Floodplain hydraulics in a glaciated bedrock river valley

David W. Ostendorf, Erich S. Hinlein and Aaron I. Judge

ABSTRACT

Data and one dimensional, unsteady theory document the average, seasonal, and diurnal hydraulics of an unconfined aquifer in the surficial floodplain deposit of the Neponset River, which flows through a glaciated bedrock valley in eastern Massachusetts. The 20 m thick silty sand deposit has a permeability of $1.4 \times 10^{-11}$ m$^2$, a porosity of 0.37, a 600 m halfwidth, and an infiltration coefficient of 0.39. The steady water table is parabolic with a 0.60 m value at the valley wall that implies an average 33-year travel time across the floodplain in response to an average recharge rate of $7.0 \times 10^{-7}$ m/s. Seasonal hydraulics are governed by the floodplain porosity and marked by periodicity of the river (0.48 m amplitude) and recharge (1.9 $\times$ $10^{-8}$ m/s amplitude), which maintain flow from the floodplain into the river year round. Attenuation of the diurnal fluctuations suggests that the specific yield ranges from 0.05 to 0.14, and yields horizontal flow excursions of 1 m scale near the riverbank.

Key words | floodplains, groundwater, hyporheic zone, stratified drift, unconfined aquifers

NOMENCLATURE

$A$ amplitude of seasonal river stage fluctuation (m)  
$A_W$ watershed area of Neponset River (m$^2$)  
$b$ unconfined aquifer thickness (m)  
$C$ infiltration coefficient  
$E$ amplitude of seasonal recharge fluctuation (m/s)  
$F_A$ average evapotranspiration rate (m/s)  
$F_C$ characteristic evapotranspiration rate (m/s)  
$f_S$ spatial dependence of seasonal partition (m)  
$g$ gravitational acceleration (m/s$^2$)  
$k$ permeability of floodplain deposit (m$^2$)  
$L$ halfwidth of floodplain (m)  
$n$ porosity  
$P$ precipitation intensity (m/s)  
$P_A$ average precipitation intensity (m/s)  
$P*$ Laplace transform of precipitation intensity (m)  
$Q_A$ average discharge of Neponset River (m$^3$/s)  
$Q_D$ discharge withdrawn from Neponset River by upstream users (m$^3$/s)  
$r_{1,2,3}$ moduli of hyperbolic functions  
$S_V$ specific yield  
$s$ Laplace transform variable (s$^{-1}$)  
$T_A$ average travel time across the floodplain (s)  
$T_C$ characteristic pressure transient response time of the floodplain (s)  
$t$ time (s)  
$U$ seasonally constant rate of river stage change (m/s)  
$u$ seepage velocity (m/s)  
$u_A$ average seepage velocity (m/s)  
$u_{AO}$ average seepage velocity at the riverbank (m/s)  
$u_{DO}$ diurnal seepage velocity at the riverbank (m/s)  
$u_{SO}$ seasonal seepage velocity at the riverbank (m/s)  
$x$ horizontal distance from the riverbank (m)  
$x_{DO}$ diurnal excursion length at the riverbank (m)  
$x_{FO}$ diurnal excursion length fluctuation at the riverbank (m)  
$\phi$ seasonal phase shift between recharge and river stage (rad)  
$\epsilon$ recharge (m/s)  
$\epsilon_A$ average recharge (m/s)  
$\epsilon_D$ diurnal recharge fluctuation (m/s)  
$\epsilon_S$ seasonal recharge fluctuation (m/s)  
$\lambda$ linear reservoir decay constant (s$^{-1}$)  
$\eta$ water table elevation (m)  

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Floodplains couple surface and subsurface hydrology, and are the focus of interdisciplinary study as a result. Classical baseflow recession curves (Chow et al. 1988) and the attenuation of river floods in natural (Cooper & Rorabaugh 1965) and urban (Turner-Gillespie et al. 2003) settings all reflect a surface water emphasis of the interaction, as do pure (Karwan & Saiers 2009) and contaminated (Wyzga & Ciszewski 2010) sediment transport and deposition on floodplains. Hourly and diurnal timescales and 1–10 m length scales in the hyporheic zone characterize the flood hydrograph analyses, while baseflow decay constants are seasonal in scale, and implicitly model flow over the entire width of the floodplain.

Unconfined aquifers operate at longer timescales than the rivers at their boundaries, since hydraulic diffusion limits the penetration of short lived disturbances into the soil (Eagleson 1970). Groundwater models are well established, and range in complexity from the linear reservoir of Gelhar & Wilson (1974), through the closed form, one dimensional diffusion analyses of Cooper & Rorabaugh (1965) and Pinder et al. (1969), and the closed form, two dimensional diffusion analyses of Moench & Barlow (2000) and Malama & Johnson (2010), to numerical solutions that incorporate heterogeneity and two dimensional flow near the streambed (Mehl & Hill 2010). The linear reservoirs ignore spatial dependence, diffusive solutions idealize spatial dependence, while numerical models incorporate heterogeneity and anisotropy, where justified by a spatially resolved database that may include pump tests (Rotting et al. 2006) or other local estimates of aquifer properties. River (Singh & Sagar 1977) or tidal (Alcolea et al. 2007) stage, and infiltration or evapotranspiration (Shah & Ross 2009) introduce transience to a hydraulically connected aquifer (Vazquez-Sune et al. 2007) as known boundary and recharge conditions over varying timescales. Of the many other groundwater papers that could be cited, Boutt & Fleming (2009) present a study particularly relevant to the present investigation due to similarities of geologic setting. These authors use a numerical model and piezometers at a 10 m length scale to assess the response of stratified drift deposits to daily regulation of the Deerfield River stage in western Massachusetts by operators of a downstream dam. The Deerfield flows through a glaciated bedrock valley, as does the Neponset River, which is considered here.

In the latter regard, the present investigation adds a well characterized case study of floodplain hydraulics in a stratified drift deposit to this established and continuing literature, with a subsurface focus and a cascade of steady, seasonal, and diurnal timescales. The floodplain deposit is the uppermost of three stratified drift deposits resulting from glaciation of a bedrock valley in eastern Massachusetts. Hydraulics of the aquitard (Ostendorf et al. 2010) and confined aquifer (Ostendorf & Kilbridge 2009) that underlie the floodplain deposit have been documented and analyzed separately. The present model rests on the assumptions of small disturbances, simple geometry, and unconfined aquifer homogeneity that distinguish the Cooper & Rorabaugh (1965) and Pinder et al. (1969) analyses, so that the analytical solutions are variants of the one dimensional diffusion equation (Carslaw & Jaeger 1973) across the three timescales. The steady and seasonal timescales add importance to recharge in the present study, and complement the hyporheic focus of Boutt & Fleming (2009) and Swamee & Singh (2005) on diurnal river stage effects. Arbitrary river stage is featured in this study through the convolution integral approach of Singh & Sagar (1977) and Moench & Barlow (2000), in contrast to the Fourier series analysis of Swamee & Singh (2003) and daily oscillations of Boutt & Fleming (2009). Stormscale recharge is modeled as a nested convolution integral, providing a simple linkage between surface and subsurface hydrology. Most importantly, data sets

\[ \eta_A \] average water table elevation (m)
\[ \eta_D \] diurnal water table fluctuation (m)
\[ \eta_l \] Laplace transform of the diurnal water table fluctuation (m-s)
\[ \eta_{River} \] diurnal river stage fluctuation (m)
\[ \eta_{River}^{\prime} \] Laplace transform of the diurnal river stage fluctuation (m-s)
\[ \eta_R \] river stage fluctuation (m)
\[ \eta_S \] seasonal water table fluctuation (m)
\[ \theta_{1,2,3} \] phases of hyperbolic functions
\[ \nu \] groundwater kinematic viscosity (m²/s)
\[ \omega \] annual frequency (rad/s)
calibrate and confirm the aquifer property values down the cascade. Other examples of the cascade calibration of glacial deposit properties can readily be cited: Ostendorf et al. (2004a) successfully calibrate subsurface hydraulics at steady, seasonal, and diurnal timescales in a till drumlin in eastern Massachusetts. Reynolds (1987) calibrates glacial outwash properties across seasonal and diurnal transience in upstate New York. Ostendorf et al. (2004b) calibrate hydraulic properties of glacial outwash material in southeastern Massachusetts at decadal, annual, and seasonal scales.

MODELS

Average hydraulics

The floodplain is assumed homogeneous and thick, with water table elevation \( \eta \) (above the average river stage) and recharge \( \varepsilon \) comprised of average values (\( A \) subscript) with seasonal (\( S \) subscript) and diurnal (\( D \) subscript) fluctuations:

\[
\eta = \eta_A(x) + \eta_S(x, t) + \eta_D(x, t) \quad (\eta < b)
\]

\[
\varepsilon = \varepsilon_A + \varepsilon_S(t) + \varepsilon_D(t)
\]

with time \( t \) and distance \( x \) from the riverbank. The water table elevation is much smaller than the unconfined aquifer thickness \( b \), which is approximated as the vertical distance between the flat bottom of the floodplain deposit and the average river stage, as suggested by Figure 1. The floodplain is presumed horizontal, and head refers to mean sea level and gage pressure. In the latter regard, barometric pressure fluctuations are assumed to apply synoptically and without attenuation to all locations in the floodplain deposits, and are ignored as a consequence. The water table transience reflects recharge \( \varepsilon \) unsteadiness as well as a varying river stage \( \eta_{\text{river}} \) at the \( x \) origin (Figure 1). The recharge is due to highway and ambient runoff, and also reflects evapotranspiration. The amount of recharge is spatially uniform. The floodplain is of width \( 2L \) and is bound by impermeable vertical walls, so that the boundary conditions for all the partitions are:

\[
\frac{\partial \eta}{\partial x} = 0 \quad (x = L)
\]

\[
\eta = \eta_{\text{river}} \quad (x = 0)
\]

Each partition is flat at the wall, as is their sum (Equation (2a)). The average floodplain hydraulics balance flux and steady recharge for a small disturbance:

\[
-\frac{kgb}{v}\frac{d^2 \eta_A}{dx^2} = \varepsilon_A
\]

\[
\eta_A = 0 \quad (x = 0)
\]

with soil permeability \( k \), groundwater kinematic viscosity \( v \), and gravitational acceleration \( g \). Equations (2a) and (3) imply a parabolic average water table partition:

\[
\eta_A = \frac{\varepsilon_A v x}{k gb} \left( L - \frac{x}{2} \right)
\]

Seasonal hydraulics

The seasonal hydraulics are assumed sinusoidal, with annual frequency \( \omega \) and a phase shift \( \varphi + \pi/2 \) between the
recharge and the river stage:

\[
\eta_s = \frac{\partial \eta_s}{\partial t} - \frac{kgb \, \partial^2 \eta_s}{\nu \, \partial x^2} = E \exp \left[ i \left( \omega t + \frac{\pi}{2} + \varphi \right) \right]
\]

(5a)

\[
\eta_s = A \exp \{i \omega t \} \quad (x = 0)
\]

(5b)

The porosity \( \eta \) of the floodplain deposit characterizes dewatering of the soil for the unconfined aquifer at seasonal timescales. The amplitudes of the recharge and river seasonal sinusoids are \( E \) and \( A \), respectively.

The variables separate and the solution is complex, with known temporal function and unknown spatial dependence \( \dot{f}_s(x) \). Floodplain deposit friction attenuates the seasonal transience from the recharge and the river:

\[
\eta_s = f_s(x) \exp \{i \omega t \}
\]

(6a)

\[
in \, \eta_s = \frac{kgb \, d^2 f_s}{\nu \, \partial x^2} = iE \exp \{i \varphi \}
\]

(6b)

\[
f_s = A \quad (x = 0)
\]

(6c)

\[
\frac{d f_s}{d x} = 0 \quad (x = L)
\]

(6d)

The spatial dependence is hyperbolic:

\[
f_s = \frac{E}{\eta_{so}} \exp \{i \varphi \} + \left[ A - \frac{E}{\eta_{so}} \exp \{i \varphi \} \right] \times \frac{\sinh \left( (1 + i) \frac{\eta_{so}}{\sqrt{2kgb}} x \right)}{\cosh \left( (1 + i) \frac{\eta_{so}}{\sqrt{2kgb}} \right)} - \tan h.
\]

\[
\times \left[ (1 + i) L \frac{\eta_{so}}{\sqrt{2kgb}} \sinh \left( (1 + i) \frac{\eta_{so}}{\sqrt{2kgb}} L \right) \right]
\]

(7)

The imaginary part of Equations (6a) and (7) is the seasonal solution,

\[
\eta_s = \frac{E}{\eta_{so}} \sin(\omega t + \varphi) + A[r_1 \sin(\omega t + \varphi) + \theta_1]
\]

\[
- r_2 r_3 \sin(\omega t + \theta_2 + \theta_3) - \frac{E}{\eta_{so}} [r_1 \sin(\omega t + \varphi + \theta_1)
\]

\[
- r_2 r_3 \sin(\omega t + \varphi + \theta_2 + \theta_3)]
\]

(8)

with moduli \( r_{1,2,3} \) and phase \( \theta_{1,2,3} \) of the hyperbolic functions \( \sinh \) and \( \cosh \), respectively. Floodplain deposit characterizes dewatering of the soil for the diurnal, respectively.

Diurnal hydraulics

The river stage and evapotranspiration and storm runoff disturb the floodplain hydraulics on a diurnal scale that is aperiodic:

\[
S_Y \frac{\partial \eta_D}{\partial t} - \frac{kgb \, \partial^2 \eta_D}{\partial x^2} = \epsilon_D(t)
\]

(10a)

\[
\eta_D = 0 \quad (t = 0)
\]

(10b)

\[
\eta_D = \eta_{D\text{river}}(t) \quad (x = 0)
\]

(10c)

with specific yield \( S_Y \) (Fetter 2001) of the floodplain deposit used to characterize dewatering of the soil for the diurnal scale.
timescale. The initial diurnal disturbance is zero and the diurnal river fluctuation \( \eta_{\text{River}}(t) \) is assumed known. The derivatives in Equation (10a) are estimated by algebraic equivalents in order to assess the time \( T_C \) for pressure transients to reach the floodplain boundary (Gelhar & Wilson 1974):

\[
\frac{S_V}{T_C} \approx \frac{kgb}{vE^2} \quad (T_C \gg 10^5 \text{ sec}) \quad (11a)
\]

\[
\frac{\partial \eta_{\text{D}}}{\partial x} = 0 \quad (x \to \infty) \quad (11b)
\]

Equation (11a) is assumed to be satisfied, so that the diurnal transience of the river does not reach the valley wall, and boundary condition Equation (2a) is replaced with the simpler Equation (11b).

The Laplace transform \( \hat{\eta}_{\text{D}} \) of Equations (10) becomes an ordinary differential equation subject to Equation (11b), with the resulting solution:

\[
\hat{\eta}_{\text{D}} = \eta_{\text{River}} \exp \left(-x \sqrt{\frac{S_V S_Y}{kgb}} \right) \quad (12)
\]

\[
+ \left[ 1 - \exp \left(-x \sqrt{\frac{S_V S_Y}{kgb}} \right) \right] \left( \frac{\epsilon_D}{S_Y} \right)
\]

The Laplace transform variable is \( s \) and transformed recharge and river partitions are \( \epsilon_D \) and \( \hat{\eta}_{\text{River}} \). A convolution integral inverts Equation (12) (Fodor 1965) with the result:

\[
\eta_{\text{D}} = \int_0^t \frac{d\eta_{\text{River}}}{d\tau} \text{erfc} \left[ \frac{x}{2 \sqrt{kgb} (t-\tau)} \right] d\tau
\]

\[
+ \frac{\epsilon_D(t)}{S_Y} \text{erf} \left[ \frac{x}{2 \sqrt{S_Y V} (t-\tau)} \right] d\tau \quad (15)
\]

Equations (1), (4), (8), and (13) superimpose the average, seasonal, and diurnal hydraulics of the floodplain when the river stage and recharge are specified across the cascade of timescales.

### A linear reservoir for diurnal runoff

The contribution of recharge to the diurnal convolution integral in Equation (13) is simple for a constant \( \epsilon_D(\tau) \). Thus, a characteristic evapotranspiration rate \( F_C \) simulates the interaction of seasonally constant (negative) recharge with the water table between storms. The response of the floodplain to the infiltration fraction \( C \) of precipitation of intensity \( P(\tau) \) is another matter, however, and a simple model is needed to relate rainfall input to recharge output. The focus of the present investigation on analytical groundwater models demands a simple and closed form account of the runoff and infiltration process, rather than a detailed numerical model of the catchment.

Accordingly, the local floodplain catchment is modeled as a linear reservoir that routes input \( CP - F_C \) with a decay constant \( \lambda \):

\[
\frac{1}{\lambda} \frac{d\epsilon_D}{d\tau} + \epsilon_D = CP(\tau) - F_C \quad (\omega \ll \lambda) \quad (14a)
\]

\[
\epsilon_D = -F_C \quad (\tau = 0) \quad (14b)
\]

The decay constant is much larger than the annual frequency since diurnal fluctuations are modeled with this systems approach, while \( \omega \) is seasonal in scale. The Laplace transform (starred variables) of Equation (14a) has a simple solution upon inversion to real time:

\[
\epsilon_D^* = \frac{\lambda CP^*}{s + \lambda} - \frac{F_C}{s} \quad (15a)
\]

\[
\epsilon_D(\tau) = \lambda C \int_0^\tau P(T) \exp[-\lambda(\tau - T)]dT - F_C \quad (15b)
\]

Equations (13) and (15b) suggest that nested convolution integrals describe the floodplain response to a hyetograph. The catchment hydraulics delay and attenuate runoff through \( \lambda \), while the floodplain hydraulics delay and attenuate the recharge through \( kgb/(S_Y V x^2) \).
SITE DESCRIPTION AND METHODS

Site description

This theory is applied to the western floodplain of the Neponset River, which flows with an average discharge $Q_A$ of 5.68 m$^3$/s (USGS 2010) through a glaciated bedrock valley in eastern Massachusetts (Klinger 1996). The discharge is measured at the Greenlodge gage (no. 01105554), which drains a watershed area $A_W$ of 2.17 $\times$ 10$^8$ m$^2$ that receives an average precipitation intensity $P_A$ of 3.87 $\times$ 10$^{-8}$ m/s, as measured at the Blue Hill meteorological observatory (NCDC 2006), located within 2 km of the site. Twelve upstream communities withdraw Neponset River water for domestic supply and other consumptive uses, resulting in a net export $Q_E$ of 1 m$^3$/s out of the watershed (NRWA 2004). An average evapotranspiration rate equal to 7.92 $\times$ 10$^{-9}$ m/s ($F_A$) follows as the difference between $P_A$ and $(Q_A + Q_E)/A_W$. Table 1 lists this and many other adopted and calibrated parameter values. The Greenlodge gage exhibits average monthly values that range from 1.93 m$^3$/s in August to 8.53 m$^3$/s in February, based upon the 2005–2008 calendar year period of record presented on the USGS (2010) website. Figure 2 displays these observed monthly average discharges as symbols, along with a sinusoid fit through the seasonally fluctuating data. The latter is zero on November 30, which is set as the temporal origin for seasonal calibration. The assumption of sinusoidal discharge (Figure 2) and stage (Equation (5b)) behavior implies a linear stage-discharge relationship over the range of seasonal fluctuations.

Ice contact stratified drift deposits of markedly different grain size distribution (Flint 1971) fill the valley. The lowest and most permeable of these deposits comprises the Fowl Meadow Aquifer, which is used as a public water supply in the study area (Ostendorf & Kilbridge 2009). A supply well withdraws an average groundwater discharge of 0.042 m$^3$/s from the aquifer as a primary component of the supply. The Fowl Meadow Aquifer is confined in the study area, with a site averaged thickness of 10 m and permeability of 10$^{-11}$ m$^2$. It is underlain by bedrock (Chute 1966) and overlain and protected by an aquitard of 10 m thickness and permeability ranging from 10$^{-13}$ to 10$^{-17}$ m$^2$ (Ostendorf et al. 2010). The latter deposit decouples the aquifer from the shallowest of the stratified drift deposits, which is the

Table 1 | Adopted and calibrated parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Neponset River discharge ($Q_A$)</td>
<td>5.68 m$^3$/s</td>
<td>USGS (2010)</td>
</tr>
<tr>
<td>Average precipitation intensity ($P_A$)</td>
<td>3.87 $\times$ 10$^{-8}$ m/s</td>
<td>NCDC (2006)</td>
</tr>
<tr>
<td>Average evapotranspiration ($F_A$)</td>
<td>7.92 $\times$ 10$^{-9}$ m/s</td>
<td>Water budget</td>
</tr>
<tr>
<td>Zero seasonal river fluctuation date</td>
<td>November 30</td>
<td>Calibrated</td>
</tr>
<tr>
<td>Floodplain halfwidth ($L$)</td>
<td>600 m</td>
<td>Topography</td>
</tr>
<tr>
<td>Elevation of bottom of floodplain deposit</td>
<td>8 m below mean sea level</td>
<td>Grain size data</td>
</tr>
<tr>
<td>Porosity ($n$)</td>
<td>0.37</td>
<td>Moisture content data</td>
</tr>
<tr>
<td>Groundwater kinematic viscosity ($\nu$)</td>
<td>1.3 $\times$ 10$^{-6}$ m$^2$/s</td>
<td>White (2008)</td>
</tr>
<tr>
<td>Floodplain permeability ($k$)</td>
<td>1.4 $\times$ 10$^{-11}$ m$^2$</td>
<td>Slug tests</td>
</tr>
<tr>
<td>Floodplain aquifer thickness ($b$)</td>
<td>20 m</td>
<td>Steady calibration</td>
</tr>
<tr>
<td>Average recharge ($\varepsilon_A$)</td>
<td>6.98 $\times$ 10$^{-9}$ m/s</td>
<td>Steady calibration</td>
</tr>
<tr>
<td>Infiltration coefficient ($C$)</td>
<td>0.39</td>
<td>Steady calibration</td>
</tr>
<tr>
<td>Seasonal river stage amplitude ($A$)</td>
<td>0.479 m</td>
<td>Seasonal calibration</td>
</tr>
<tr>
<td>Phase shift ($\phi$)</td>
<td>-0.321 rads</td>
<td>Seasonal calibration</td>
</tr>
<tr>
<td>Seasonal recharge amplitude ($E$)</td>
<td>1.90 $\times$ 10$^{-8}$ m/s</td>
<td>Seasonal calibration</td>
</tr>
<tr>
<td>Runoff decay constant ($\lambda$)</td>
<td>5.2 $\times$ 10$^{-5}$ s$^{-1}$</td>
<td>Diurnal calibration</td>
</tr>
<tr>
<td>Characteristic evapotranspiration ($F_C$)</td>
<td>1.79 $\times$ 10$^{-8}$ m/s</td>
<td>8/9 diurnal calibration</td>
</tr>
<tr>
<td>Characteristic evapotranspiration ($F_C$)</td>
<td>1.30 $\times$ 10$^{-7}$ m/s</td>
<td>10/9 diurnal calibration</td>
</tr>
</tbody>
</table>
fl}

The western floodplain extends 600 m (L) from the river to the bedrock valley wall. Interstate Highway 95 crosses the Neponset River and the Metropolitan Transit Authority railroad runs along the floodplain in the midst of the study area, as indicated in Figure 3.

Seven shallow monitoring wells and five multidepth well clusters characterize the floodplain deposit hydraulics. Figure 3 displays the monitoring well and cluster well locations, while Table 2 lists some of their attributes, including date of construction, diameter, screen length, mean sea level elevation of screen section and ground surface, distance from the Neponset River bank, and average head. The shallow monitoring wells were installed by prior investigators, while the cluster wells were installed by the University of Massachusetts between 2006 and 2009. The clusters (A–E) delineate the three deposits, and individual cluster wells are distinguished by the second letter in their designation.

Figures 4 and 5 display grain size distributions (ASTM 1996a) of split spoon samples taken from boreholes during cluster construction. The floodplain, which is marked by an appreciable sand fraction in the grain size distribution, extends from the ground surface down to an average elevation of 8 m below mean sea level. The deposit is a silty sand. Moisture contents (ASTM 1996b) determined in saturated split spoon samples from the floodplain deposit ranged from 20 to 25% by mass. The midrange value implies a porosity of 0.37 (n) in the floodplain deposit, based on a grain specific gravity of 2.65.

Overdamped (Bouwer & Rice 1976), underdamped (Ostendorf et al. 2005), and extended (Ostendorf & DeGroot 2010) slug tests were run in many of the monitoring and cluster wells – the floodplain is several orders of magnitude more permeable than the underlying aquitard, and an order of magnitude less permeable than the aquifer. Table 2 lists overdamped slug test permeabilities for the floodplain deposits – these range from $3.5 \times 10^{-13}$ to $5.3 \times 10^{-11}$ m². An average of $1.4 \times 10^{-11}$ m² is adopted for this study, as listed in Table 1. The E cluster wells have the lowest ground surface elevation (12.62 m above mean sea level), the other monitoring and cluster wells have ground surface elevations that range from 13.16 to 15.27 m above mean sea level. The cluster wells and monitoring wells in lower ground surface elevations (1S and 3S) included riser pipe sections extending from 0.5 to 1 m above the ground surface, precluding flooding from rare excursions of the water table above the ground surface.

**METHODS**

The cluster wells were of 5 cm diameter PVC construction, with screen lengths of 1.5 m set in 2.1–2.4 m long uniform
medium sand packs of 10 cm diameter. The cluster wells were installed through 10 cm diameter hollow stem augers, with bentonite chip seals above the sand pack and a steel protective pipe with locking cap set in a concrete collar at the ground surface. The cluster wells were screened at different elevations in separate, but closely spaced boreholes, and were intended to resolve the vertical concentration and head profiles at a given horizontal location. Each cluster included a logging well equipped with a model 600XLM unvented pressure transducer (YSI; Yellow Springs, OH) programmed to sample gage pressure at 900 s intervals. The logging cluster wells were screened near the bottom of the floodplain deposit, and were paired with a manually sampled cluster well screen at the same interval. The pairs are included in Table 2. The transducer logs were downloaded in the field each month using a laptop computer and Eco-watch software (YSI; Yellow Springs, OH), calibrated each month with a manual determination of water level in the logging well. Barometric pressure was also measured at 900 s intervals in the field with a model PTU 300 unvented pressure transducer (Vaisala; Helsinki, FN). The logging cluster well pressure data have barometric pressure added without attenuation, so that the reported head data are consistent with the manual observations. Neponset River stage data were sampled using a model H55 bubbler flow meter (Hydrological Services; Bradenton, FL) installed to augment the USGS (2010) gage. This also measured gage pressure, since the air bubbles equilibrated to current barometric conditions. Precipitation was sampled on site at 15 min intervals, using a model 385 Met One heated rain gage (Campbell Sciences; Logan, UT). The site was powered with electricity from an adjacent utility line, and the stage and precipitation data were downloaded to the University of Massachusetts laboratory in Amherst over dedicated land lines.

The shallow monitoring wells and the unlogged cluster wells were manually sampled on a monthly basis from April 2007 through May 2010. Water levels in each well were measured using a water level sensor (Slope Indicator: Seattle, WA) or a model Dipper T electric water level meter (Heron Instruments, Inc.; Burlington, ONT) prior to purging

<table>
<thead>
<tr>
<th>ID</th>
<th>Constructed</th>
<th>Elevations</th>
<th>Length (m)</th>
<th>Diameter (cm)</th>
<th>x (m)</th>
<th>Head (m)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>11/7/06</td>
<td>0.59 a (14.86)b</td>
<td>1.58</td>
<td>5.08</td>
<td>16</td>
<td>12.13</td>
<td>9.0 × 10⁻¹² m²</td>
</tr>
<tr>
<td>AC</td>
<td>11/9/06</td>
<td>0.67</td>
<td>1.58</td>
<td>5.08</td>
<td>16</td>
<td>12.66</td>
<td>7.4 × 10⁻¹³ m²</td>
</tr>
<tr>
<td>BD</td>
<td>8/28/07</td>
<td>2.27 (13.58)</td>
<td>1.58</td>
<td>5.08</td>
<td>417</td>
<td>12.23</td>
<td>6.0 × 10⁻¹² m²</td>
</tr>
<tr>
<td>BE</td>
<td>8/28/07</td>
<td>2.41</td>
<td>1.58</td>
<td>5.08</td>
<td>66</td>
<td>12.23</td>
<td>3.2 × 10⁻¹² m²</td>
</tr>
<tr>
<td>CB</td>
<td>10/25/06</td>
<td>0.47 (14.78)</td>
<td>1.58</td>
<td>5.08</td>
<td>66</td>
<td>12.33</td>
<td>3.5 × 10⁻¹³ m²</td>
</tr>
<tr>
<td>CC</td>
<td>10/30/06</td>
<td>0.52</td>
<td>1.58</td>
<td>5.08</td>
<td>66</td>
<td>12.33</td>
<td>12.12</td>
</tr>
<tr>
<td>DD</td>
<td>10/24/06</td>
<td>–0.18 (14.51)</td>
<td>1.58</td>
<td>5.08</td>
<td>167</td>
<td>12.33</td>
<td>2.1 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>DE</td>
<td>10/25/06</td>
<td>–0.01</td>
<td>1.58</td>
<td>5.08</td>
<td>167</td>
<td>12.33</td>
<td>5.3 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>ED</td>
<td>8/15/07</td>
<td>1.13 (12.62)</td>
<td>1.58</td>
<td>5.08</td>
<td>356</td>
<td>12.33</td>
<td>1.6 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>EE</td>
<td>8/14/07</td>
<td>1.19</td>
<td>1.58</td>
<td>5.08</td>
<td>356</td>
<td>12.33</td>
<td>2.0 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>OB5A</td>
<td>11/14/89</td>
<td>10.4 (14.31)</td>
<td>0.91</td>
<td>5.08</td>
<td>14</td>
<td>12.12</td>
<td>1.5 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>82S</td>
<td>7/14/97</td>
<td>11.3 (14.80)</td>
<td>4.57</td>
<td>5.08</td>
<td>251</td>
<td>12.48</td>
<td>2.1 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>7S</td>
<td>8/3/97</td>
<td>10.7 (15.27)</td>
<td>6.10</td>
<td>5.08</td>
<td>200</td>
<td>12.57</td>
<td>5.3 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>6S</td>
<td>8/4/97</td>
<td>10.6 (14.11)</td>
<td>4.57</td>
<td>5.08</td>
<td>463</td>
<td>12.58</td>
<td>1.6 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>5S</td>
<td>8/8/97</td>
<td>10.2 (13.73)</td>
<td>4.57</td>
<td>5.08</td>
<td>524</td>
<td>12.58</td>
<td>2.0 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>3S</td>
<td>7/10/09</td>
<td>10.3 (13.50)</td>
<td>3.05</td>
<td>5.08</td>
<td>407</td>
<td>12.95</td>
<td>1.5 × 10⁻¹¹ m²</td>
</tr>
<tr>
<td>1S</td>
<td>7/13/09</td>
<td>8.28 (13.16)</td>
<td>1.58</td>
<td>5.08</td>
<td>322</td>
<td>12.73</td>
<td>12.12</td>
</tr>
</tbody>
</table>

aMidscreen elevation.
bGround surface elevation, meters above mean sea level.
cScreen section attribute.
dAverage head.

Table 2 | Attributes of monitoring and cluster wells in floodplain deposit
for groundwater quality sampling. Observed depth to the free surface was subtracted from the known mean sea level elevation of the top of the riser pipe in the monitoring well in the establishment of the manually measured water level elevation.

**RESULTS AND DISCUSSION**

**Average calibration**

Monthly data for seven monitoring wells and five manually sampled cluster wells are averaged over their periods of record, with the results listed in Table 2. A nested Fibonacci search (Beveridge & Schechter 1970) calibrates an average recharge of \(6.98 \times 10^{-9} \text{ m/s (} \epsilon_A)\) and an average head of 12.1 m at the origin, based on a groundwater kinematic viscosity of \(1.5 \times 10^{-6} \text{ m}^2/\text{s (White 2008). The calibrated average head is subtracted from the observed averaged heads in order to compute the measured } \eta_A \text{ (symbols in Figure 6) for comparison with the prediction (curve in Figure 6) of Equation (4). The search minimizes the root mean square of the difference between the measured and predicted average water table fluctuation. The 0.14 m root mean square error is 23% of the 0.60 m average head at the valley wall, which suggests reasonable calibration accuracy, in view of the model simplicity. The average head at the origin implies an unconfined aquifer thickness of 20 m (} b \text{) – the floodplain deposits are much thicker than the water table fluctuations, justifying the linearization of the unconfined aquifer hydraulics and the simple superposition of the partitions. Darcy’s law offers an estimate of the seepage velocity } u \text{ at the average and seasonal scales:}

\[
u = -\frac{kg}{\nu n} \frac{\partial \eta}{\partial x} \quad \text{(average, seasonal)}
\]
Equations (4) and (16) yield a straightforward estimate of $u_A$ and the average travel time $T_A$ across the floodplain:

$$u_A = \frac{\varepsilon_A x - L}{bn}$$

(17a)

$$T_A = \frac{1}{C} \int_0^L \int_0^\infty \frac{dx'}{\mu_A|}$$

(17b)

$$T_A = \frac{bn}{\varepsilon_A}$$

(17c)

Table 1 suggests that the average seepage velocity varies from zero at the valley wall to a maximum of $-5.7 \times 10^{-7}$ m/s at the riverbank, into the Neponset River. The average travel time is $1.1 \times 10^9$ s, or 33 years. Groundwater moves slowly across the floodplain. The $\varepsilon_A$ calibration implies an infiltration fraction $C$ through its definition as the difference between infiltration and evapotranspiration rates:

$$\varepsilon_A = CP_A - F_A$$

(18)

The values in Table 1 imply an infiltration fraction of 0.39 (C).

### Seasonal calibration

The individual monthly heads at each monitoring or cluster well are averaged in order to establish a seasonal data base, with the results plotted as symbols in Figures 7 and 8. These observed monthly residuals are the difference between the monthly averages and the observed annual average heads (the latter are listed in Table 2). The annual frequency ($\omega$) is $1.99 \times 10^{-7}$ rad/s. These seasonal data calibrate Equation (8) through a nested Fibonacci search for optimal $A$, $E$, and $\varphi$, with the results summarized in Table 1 and plotted as curves in Figures 7 and 8.

The data and theory clearly document an attenuated seasonal amplitude of floodplain transience with distance from the Neponset River. The Neponset River exhibits a seasonal stage amplitude fluctuation of 0.479 m (A). The calibration error of 15 cm is 31% of this amplitude, which represents reasonable accuracy in view of the model simplicity and the use of a common set of parameter values for all 12 wells. The calibrated value is close to the scale of the seasonal variation found in cluster well AC, located 16 m from the river.
Monitoring well 5S, by contrast, is located 524 m from the river and exhibits a 0.2 m seasonal amplitude (Figure 8(f)). The calibrated phase shift of \(-0.321\) rads (\(\phi\)) suggests that recharge and river discharge are nearly in phase – the minimum seasonal Neponset River discharge of early September implied by Figure 2 corresponds to minimum seasonal recharge induced by maximum evapotranspirative losses in June. The calibrated \(E\) value of \(1.90 \times 10^{-8}\) m/s is larger than \(e_A\), so that evapotranspiration exceeds infiltration for significant parts of the water year.

Equations (6a), (7), and (16) specify the seasonal seepage velocity at the origin \(u_{SO}\):

\[
u_{SO} = r_2 \sqrt{\frac{kg_0}{b MV}} A \sin \left( o t + \frac{\pi}{4} + \theta_2 \right) - \frac{E}{k_0} \sin \left( o t + \phi + \frac{\pi}{4} + \theta_2 \right)
\]

(19)

**Figure 7** | Observed (circles) and calibrated (curves) seasonal head fluctuation: (a) monitoring well OB5a, (b) cluster well AC, (c) cluster well CB, (d) cluster well DD, (e) monitoring well 7S, and (f) monitoring well 82S. Root mean square error is 15 cm.
Figure 9 displays the variation of the average $u_{AO}$ plus seasonal seepage velocity at the Neponset River implied by Table 1 and Equations (17a) and (19). Water flows from the floodplain into the river all year round, with a maximum value of $-1.0$ μm/s in June and a minimum of $-0.1$ μm/s in December.

**Diurnal calibration**

The calibration proceeds down the cascade to the diurnal partition, and uses the continuous data and convolution integrals to either estimate diurnal model parameters or confirm values established by the longer timescales. Table 1 and
Equation (11a) suggest that disturbances from the Neponset River propagate to valley wall over a duration \( T_C \) of \( 2 \times 10^7 \) s, based on a specific yield of order \( 10^{-1} \), comparable to the order of magnitude of the floodplain deposit porosity. This dominates diurnal transience, so that Equation (11b) and the subsequent theory is justified at the field site. A 10 day dry period during the beginning of August 2009 is used first because it precludes the complications of precipitation and runoff modeling. Figure 10(a) displays (as symbols) the uniform decline of Neponset River stage and the subsequent theory is justified.

Continuous (15 min frequency) precipitation (Figure 11(b)) and river head (Figure 11(a)) data for July 2009 were input to the storm scale convolution integrals in Equations (13) and (15b), along with the calibrated parameter values of Table 1. The theory (curves) and July 2009 data (symbols) shown in Figures 11(c)–(f) and 12(a) calibrate a common decay constant \( \lambda \) of \( 5.2 \times 10^{-5} \) s\(^{-1}\) that is two orders of magnitude larger than \( \omega \) – distinguishing diurnal from seasonal fluctuations. The value compares favorably to the \( 3.5 \times 10^{-3} \) s\(^{-1}\) value used by Ostendorf et al. (2001) to calibrate a linear reservoir for interflow contributions to a hydrograph of an access road drainage system. The root mean square error of the calibrations ranges from 5 to 11 cm, indicating reasonable model accuracy.

Figures 12(b) and 13 show the results of a second diurnal calibration (of \( F_A \)) using Tables 1, 3 and the diurnal model and the data from October 2009. The data document another wet month with a total of 0.16 m precipitation total (Figure 13(b)), but one which occurred during a period of seasonally rising water table and Neponset River stage (Figures 2 and 13(a)). The curves in Figures 12(b) and 13(c)–(f) represent another successful calibration of the diurnal theory (3–9 cm root mean square errors), and confirm
that specific yield governs storage release from floodplain deposits at a diurnal scale, as noted by Reynolds (1987) and Boutt & Fleming (2009). The October characteristic evapotranspiration rate of $1.3 \times 10^{-18} \text{m/s}$ is less than the August value, as expected for a Fall sampling period with cooler temperatures and lower gradients of water vapor pressure. The August and October evapotranspiration rates both exceed the independently computed $F_A$ value of $7.92 \times 10^{-9} \text{m/s}$. The latter reflects annual budgets for the entire watershed – one would anticipate that winter values

**Table 3** | Diurnally calibrated specific yields for cluster wells using July 2009 data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AB</th>
<th>BE</th>
<th>CC</th>
<th>DE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific yield ($S_Y$)</td>
<td>0.14</td>
<td>0.095</td>
<td>0.052</td>
<td>0.048</td>
<td>0.10</td>
</tr>
</tbody>
</table>

![Figure 10](https://iwaponline.com/hr/article-pdf/43/6/870/370123/870.pdf)
of $F_C$ would fall well below the $F_A$ estimate in order to reconcile the diurnal calibrations with the annual estimate.

Equations (13) and Darcy’s law permit the estimation of the diurnal seepage velocity fluctuation at the Neponset River bank $u_{DO}$:

$$u_{DO} = \frac{k_g S_Y v}{\eta_D} \left( x = 0 \right)$$  \hspace{1cm} (21a)

$$u_{DO} = \sqrt{\frac{k_g}{\pi S_Y b} \left[ \int_0^t \left( \frac{d\eta_{DRiver}}{d\tau} - \frac{e_D(\tau)}{S_Y} \right) d\tau \right]} \frac{\sqrt{t - \tau}}{C_0}$$  \hspace{1cm} (21b)

Figure 14(a) displays $u_{DO}$ for July 2009 – the seepage velocity fluctuates from $-10^{-2}$ to $10^{-2}$ mm/s. Equation 21(b) in turn describes the excursion length $x_{DO}$ of groundwater...
molecules induced by the diurnal fluctuation:

\[ x_{DO} = \int_0^t u_{DO} dt' \]  

The excursion length for July 2009 is plotted as symbols in Figure 14(b), along with a straight line which represents the average seepage velocity at the bank. The difference between the symbols and the line in Figure 14(b) represents an aperiodic horizontal excursion length fluctuation \( x_{FO} \) of water molecule trajectory from its average position during the month. The storms generate three excursions of 1 m amplitude during the month, with weeklong persistence. The stormscale aperiodicity adds to aquifer heterogeneity (Freeze & Cherry 1969) in generating groundwater dispersion. This augmented dispersivity, which has been established on theoretical grounds by Cirpka & Attinger (2003), has been noted in groundwater plumes downgradient of a highway infiltration basin in glacial outwash receiving focused storm runoff (Ostendorf et al. 2008), estuarine deposits with tidal forcing (Yim & Mohsen 1992), as well as the present context of recharge rate and surface water level transience at a hydrologic boundary (Kim et al. 2000). Indeed, Boutt & Fleming (2009) simulate mass transport at a diurnal scale in the hyporheic zone of the Deerfield River using meter scale dispersivity estimates, induced by 0.5 m amplitudes of river stage. Figure 14(c) endorses the scale of this dispersivity estimate, but should be viewed with caution. A two dimensional theory compared with transverse contaminant concentration gradients would inspire more confidence, since transverse mixing is more sensitive to transience than its longitudinal counterpart (Cirpka & Attinger 2003). Longitudinal dispersivity increases strongly with spatial heterogeneity in field studies, and a 0.5 m scale calibration is common without transience.

DISCUSSION

The diurnally calibrated range of specific yields for the silty sand of the Neponset River floodplain deposit is consistent with the broad variation found in the literature. Thus the 0.048–0.14 values of Table 3 fall within the literature based ranges of 0.10–0.28 for fine sand and 0.03–0.19 for silt (Fetter 2001). One must be cautious in \( S_Y \) estimation because even calibrated field studies in similar material offer different ranges of \( S_Y \) values. Boutt & Fleming (2009) find a 0.21 value for periodic oscillations of the Deerfield River, which resembles the 0.20 specific yield calibration used by Barlow et al. (2000) to interpret observed flood wave attenuation of the Cedar River in Iowa. However, other results vary: Chen et al. (1999) use pump test data to document \( S_Y \) values ranging from 0.035 to 0.12 in the Republican River of Nebraska, while Reynolds (1987) calibrates specific yields that range from 0.015 to 0.034 to explain flood wave attenuation in the Tioughnioga River. All four of these studies characterize alluvial sand and gravel aquifers of \( 10^{-10} \) m$^2$ permeability and glacial origin,
subject to diurnal transient forcing. Specific yield emerges as a site related characteristic requiring field data for its estimation, particularly for the interpretation of diurnally varying hydraulics and associated mass transport across the hyporheic zone.

The calibrated hydraulics have implications for future research. The one dimensional model might serve as a boundary condition for the two dimensional, unsteady flow of groundwater through the hyporheic zone that details the interaction of the river bottom and the unconfined aquifer. In this regard, the Neponset River is only 1–3 m deep, while the floodplain deposit is an order of magnitude larger – so that there is an appreciable upward component of seepage velocity near cluster well A and monitoring well OB5A. A finer grid of monitoring wells and seepage meters might inform this more detailed analysis, which

Figure 13 | Observed (symbols) and calibrated (curves) diurnal fluctuations for October 2009 (root mean square error): (a) Neponset River stage, (b) precipitation intensity, (c) cluster well AB (9 cm), (d) cluster well CC (4 cm), (e) cluster well DE (4 cm), and (f) cluster well EE (5 cm).
could use the existing two dimensional, closed form derivation of Moench & Barlow (2000) and a specific yield of 0.14 as a starting point.

In the latter regard, the hydraulic data of the present investigation were accompanied by groundwater quality data at comparable frequencies – including major ion analysis of the manually sampled observations and continuous logs of specific conductivity in the cluster wells. Thus a future submission might consider the seasonal and average contaminant transport through the floodplain – with advection established by the calibrated, one dimensional hydraulics established here. However, dispersivity would vary with distance from the river – the attenuated diurnal hydraulics would lead to horizontal excursions smaller than the hyporheic Figure 14. Recharge concentration would vary with distance from Interstate 95, since the highway runoff has more ionic activity than the ambient runoff. This would impart a second horizontal dimension to the mass transport model – and the surface introduction of the recharge would introduce vertical concentration gradients if dispersivity is insufficient to mix the runoff over the 20 m depth of the floodplain deposit.

CONCLUSIONS

A consistent set of property values and extensive database calibrate a suite of closed form models of unconfined aquifer hydraulics across average, seasonal, and diurnal timescales in the floodplain of the Neponset River, which flows in a glaciated bedrock valley in eastern Massachusetts. The $1.4 \times 10^{-11} \text{ m}^2$ permeability, 20 m thickness, 0.37 porosity, and 600 m halfwidth of the floodplain deposit imply a 33-year travel time in response to an average recharge rate of $7.0 \times 10^{-9} \text{ m/s}$ – groundwater moves slowly across the floodplain. An annual water budget suggests that 39% of the precipitation recharges the floodplain deposit, of which $7.9 \times 10^{-9} \text{ m/s}$ returns as average evapotranspiration.
The river and recharge introduce transience to the floodplain at seasonal and diurnal scales. Three years of manually measured monthly data from 12 wells suggest that the seasonal transience is periodic and the recharge and river are nearly in phase. The 0.48 m amplitude of the river sinusoid and 1.9×10⁻⁸ m/s amplitude of the recharge sinusoid do not overcome their steady counterparts, so that the groundwater flows towards the river in all seasons, when considered on a monthly average basis. Seasonal storage is governed by porosity – the floodplain deposit pores all drain within several months. This is decidedly not the case for diurnal transience however, and specific yields are required to reconcile the observed and modeled attenuation of 1 m amplitude flood waves of the Neponset River across the floodplain and the accompanying evapotranspiration and storm precipitation. A linear reservoir routes the latter to the water table with a decay constant of 5.2×10⁻⁵ s⁻¹. Continuous logs of water levels from five cluster wells and the Neponset River stage, with supporting meteorological data document the diurnal hydraulics for wet months (July 2009 and October 2009) and an intervening dry period (first weeks of August 2009). The calibrated specific yield varies from 0.04 to 0.14 for the five cluster wells, but each well retains its specific yield for the three test periods. The characteristic evapotranspiration rates are common to the five cluster wells, but fall from a July value of 1.8×10⁻⁸ m/s to an October value of 1.3×10⁻⁸ m/s – plausible behavior in light of the pan evaporation rate for Massachusetts and the annual average value, which reflects independent, water budget assessment. The diurnal hydraulics imply meter scale horizontal excursions of groundwater molecules near the riverbank, which could augment dispersive transport in the hyporheic zone.

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