Soft Dilaton Theorem in String Field Theory

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The soft dilaton theorem is derived in covariant and light-cone gauge closed string field theories. We have to take special care of the singularity of the diagrams where the zero-momentum dilaton is attached to one of the external on-shell lines.

§ 1. Introduction

The soft dilaton theorem\(^1,2\) is one of the most fundamental properties in string theory. It relates an amplitude with a zero-momentum dilaton to the amplitude without the dilaton. Concretely, it reads

\[
\lim_{k \to 0} A^{(N+1)}(k; p_1, p_2, \ldots, p_N) = c g (\sqrt{\alpha'})^{(d-2)/2} \left[ \sqrt{\alpha'} \frac{\partial}{\partial \sqrt{\alpha'}} - \frac{1}{2} (d-2) g \frac{\partial}{\partial g} \right] A^{(N)}(p_1, p_2, \ldots, p_N),
\]

(1)

where \(A^{(N+1)}(k; p_i)\) on the l.h.s. is the \(N+1\) point amplitude of one dilaton with momentum \(k\) and \(N\) other particles which are on-shell and massless and carry momenta \(p_i (i=1-N)\), and \(A^{(N)}(p_i)\) on the r.h.s. is the \(N\) point amplitude of the latter \(N\) particles only. On the r.h.s. of Eq. (1), \(\alpha'\) and \(g\) are the slope parameter and the (dimensionless) string coupling constant, respectively, \(d(=26)\) is the space-time dimension, and \(c\) is some number. Equation (1) tells that the constant shift of the dilaton field \(\varphi \to \varphi + \epsilon\) in string theory is equivalent to the change of \(\alpha'\) and \(g\),

\[
\sqrt{\alpha'} \to \sqrt{\alpha'} e^\epsilon, \quad g \to g e^{-\frac{d-2}{2} \epsilon},
\]

(2)

which keeps the gravitational constant \(\kappa = g (\sqrt{\alpha'})^{(d-2)/2}\) invariant.

The purpose of this paper is to derive this soft dilaton theorem in the framework of string field theory (SFT); the covariant closed SFT of the Kyoto group\(^3\) and the light-cone gauge SFT.\(^4\) In particular, we resolve the (apparent) ambiguity pointed out recently by Kugo-Zwiebach\(^5\) and Kawano\(^6\) concerning the dilatation property in the covariant closed SFT of the Kyoto group.

§ 2. Dilatation property of SFT

The issue of dilaton in SFT was first discussed by Yoneya\(^7\) in the light-cone gauge SFT.\(^4\) His argument was then applied by Nagoshi and the present author\(^8\) to the covariant closed SFT developed by the Kyoto group.\(^3\) Let us first recapitulate Ref. 8) for the covariant SFT in a slightly modified form. The point is the following two basic properties of the 3-string vertex \(V(1, 2, 3)\):
\[ \int d^1 \langle \text{Dil}(1) \mid V(1, 2, 3) \rangle = \frac{1}{4\sqrt{d-2}} \{ \mathcal{D}^{(2)}, Q^{(2)}_b \} \mid R(2, 3) \rangle, \] (3)

\[ \sum_{r=1,2,3} \mathcal{D}^{(r)} \mid V(1, 2, 3) \rangle = \frac{d-2}{2} \mid V(1, 2, 3) \rangle, \] (4)

where \( \int d^1 \) denotes the integration over the zero-modes \((p, \bar{c}, a)\) of the string 1, and \( |\text{Dil}⟩ \) is the dilaton state with vanishing 26-momentum \( p_\mu \) and the string-length parameter \( a \):

\[ |\text{Dil}(p, \bar{c}, a)⟩ = -\frac{1}{\sqrt{d-2}} (a^{\pm} \bar{c}^{-} \bar{c}^{+} + c^{-} \bar{c}^{+} \bar{c}^{+} + c^{-} \bar{c}^{+} \bar{c}^{+}) |0⟩ \]

\[ \times (2\pi)^{d+1} \delta^d(p) \frac{1}{2} [\delta(a+0) + \delta(a-0)]. \] (5)

\( \mathcal{D} \) in Eqs. (3) and (4) is the dilatation operator given by

\[ \mathcal{D} = \mathcal{D}_x + \mathcal{D}_{gh} + N_{gh} - 2 \mathcal{D}_a \] (6)

with

\[ \mathcal{D}_x = -\frac{1}{2} \int_{-\infty}^{\infty} d\sigma \left\{ X^\mu(\sigma), -\frac{\partial}{\partial X^\nu(\sigma)} \right\} = \frac{1}{2} \left\{ p_\mu, \frac{\partial}{\partial p_\mu} \right\} + \frac{1}{2} \sum_{n \neq 0} n a_n^{(+)} \cdot a_n^{(-)}, \]

\[ \mathcal{D}_{gh} = \frac{1}{2} \left[ \bar{c}_0 \frac{\partial}{\partial \bar{c}_0} \right] + \sum_{n \neq 0} \bar{c}_n^{(\pm)} c_n^{(\mp)}, \]

\[ N_{gh} = \frac{1}{2} \left[ \bar{c}_0 \frac{\partial}{\partial \bar{c}_0} \right] + \sum_{n \neq 0} c_n^{(\pm)} \bar{c}_n^{(\pm)}, \]

\[ \mathcal{D}_a = \frac{1}{2} \left\{ a, \frac{\partial}{\partial a} \right\}. \] (7)

Equations (3) and (4) imply that the covariant closed SFT action,

\[ S = \frac{1}{2} \phi \cdot Q_b \phi + \frac{1}{3} g \phi^3, \] (8)

has an invariance under the following (infinitesimal) transformation of both the string field \( \phi \) and the coupling constant \( g \):

\[ \delta_b \phi = \mathcal{D} \phi + \frac{2}{g} \sqrt{d-2} \mid \text{Dil} \rangle, \] (9)

\[ \delta_b g = \frac{d-2}{2} g. \] (10)

The light-cone gauge closed SFT vertex also has the properties (3) and (4) with \( \mathcal{D} \) and \( Q_b \) replaced by

\[ \mathcal{D}_{LC} = \frac{1}{2} \left\{ \bar{p}_-, \frac{\partial}{\partial \bar{p}_-} \right\} + \frac{1}{2} \left\{ \bar{p}_+, \frac{\partial}{\partial \bar{p}_+} \right\} + \frac{1}{2} \left\{ \bar{p}_- \frac{\partial}{\partial \bar{p}_-} \right\} + \frac{1}{2} \sum_{n \neq 0} n a_n^{(+)} \cdot a_n^{(-)} \] (11)

and
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respectively. (Our conventions are that $v_{\pm} = -(v_{0} \pm v_{d-1})/\sqrt{2}$ for any vector $v_{\mu} = (v_{0}, v_{1}, \cdots, v_{d-1})$ and $v$ denotes the transverse components $v_{i}$ ($i = 1 \cdots d-2$). The string-length $a$ in the light-cone gauge SFT is given by $a = 2p_{+}$.) Therefore, the light-cone gauge closed SFT action $S_{LC}$,

$$S_{LC} = \frac{1}{2} \Phi \cdot L_{LC} \Phi + \frac{1}{3} g \Phi^{3},$$

is invariant under the transformation (9) and (10) with $\mathcal{D}$ replaced by $\mathcal{D}_{LC}$.

At first sight, the invariance under the transformation (9) and (10) (and the corresponding one in the light-cone gauge SFT) seems to directly imply the equivalence of the dilaton shift with the transformation (2) since the dilatation operator should effect the scale change, which is equivalent to the rescaling of the length unit $\sqrt{a}$ in string theory. However, the matter is not so simple. Let us explain the reason along with the modifications made in this paper from Refs. 7 and 8).

First in the covariant SFT, the problem is that the coefficient $(d-2)/2$ in Eqs. (4) and (10) may be varied arbitrarily by changing the coefficient of $\mathcal{D}_{a}$ in $\mathcal{D}$ (6). In fact, since $\sum_{r} \mathcal{D}_{a}^{(r)}|V\rangle = (1/2)|V\rangle$, another dilatation operator

$$\mathcal{D}(a) = \mathcal{D}_{x} + \mathcal{D}_{a} + N_{a} + a \mathcal{D}_{a},$$

with a parameter $a$ satisfies

$$\sum_{r=12,3} \mathcal{D}_{a}^{(r)}(a)|V(1, 2, 3)\rangle = \frac{d + a}{2} |V(1, 2, 3)\rangle,$$

and therefore the action (8) is still invariant under (9) with this new $\mathcal{D}(a)$ and $\delta_{bg} = ((d + a)/2)g$ instead of (10) (note that $[\mathcal{D}_{a}, Q_{a}] = 0$). This is the ambiguity pointed out in Refs. 5 and 6). In Ref. 8) the choice $a = 0$ was made. This choice looks natural since the dilatation operator $\mathcal{D}(a = 0)$ does not effect the dilatation of (unphysical) string-length parameter $a$. However, in this case we have $\delta_{bg} = (d/2)g$, which does not agree with Eq. (2).

Second, in the light-cone gauge SFT, the problem is the wrong sign of the $(1/2)(p_{+}, \partial / \partial p_{+})$ term in $\mathcal{D}_{LC}$ (11). It makes the correspondence to the rescaling of $\sqrt{a}$ in Eq. (2) obscure. The same problem persists even if we adopt the dilatation operator of Ref. 7) (Eq. (8) of Ref. 7)) which completely lacks the light-cone momenta part. We adopt Eq. (11) here since it is more convenient for our present purpose. (Note that the difference, $p_{-}(\partial / \partial p_{-}) - p_{+}(\partial / \partial p_{+})$, commutes with $L_{LC}$ (12).)

Therefore, it is interesting to see how the soft dilaton theorem Eq. (1) is derived in SFTs using the properties Eqs. (3) and (4) of the 3-string vertex. We find that the above problems in two SFTs are both resolved by a careful treatment of the diagrams where the soft dilaton is attached to one of the external on-shell lines.

§ 3. Derivation of the soft dilaton theorem

Let us consider all the Feynman diagrams in the covariant closed SFT Eq. (8)
which contribute to the l.h.s. of Eq. (1). These diagrams are classified into two sets (see Fig. 1). One (called $I$) is the set of diagrams where the soft dilaton is attached to an internal line. The other set (called $E$) consists of diagrams where the soft dilaton is attached to one of the $N$ external lines. In the following we adopt the Siegel gauge $\bar{c}_0\Phi = 0$.

Note the following points on the Feynman rule:

i) The propagator is given by $\bar{c}_0/L_p$ with $L_p = -p^2/2 - 2(N_+ + N_-) + 4$, and there is a $\bar{c}_0$-integration $\int d\bar{c}_0$ for every (internal as well as external) line.

ii) The vertex with an external soft dilaton is given by Eq. (3).

The effect of the soft dilaton insertion in the $I$-diagrams is simple. Since $[\mathcal{D}, \bar{c}_0] = [\mathcal{D}, \partial/\partial \bar{c}_0] = 0$ and $Q_b$ is given as

$$Q_b = L_0 \frac{\partial}{\partial \bar{c}_0} + \bar{Q}_b + M\bar{c}_0,$$

(16)
every internal line with a soft dilaton insertion is expressed as

$$\int d\bar{c}_0 V \frac{\bar{c}_0}{L} [\mathcal{D}, Q_b] \frac{\bar{c}_0}{L} V = \int d\bar{c}_0 \left( V \bar{c}_0 \mathcal{D} V - V \mathcal{D} \bar{c}_0 \frac{1}{L} V \right),$$

(17)

where the two $V$s are the 3-string vertices connected by the internal line (see Fig. 1). Namely, the effect of the soft dilaton insertion to the internal lines is to operate $\mathcal{D}$ on every internal leg of the 3-string vertices one by one.

Next let us consider the $E$-diagrams. Here, we see that a naive application of the formula (3) is inappropriate. In fact, it gives, corresponding to Eq. (17),

$$\int d\bar{c}_0 \bar{c}_0[\mathcal{D}, Q_b] \frac{\bar{c}_0}{L} V = \int d\bar{c}_0 \bar{c}_0 \left( \mathcal{D} V - L_0 \frac{1}{L} V \right),$$

(18)

where the first $\bar{c}_0$ is the one associated with the external state. In Eq. (18) we have to take the on-shell limit $L_0 \to 0$ for the external momentum. However, this limit is not well-defined for the second term on the r.h.s. of Eq. (18) due to the presence of $1/L$.

In the correct treatment of Eq. (1) we have to first put $p_i(i = 1 - N)$ on the mass-shell and then take the zero 26-momentum limit $k_\mu \to 0$ of the dilaton. Then since the propagator on the l.h.s. of Eq. (18) has the $O(1/k)$ singularity $1/L_p + k \to -1/p \cdot k$ for the on-shell external momentum $p$, we have to keep the $O(k)$ term in the dilaton vertex (3) (cf., the derivation of the soft pion theorems\textsuperscript{9}). Let $V_{\text{full}}(k)$ denote the dilaton vertex with 26-momentum $k_\mu$ and vanishing string-length $a \to 0$, i.e., $V_{\text{full}}(k = 0) = [\mathcal{D}, Q_b]$ in Eq. (3). Then we have (see the Appendix)
where $\bar{N}_{\text{eh}}$ is the oscillator part of $N_{\text{eh}}$ of Eq. (7), and the operator $T_\mu$ is given by

$$T_\mu = \sum_{\pm} \sum_{n+i,n'\neq 0} \frac{1}{n} \alpha_{\mu}^{(\pm)}(\alpha_{\mu}^{(\pm)} \cdot \alpha_{\mu}^{(\pm)} + 2i \gamma^{(\pm)} \cdot \gamma^{(\pm)}).$$

In Eq. (19) we have dropped the terms which do not contribute to the l.h.s. of Eq. (1). They are i) the terms not proportional to $\partial/\partial \bar{c}_0$, ii) the $O(k)$ terms which do not commute with the mass-operator $N_+ + N_-$, and iii) the $O(k^2)$ terms.

Now let us reconsider the $E$-diagrams using Eq. (19). Take a diagram where the soft dilaton attaches to the external leg of one of the $N$ particles with on-shell momentum $p$ satisfying $p^2 = -m^2$ (i.e., $p$ is one of $\Pi_i$ in Eq. (1)). Then, we have

$$V_{\text{Dil}}(k) \frac{\bar{c}_0}{L_i} \rightarrow \mathcal{D} \left\{ \frac{1}{2} \left( \frac{p_i \cdot \partial}{\partial p_i} \right) \right\} - \bar{N}_{\text{eh}} - 1 + \frac{1}{k \cdot p} \left( m^2 + k \cdot T \right),$$

where $V_{\text{Dil}}(k)$ on the l.h.s. should be understood to be Eq. (19) without the factor $\exp\left(k_\mu (\partial/\partial p_\mu)\right)$. In deriving Eq. (21) we have used the fact that only the $(1/2)(p_i \cdot \partial/\partial p_i) + \bar{N}_{\text{eh}} - 2 \mathcal{D} \alpha$ part in $\mathcal{D}$ (6) can contribute to $L_{p+k} \mathcal{D}(1/L_{p+k})$ since the other terms change the mass-level and their contribution vanishes safely in the limit $k \to 0$ for on-shell $p$ with $L = 0$. Note that the term $2(p \cdot k) \mathcal{D} \alpha$ of Eq. (19) contributes $2 \mathcal{D} \alpha$ to Eq. (21), which just cancel the same term from $L_{p+k} \mathcal{D}(1/L_{p+k})$.

Now take a diagram $G$ contributing to the r.h.s. of Eq. (1) and consider all the corresponding diagrams contributing to the l.h.s., i.e., all the $I$- and $E$-diagrams obtained from the diagram $G$ by a soft dilaton insertion. Denoting by $\bar{A}^{(N)}$ the amplitude multiplied by the momentum and the $\alpha$ conservation delta-functions, i.e., $\bar{A}^{(N)}(p_i) = A^{(N)}(p_i) \times (2\pi)^{d/2} \delta(\Sigma_i p_i) \delta(\Sigma_i \alpha_i)$, we have

$$\bar{A}^{(N+1)}(k; p_i) \sim k_{-1} \frac{d-2}{2} v_c$$

$$+ \sum_{i=1}^{N} \left( \frac{1}{2} \left( \frac{p_i \cdot \partial}{\partial p_i} \right) - 1 \frac{1}{k \cdot p_i} \left( m_i^2 + k \cdot T \right) \right) \bar{A}^{(N)}(p_i),$$

where the subscript $G$ on $\bar{A}$ indicates that we are considering only the contribution of the diagram $G$ (and its soft dilaton insertion), and $v_c$ is the number of the 3-string vertices in $G$. Note that the $I$-diagrams and the $\mathcal{D}$ term on the r.h.s. of Eq. (21) change every 3-string vertex $V$ in $G$ into $V \mathcal{D}^{(r)} V$ one by one, which owing to Eq. (4) gives the $v_c$ term in Eq. (22). Note also that $\bar{N}_{\text{eh}} = 0$ in Eq. (21) for the physical external state.

The last term of Eq. (22) proportional to $1/(k \cdot p_i)$ vanishes for massless external states owing to $T_\mu \langle \text{massless} \rangle = 0$. The soft dilaton theorem, Eq. (1), for massless external states is immediately obtained from Eq. (22) by making the substitution $v_c = g(\partial/\partial g)$, restoring $a'$ (which has been set equal to one), using the dimension counting formula
and summing over the diagrams $G$.

If we express the r.h.s. of Eq. (21) in terms of $\mathcal{D}(a)$ of Eq. (14) instead of $\mathcal{D} (=\mathcal{D}(a)-(2+a)\mathcal{D}_a)$ and proceed in the same manner using Eq. (15), the quantity operating on $\bar{A}_c^{(\nu)}(\tilde{p}_i)$ on the r.h.s. of Eq. (22) is replaced by

$$-rac{d+a}{2} v_c + \sum_{i=1}^{\nu} \left( \tilde{p}_i, \frac{\partial}{\partial \tilde{p}_i} \right) -1 + (2+a) \mathcal{D}_a^{(\nu)} + \frac{1}{k \cdot \tilde{p}_i} \left( \ldots \right).$$

For the tree diagrams the dependence of $\bar{A}(\nu)$ on $a_i$ is only through $\delta(\Sigma a_i)$, and we have

$$\sum_{i=1}^{\nu} \mathcal{D}_a \bar{A}_c^{(\nu)} = (-1+N/2) \bar{A}_c^{(\nu)}.$$ Using the (tree) relation $N=v_c+2$, the $a$-dependence disappears and we obtain the same soft dilaton theorem as Eq. (1).

For the loop diagrams we have two problems. One is that we still do not have a complete quantization procedure for the closed SFT of Eq. (8) applicable to higher loops, although there are promising proposals made in Ref. 10. (The argument of this paper leading to Eqs. (22) and (24) applies even if we add the quantum correction of Ref. 10.) The other problem is that the $l$-loop diagram contains a divergent factor $(\int da/2\pi)^l$ which should formally be absorbed into $\hbar$. If we assume that there is a suitable regularization method in which the naive $a$-dimension counting formula

$$\sum_{i=1}^{\nu} \mathcal{D}_a \bar{A}_c^{(\nu)} = (-1+ l_c + N/2) \bar{A}_c^{(\nu)},$$ holds ($l_c$ is the number of loops in the diagram $G$), we again obtain the soft dilaton theorem Eq. (1) using the relation $l_c=(v_c-N+2)/2$.

The derivation of the soft dilaton theorem (1) in the light-cone gauge SFT is quite similar to the covariant case given above. It is simplest to consider a dilaton carrying a momentum $k\mu=(k^+=\epsilon/2, k^-=0, k)$. Then in the limit $\epsilon \to 0$ the dilaton vertex in the light-cone gauge SFT is given by an expression similar to Eq. (19),

$$V_{\text{diff}}^l(k) = \left[ (\mathcal{D}_+, L_{\text{lc}}(p+k)) + 2(k \cdot p) \mathcal{D}_+ + k \cdot T_{\text{lc}} \right] e^{k \cdot (\partial/\partial p)} + \ldots$$

where $\mathcal{D}_+ = (1/2)(\partial_+, \partial/\partial p_+)$, and $T_{\text{lc}}$ is given by Eq. (20) with the ghost and the light-cone oscillators removed. Therefore, corresponding to Eq. (21), we have

$$V_{\text{diff}}^l(k) \frac{1}{L_{\text{lc}}(p+k)} \frac{1}{k \cdot \tilde{p}_i} \mathcal{D}_+ - \frac{1}{2} \left( \partial_{\tilde{p}_i} \frac{\partial}{\partial \tilde{p}_i} \right) -1 + \frac{1}{k \cdot p} (m^2 + k \cdot T_{\text{lc}}).$$

Note that the $(1/2)(\partial_+, \partial/\partial p_+)$ term, which had the opposite sign to the other components in $\mathcal{D}$ (6), now has the correct sign in the last round bracket of Eq. (26) owing to the contribution of the $2(k \cdot \tilde{p}_i) \mathcal{D}_+$ term of (25). The rest of the argument leading to the soft dilaton theorem (1) is exactly the same as in the covariant case.

Appendix

In this appendix we summarize the point in the derivation of the $O(k)$ part of the dilaton vertex (19). See Appendix B of Ref. 8) for the derivation of the $O(1)$ part.

Let us consider the l.h.s. of Eq. (3) for a dilaton state $\langle \text{Dil}(1) \rangle$ carrying the 26-momentum $p_{\mu}=k\mu$ and the string-length $a_1=\epsilon$. We take the limit $\epsilon \to 0$, and expand in powers of $k$. The necessary formulas of the Taylor expansion in terms of
$y = \epsilon / \alpha_0$ of the quantities in the vertex $V(1, 2, 3)$ are given in Appendix A of Ref. 8) except the following two:

$$\alpha_0 = e \left( \ln \frac{y}{e} \right) - \frac{y}{2} + \cdots ,$$

$$\bar{N}_n = \frac{1}{n!} \left( \frac{n}{e} \right)^{n-1} \frac{1}{\epsilon \alpha_0} \left[ \text{sgn}(y) \right]^n \left(1 - \frac{y}{2} + \cdots \right),$$

where the dots $\cdots$ denote the $O(y^2)$ terms. Note, in particular, that Eq. (28) and $P = \alpha_0 k - \epsilon \alpha_0$ gives the expansion

$$\bar{N}_1 \alpha^{(\pm)} \cdot P = \frac{y}{e} \left[ \alpha_0^{(\pm)} \left( \frac{1}{y} k - \left( p_3 + \frac{k}{2} \right) + \frac{y}{2} p_3 \right) \right],$$

which contains the $O(y^2)$ term for a non-vanishing $k_n$. (The $O(y^2)$ part of Eq. (28) also contributes to the $O(y^2)$ term of Eq. (29). However, it is multiplied by $k_n$ and is unimportant here.) Then a straightforward calculation gives

$$V_{DL}(k) = \left[ (D, L_{\rho + \gamma}) + k_n Q_n \right] e^{\delta(k \rho_0 + \lambda_2 \rho_0)} \frac{\partial}{\partial \rho_0} + \cdots$$

with $Q_n$ given by

$$Q_n = 2 \left( \frac{\alpha_0}{e} - \bar{N}_n \right) p_{\nu} - 2 \sum_{n \neq 0} \sum_{n_{i+1}, m \neq 0} \frac{1}{n} \alpha_0^{(\pm)} \alpha_n^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}$$

$$+ \sum_{n \neq 0} \sum_{n_{i+1}, m \neq 0} \frac{1}{n} \alpha_0^{(\pm)} \alpha_n^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}$$

$$- 2 \sum_{n \neq 0} \sum_{n_{i+1}, m \neq 0} \frac{1}{n} \alpha_0^{(\pm)} \alpha_n^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} + i \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}.$$

The effect of the exponential term in Eq. (30) is to replace $\delta(p_3 + p_3) \delta(e + \alpha_0 + \alpha_0)$ in $|R(2, 3)\rangle$ by $\delta(k + p_3 + p_3) \delta(e + \alpha_0 + \alpha_0)$. Among many terms in $Q_n$ we shall comment on the origin of $\bar{N}_n$. It came from the $(r, s) = (2, 3)$ and $(3, 2)$ part of $i \sum \sum \sum_{N_{n_{i+1}} m \neq 0} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}$ by taking the $y$ part of $\gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} = i n(1 + y) c_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}$ and $\gamma_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)} = i n(1 - y) c_{(\pm)}^{(\pm)} \gamma_{(\pm)}^{(\pm)}$, and the $y^0$ part of $\bar{N}_{n_{i+1}}$ ($y$ is canceled by $1/y$ from Eq. (29)). The $D$ term in Eq. (19) is obtained by taking the symmetric limit $(\lim_{\epsilon+0} + \lim_{\epsilon-0})/2$ in Eq. (30).

References