

time behavior of four out of the five heats. However, the fifth heat, 813, showed a remarkable resistance to the deleterious effects of tensile hold times in both the aged and annealed conditions, with an accompanying increased resistance to intergranular crack propagation. Thus it can be noted that the results of Maiya are quite similar to those reported by Brinkman and Korth [5]. However, it appears that more long term test data and microstructural work is needed to understand why the heat-to-heat variations observed in tensile and creep properties are not as pronounced in fatigue and creep-fatigue properties.

Maiya has also raised the question in the discussion as to whether heat-to-heat variations also existed in the aged condition. Fig. 1 shows our recent results on the influence of long-term aging on creep properties of various heats of type 304 stainless steel. The figure shows that the weak, medium, and strong heats retain their characteristic behavior even after aging; these observations are also consistent with our results on materials tested in as-received and reannealed conditions.

#### Additional References

4 Sikka, V. K., McCoy, H. E., Jr., Booker, M. K., and Brinkman, C. R., *JOURNAL OF PRESSURE VESSEL TECHNOLOGY, TRANS. ASME, Vol. 96, Series J, No. 4, Nov. 1975*, pp. 243-251.

5 Brinkman, C. R., and Korth, G. E., "Heat-to-Heat Variations in the Fatigue and Creep-Fatigue Behavior of AISI Type 304 Stainless Steel at 593°C," *Journal of Nuclear Materials, Vol. 48, 1973*, pp. 293-306.

### Elastoplastic Analysis of Space Framework: Initial Force Schemes<sup>20</sup>

**S. D. GUPTA.**<sup>21</sup> The basic equations described in the paper are a particular case (for a perfectly plastic material) of the formulation given in Gupta and Hollmeier.<sup>22</sup> This formulation is more comprehensive and includes strain hardening properties of the material.

The paper describes an initial force scheme which the author claims different from the initial force method of Gupta and Hollmeier. Steps of the Iterative Scheme are the same as those of the initial force method of Gupta and Hollmeier with the exception that the footnote reference gives more details of the method. Coincidence of the Iterative Scheme of the paper with the initial force method of Gupta and Hollmeier has also been supported by author's own statement on p. 4: "For plane frameworks, the element forces and load parameters corresponding to the subsequent yielding are identical to those obtained by using the classical limit analysis." One finds why the author did not figure out the difference of his Iterative Scheme from the initial force method of Gupta and Hollmeier. Also the paper did not make an attempt to compare the results with that of Gupta and Hollmeier.

Four out of five example problems given in the paper are identical with those of Gupta and Hollmeier. The example problems have configurations and geometric properties and loading conditions which have been arbitrarily chosen by Gupta and Hollmeier. The paper should mention clearly that these example problems have been taken from Gupta and Hollmeier. In spite of taking these examples problems from Gupta and Hollmeier, the paper avoids any discussion of the results.

Throughout the paper, the deformations due to shear forces have been neglected, but the paper ignores any such statement

<sup>20</sup>By K. N. Lee, published in the May 1975 issue of the *JOURNAL OF PRESSURE VESSEL TECHNOLOGY, TRANS. ASME, Vol. 96, Series J, No. 2, p. 90.*

<sup>21</sup>Senior Engineering Programmer, Bechtel International Corp., San Francisco, Calif. Assoc. Mem. ASME.

<sup>22</sup>Gupta, S. D., and Hollmeier, R. J., "Elasto-Plastic Analysis of Space Frames," ASME Paper No. 74-PVP-49, presented at 1974 Pressure Vessel and Piping Conference held in Miami, Fla., June 1974.

There is a discrepancy in the stated value of  $C_{44}$ ,  $C_{55}$  and  $C_{66}$  in the paper.

Overall, most of the material of the paper (especially the original contribution) has already been published in Gupta and Hollmeier and one sees no reason why the publication of the paper has been warranted.

#### Author's Closure

The basic attitude before doing research, in my opinion, is to find out the current stand in that specific research area. The basic equations developed by Morris and Fenves [1, 2]<sup>23</sup> in the theory of plasticity of space frameworks as well as the Drucker's normality principle in plasticity [3] form the basis of the work and findings given in my paper [4]. As stated therein, the theme of the iterative initial force scheme is identical to that of the initial stress technique developed by O. C. Zienkiewicz, et al. [6].

The quadratic representation of the yield surface is an early attempt to unify the various yield criteria developed by Morris and Fenves [1, 2], in which the yield surface varies with the location of the neutral axis. As is known, an entire smooth yield surface  $f(F_i) = K^2$  can be represented in terms of Taylor's series as:

$$f(0) + \frac{\partial f}{\partial F_i} F_i + \frac{1}{2} \frac{\partial^2 f}{\partial F_i \partial F_j} F_i F_j + \dots = K^2$$

Hence, by adopting the appropriate assumptions, it leads to the proposed quadratic form. It should be noted that the quadratic form simulates the representation in the theory of plasticity where the point-wise stress components are used.

A similar proposal also along the lines of the initial stress technique [6] has been presented in the paper [7] of Gupta, et al. However, in the key step to redistribute the unbalanced initial residue forces, the technique adopted in that paper is to consider element-wise equilibrating forces by imposing simply-supported or fixed end conditions according to the yield condition at each element end. This technique as well as the use of concentrated unbalanced initial forces does not follow along the lines of the initial stress technique [6].

In my paper [4], the technique to redistribute the unbalanced initial residue force is the standard kinematical equivalent nodal force generation scheme used in finite element analysis [5]. Assume that at nodes I, J, K, the calculated unbalanced initial residual forces are  $F_i^k$ ,  $i = 1, 4, 5, 6$ ,  $k = I, J, K$ , as shown in

<sup>23</sup>Numbers in brackets designate References at end of closure.

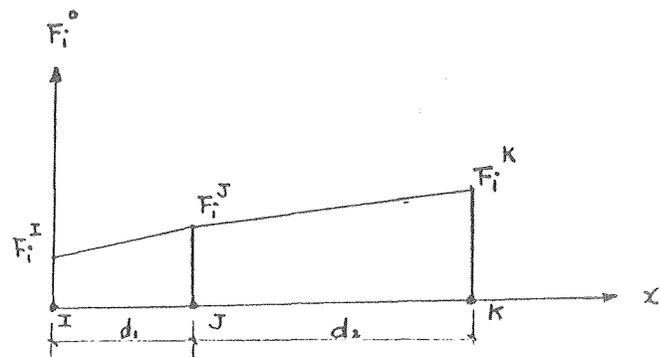


Fig. 1 Linear distribution of unbalanced initial residue forces in beam elements I-J and J-K

Fig. 1. The unbalanced initial residual forces are linearly distributed in the beam elements. The kinematical equivalent nodal force at node  $I$  is

$$P_i^I = \int_0^{d_1} g_i(x) G_i^I(x) dx \quad (\text{no sum on } i)$$

where

$$g_i(x) = \frac{F_i^J - F_i^I}{d_1} x + F_i^I$$

and  $G_i^I(x)$  are the corresponding interpolation strain functions of forces  $F_i^0$  at node  $I$ . For axial and twisting displacement fields, a linear interpolation function is used in each element and a cubic interpolation function is used for lateral deflections.

The jump of the slope of the deflection curve at the yield section, characterized in the classical plastic hinge technique, is not observed in my sample solutions. However, all the element forces and load parameters corresponding to the yielding of beam cross sections are identical to those of the classical plastic analysis of plane frameworks.

The technique presented in Gupta's paper [7] to redistribute the unbalanced initial residue forces is somewhat arbitrary and, in my opinion, falls into the trial and error category. Hence, any comparison of the numerical results in my paper [4] and the paper [7] is not warranted. Moreover, the major contribution of "Elastoplastic Analysis of Space Frameworks: Initial Force Schemes" is the self-equilibrating force scheme, which I believe is a completely new development as far as the "initial stress method" [6] is concerned. All the results obtained by the iterative initial force scheme have been validated by those obtained by the self-equilibrating force scheme.

#### References

- 1 Morris, G. A., and Fenves, S. J., "A General Procedure for the Analysis of Elastic and Plastic Frameworks," *Structural Research Series No. 325*, Department of Civil Engineering, University of Illinois, Aug. 1967.
- 2 Morris, G. A., and Fenves, S. J., "Elastic-Plastic Analysis of Frameworks," *Journal of Structural Division*, ASCE, ST5, May 1970.
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- 4 Lee, K. N., "Elastoplastic Analysis of Space Framework: Initial Force Schemes," *JOURNAL OF PRESSURE VESSEL TECHNOLOGY*, TRANS. ASME, Series J, Vol. 96, No. 2, May 1975, p. 90.
- 5 Zienkiewicz, O. C., *The Finite Element Method in Engineering Sciences*, McGraw-Hill, New York, 1971.
- 6 Zienkiewicz, O. C., Valliappan, S., and King, I. P., "Elasto-Plastic Solutions of Engineering Problems, Initial-Stress, Finite Element Approach," *Int. Journal Num. Meth. in Engineering* 1, 1969, pp. 75-100.
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### Finite Element Analysis of Perforated Plates Containing Triangular Penetration Patterns of 5 and 10 Percent Ligament Efficiency<sup>24</sup>

T. SLOT.<sup>25</sup> The author has investigated several important questions pertaining to the validity of the equivalent solid plate concept used in the analysis of perforated plates, such as tube-

sheets and certain vessel closures. Analytical procedures based on this concept were formulated by O'Donnell and Langer [1]<sup>26</sup> and subsequently included in the ASME Boiler and Pressure Vessel Code [2]. More recently, a generalization of the concept for three-dimensional stress analysis of perforated plates was formulated by the writer [3, 4]. Effective elastic constants and stress multipliers were tabulated for such purpose in [4], for ligament efficiencies ranging from 0.05 to 1.0, and for both the triangular and square penetration patterns. The new results presented by the author are a valuable addition to the literature on perforated plate analysis. Purpose of this discussion is to contribute a number of related and generally supportive observations gleaned from the writers's own involvement with the subject.

**Mechanical Loading.** In the first part of the paper, the author reports on solutions obtained for bending of a perforated plate, whose triangular penetration pattern is defined by a ligament efficiency of 0.05. Although the point is not stressed in the paper, it is important to recognize that the three-dimensional finite-element model used in the analysis has a thickness-to-pitch ratio of 2.0 and that as such the plate must be regarded as a thick perforated plate. If the rules in [2] are adhered to, then the equivalent solid plate approach can only be used in the case of bending when the plate is thick enough to meet the requirement  $t/p \geq 2.0$ . The basis for this rule is that for thinner plates, the effective elastic constants and stresses will deviate from those provided in the Code, which were determined for in-plane loading. This deviation was evident from the photoelastic experiments of Sampson [5] and Leven [6]. Subsequent theoretical work by Meijers [7] and by Grigolyuk and Fil'shtinskii [8] showed fundamental differences between solutions for in-plane loading and for bending of thin plates. Meijers also obtained three-dimensional finite-element solutions for bending in which the effect of thickness on the effective elastic constants and the stresses

<sup>26</sup>Numbers in brackets designate References at end of discussion.

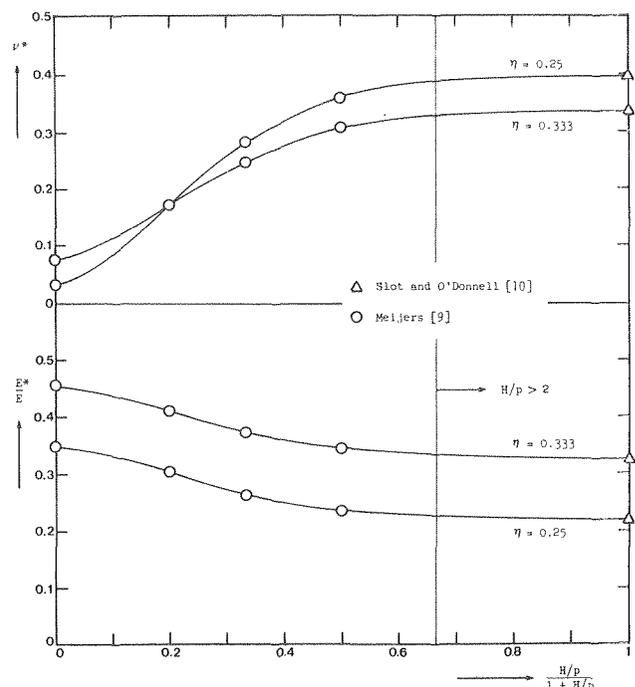


Fig. 1 Effective elastic constants for bending in relation to plate thickness ( $p$  = pitch of triangular penetration pattern,  $H$  = plate thickness,  $\nu = 0.3$ )

<sup>24</sup>By D. P. Jones, published in the August 1975 issue of the *JOURNAL OF PRESSURE VESSEL TECHNOLOGY*, TRANS. ASME, Vol. 97, Series J, No. 3, p. 199.

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