears as seen from $\mathcal{L}_V$ in (11). Furthermore, the $\mathcal{L}_\rho$ term where the substitution of $\tilde{G}(\rho) = -\cos F(\rho)$ is made gives the same contributions with that of the Skyrme term with $e=g=5.9$. Therefore the PC states are similar to those of quantized chiral soliton with a weak Skyrme term where all states go to the frozen state. \cite{6}
§ 4. Discussion and conclusions

In the preceding section we have examined the stability of chiral soliton coupled with the $\rho$ meson and quantized by taking into account the spin-isospin rotational and breathing modes in terms of three families of trial functions: $F(\rho; C)$ for the profile function of chiral field and $G(\eta; C)$ and $H(\eta; C, \beta)$ for the $\rho$ meson field. These families of trial function have a common parameter $C$, which represents the collapsed profile functions in the limit $C \to 0$ while the spreaded one in the limit $C \to \infty$. Trial functions $G(\eta; C)$ and $H(\eta; C, \beta)$ given by (13) correspond to the parity odd and parity even part of the $\rho$ meson field, respectively. The parameter $\beta$ in $H(\eta; C, \beta)$ represents the amount of parity violating contributions. The dynamical variables $R(t)$ and $B(t)$ of the breathing modes introduced through dimensionless variable $\rho = r/R(t)$ and $\eta = r/B(t)$ are assumed to make combined motion with $B(t) = aR(t)$ where $a$ is a free parameter. Varying these parameters $C$, $a$ and $\beta$, and solving the Schrödinger equation for the breathing motion we obtain the energy eigenvalue of the quantized chiral soliton as the function of these three parameters. As explicitly shown in the preceding section, the chiral soliton coupled with the $\rho$ meson is unstable even though the quantum effects of breathing mode are taking into account. The explicit decaying paths of the soliton are given by SPV states with $\beta \approx \pm 1.0$ in both limits $C \to 0$ and $C \to \infty$.

Though it might be expected that the coexistence of the kinetic term of the $\rho$ meson field and the breathing and rotational modes prevent the collapse of soliton, there appear the SPV states with almost pure gauge solution which has zero contribution from the kinetic term $\mathcal{L}_\rho$ in the limits $C \to 0$ and $C \to \infty$. Although both limits $C \to 0$ and $C \to \infty$ give the decaying paths of solitons, the mechanism of collapse is very different from each other. In the limit $C \to 0$, all the profile functions become collapsed one and all the parameters of Lagrangian (16) tend to zero. Then both energy and size of soliton go to zero. On the other hand, in the limit $C \to \infty$, the solitons fall into the frozen state at the origin, so that the size and energy go to zero.

Though the quantum effects of breathing mode are included, the situation concerning stability of soliton coupled with the $\rho$ meson does not change essentially from that in classical case. In order to stabilize the chiral soliton coupled with the $\rho$ meson it will be required to add either of the higher-derivative terms, such as the Skyrme term or the coupling with the $\omega$ meson to the Lagrangian (6).

Finally, we conclude the assumption of the combined motion of two breathing modes of the chiral field and the $\rho$ meson field, $B(t) = aR(t)$. By this collective motion we get rather small excitation energy of chiral soliton system so that this combined breathing mode gives good approximation for the description of the quantum effects of chiral soliton coupled with the $\rho$ meson.

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