2 There was excellent agreement between the experimental results and the finite-element results, with the experimental results closely approaching the fixed case.

3 At a temperature of 70 deg F and a maximum of 50-hr load duration, the acrylic hull can operate to 1000 ft with a safety factor of 1.5 based on yield and a safety factor of 2.6 based on collapse.

4 At a temperature of 70 deg F and a load duration of 50 hr or less, the top hatch and bottom plate determine the operating depths; at 50 hr or greater, the acrylic hull governs the operating depths.

5 The maximum stress concentration in the acrylic hull was at the acrylic–top-hatch interface for a fixed-boundary condition. The concentration-effective-stress to membrane-effective-stress ratio was 1.3.

6 The maximum stress concentration in the steel penetrations was in the top hatch for a fixed-boundary condition. The concentration-effective-stress to membrane-effective-stress ratio was 2.7.

7 The stress concentration in the acrylic hull due to the steel penetrations dissipated within 15 deg of the acrylic-steel interface.

## Acknowledgments

This work was sponsored by the Naval Facilities Engineering Command. Execution of the strain-gage tests and subsequent data reduction at NCEL was performed by the NCEL Deep Ocean Simulation Laboratory staff and Mr. Kit Mack of NCEL, respectively. Mr. Ed Briggs of SWRI and the SWRI Underwater Engineering Laboratory staff performed the strain-gage tests at SWRI.

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# DISCUSSION

## J. D. Stachiw[4]

Messrs. Snoey and Katona are to be commended for applying a finite element computer code to the elastic stress analysis of the complex NEMO acrylic hull whose design, fabrication, and experimental stress analysis were performed at the Naval Missile Center, Pacific Missile Range and Naval Civil Engineering Laboratory [17–20].[5] Similar computer study was performed in 1968 [21] on the same NEMO hull by the Southwest Research Institute utilizing the SEAL-SHELL-2 computer code [22]. The results are almost identical to those of this paper confirming the accuracy of the finite element elastic stress analysis for the NEMO hull performed by Messrs. Snoey and Katona. The novelty of the latter stress analysis lies in (*a*) combining the findings of the elastic stress analysis with time dependent yield stress curves for acrylic material and (*b*) using this as a basis for rating the operational depth of NEMO acrylic hull. On the basis of this synthesis, a safe operational depth of 1,000 ft has been assigned by the authors to NEMO hull for mission durations under 50 hrs. Furthermore, it is stated that at 1,000 ft depth and 50 hr mission duration, the hull possesses a safety factor of 1.5 based on yield of acrylic and 2.6 based on implosion of the hull. It is this synthesis of elastic stress analysis with plastic behavior of material that makes the conclusions of the paper rather questionable, particularly conclusion 3 assigning a 1,000 ft operational depth to the NEMO acrylic hull.

The basic shortcoming of the paper is its total neglect of (*a*) material fatigue associated with cyclic pressure loadings encountered by operational submersibles, and (*b*) deviations in sphericity and wall thickness of the fabricated hulls from nominal dimensions. The omission of these factors from the paper and particularly the lack of discussion on their relevance to assigning a safe operational depth is most disturbing. It may mislead the users of NEMO acrylic hulls into thinking that the fatigue life of NEMO hull at 1,000 ft is of no concern, and only the 50 hr mission limit need to be considered at that depth.

Nothing could be further from reality. Both the fatigue life and allowable dimensional deviations of the NEMO type hull form the basis of certification given to the NEMO submersible by the U. S. Navy [23] and to the Johnson Sea-Link submersible by the American Bureau of Shipping [24]. It was because of material fatigue and dimensional deviation considerations that only a 600 ft operational depth was assigned to the 66 in. dia, 2.5 in. thick NEMO hull by Dr. Stachiw (NEMO Project Senior Engineer) and approved by the U. S. Navy as valid. The case is similar for the Johnson Sea-Link submersible except that here the 1,000 ft operational depth assigned to the 66 in. dia, 4 in. thick acrylic hull by Dr. Stachiw, Dr. Maison, and Dr. Ottsen, has been approved by the American Bureau of Shipping. In both cases the fatigue life of the acrylic hulls has been established as a function of dimensional deviations, operational pressure, number of cycles and duration of individual cycles.

In addition to the basic shortcomings of the paper discussed above, there are also some numerical errors. For example, when in Fig. 12 all of the experimental points denoting visco-plastic buckling of NEMO models are plotted properly, the collapse depth curve is displaced 1000 ft closer to computer calculated

[4] Advanced Concepts Research Consultant, Naval Undersea R & D Center, San Diego, California. Mem. ASME.

[5] Numbers in brackets designate Additional References at end of discussion.

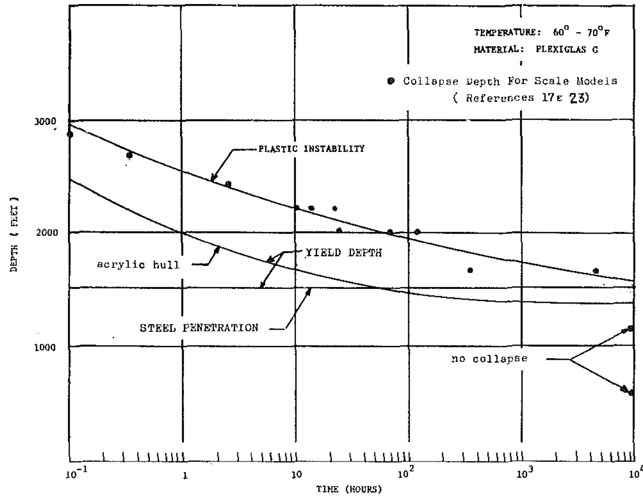


**Fig. 14 Properly plotted Fig. 12**

yield depth curve. Furthermore, the yield strength of AISI4130 annealed steel used in NEMO hulls 1, 2, and 3 is 45,850 psi [20] rather than 55,000 psi mentioned in the paragraph on Failure Criterion. Also, the hatch shown in Fig. 13 failed [20] at 2240 ft after 228 pressure cycles to 2,000 ft rather than after a single test to 2,000 ft as implied by the figure caption.

It is indeed unfortunate that the authors after concluding a good computer analysis of the acrylic hull exceeded its basic premises to the extent of assigning the hull an operational depth without taking into consideration the fatigue life of acrylic plastic under stress conditions encountered in the NEMO hull at the steel closure/acrylic interface where the stresses are the highest. However, if the readers temper the conclusions of this paper with fatigue life and dimensional deviation, considerations discussed in other publications [17–20] the paper becomes a valuable addition to acrylic hull literature.

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## Authors' Closure

The authors are grateful to Dr. Stachiw for the time and effort he spent in preparing the above discussion. Dr. Stachiw, who has provided invaluable empirical data on acrylic plastic structures, is certainly a pioneer in this field. The authors consider it a privilege to have participated in the NEMO project with Dr. Stachiw and to have complemented his acrylic hull testing program with a finite element stress analysis [10, 25].[6]

Dr. Stachiw states that the authors' stress analysis is similar to that performed by DeHart, et al. [6]. Many readers would recognize, however, that the analyses differ in four ways:

1 When Dr. DeHart, who is one of the foremost authorities on undersea structures, performed his stress analysis he used the Seal-Shell-2 computer program which is applicable to elastic thin shells of revolution. The authors, on the other hand, used Wilson's 1967 axisymmetric-solid computer code which provided a complete stress distribution across the wall thickness. Moreover, the axisymmetric-solid code is applicable for any wall thickness and has none of the kinematic assumptions, shear distribution assumptions, and contradictions inherent in thin shell theory.

2 In their analysis the authors placed particular emphasis on the acrylic-steel interface. While Dr. DeHart used a fixed continuum at the interface, the authors postulated that two boundary conditions, fixed and free, would bracket the actual case. To analyze the "free" interface condition, the authors programmed and incorporated into Wilson's code the general capability of analyzing structural boundaries that transmit only normal forces across a common interface of arbitrary orientation. The scope of this routine allows the consideration of generalized plane stress and plane strain as well as axisymmetric structures. The defined interface is completely arbitrary in that any piecewise continuous or discontinuous curve may be assumed. Moreover, the interface may be assumed fixed, i.e., a continuum, as well as free; thus, mesh numbering and configuration remain exactly the same so that the comparison of fixed versus free solutions was not confounded with mesh variations. In addition, the program was coded to allow for ease in making future additions such as iterative procedures to handle friction response and tensile cracking.

3 The authors attempted an idealization of the steel penetrations while their colleague used a simplified design for both the top and bottom steel penetrations. As will be shown later, the influence of the steel penetrations is such that an exact analysis is necessary.

4 A time-dependent, yield-stress failure criterion based on published experimental data and rational mechanics was developed and used to determine the stress analysis from the finite element results. This represented a breakthrough in acrylic hull analysis, because besides achieving excellent correlation with experimental tests, the analysis was also used to predict the structural capacity of the hull. To this end, the operating depth curve in Fig. 12 was established.

Dr. Stachiw states that Fig. 12 is in error and suggests that Fig. 14 is correct. Since the $t/b$ ratio for the scale models was not equal to the $t/b$ ratio for the full-scale hull analyzed, Dr. Stachiw's substitution of figure 14 for Fig. 12 renders the comparison invalid. The authors overcame the ratio problem in deriving Fig. 12 by using a compensation factor. The classical stability equation is as follows [26]:

$$p_{cr} = \frac{2E}{\sqrt{3(1 - \nu^2)}} \left(\frac{t}{b}\right)^2 \qquad (3)$$

where:

$p_{cr}$ = critical collapse pressure, psi
$E$ = modulus of elasticity, psi
$\nu$ = Poisson's ratio
$t$ = wall thickness of sphere, in.
$b$ = outside radius of sphere, in.

The terms in equation (3) can be separated into two categories: (*a*) material properties and (*b*) geometrical properties. Since the models and full-scale hull were fabricated of identical ma-

[6] Numbers in brackets designate References at end of paper and Additional References at end of closure.

terial, Plexiglas G, it was necessary only to scale by the geometrical size factor, $(t/b)^2$:

$$\frac{(t/b)^2 \text{ Full Scale}}{(t/b)^2 \text{ Model}} = \frac{\left(\frac{2.5}{33}\right)^2}{(0.5/7.5)^2} = 1.29$$

Thus the use of Fig. 14 is inappropriate and Fig. 12 is indeed the correct one.

Dr. Stachiw states that the yield strength of the actual steel used in NEMO was 45,850 psi, whereas the authors used the handbook value of 55,000 psi for the annealed 4130. The 45,850 psi yield strength lowers the steel penetration line in Fig. 12 to 1250 ft, while the collapse depth line remains the same. Thus the hatch becomes even more of a severe constraint to the full-depth potential of this acrylic hull.

There are two methods for improving this inadequate hatch design in the future: (*a*) increase the yield point of the hatch material or (*b*) change the hatch geometry. In the latter method, a decrease in the spherical radius of the hatch would eliminate the deleterious effect of the high bending in the "neck" section of the hatch. Ideally the radius of the centerline of the steel hatch should equal the radius for the center of pressure of the normal stresses across the thickness of the acrylic hull. The radius for the center of pressure is:

$$\bar{r} = \frac{a^3 b[ln(b/a)]}{b^3 - a^3} + \left(\frac{2}{3}\right)b \tag{4}$$

where:

$\bar{r}$ = radius for center of pressure, in.
$a$ = inside radius of sphere, in.
$b$ = outside radius of sphere, in.

Substituting the appropriate values into equation (4), the radius for the center of pressure, $\bar{r}$, is found to equal 31.4 in. Thus the centerline of the steel penetration should be located 36 percent of the wall thickness from the inside surface of the acrylic hull as shown in Fig. 15. This modification allows the flanges to function primarily as a transition zone for stresses and not to resist bending moments.

Dr. Stachiw felt that the hull analyzed by the authors should be limited to a 600 ft depth due to possible fatigue failure. In the acrylic sphere the cycling damage appears as circumferential cracks at the acrylic-steel interface. This is to be expected as the stress analysis indicates the highest effective stresses are located in this region. There are three reasons why the authors recommend a 1000 ft depth instead of a 600 ft limit:

1 Dr. Stachiw's conclusion with regard to the 600 ft depth was reached after cyclic testing of scale model hulls [2]. As indicated previously the $t/b$ ratios were different which resulted in the full-scale hull's collapse depth being 29 percent higher than the scale model. Using Lame's equations for a thick-walled sphere in conjunction with equation (1), it is found that the full-scale hull's internal membrane effective stress is 12 percent lower than that in the scale model. This reduction in effective stress is significant since a small reduction in stress level greatly increases the number of possible cycles on an $S$-$N$ curve.

2 As mentioned in the main text, the normal force gradient across the thickness of the acrylic gives rise to a frictional shear force. These resulting imposed strains across the thickness, in conjunction with the already present Poisson strain due to compression of the hull, will create high tensile strains in the acrylic. Now if a stress discontinuity, such as an O-ring groove, is superimposed, resulting in even higher localized stresses, then the fatigue life of the hull will certainly be lowered. And in fact that is exactly what happened in the scale models; the cracks and crazing appeared first at the location of the O-ring groove [2]. It is hypothesized that the O-ring groove's effects will become progressively more deleterious with longer load durations since

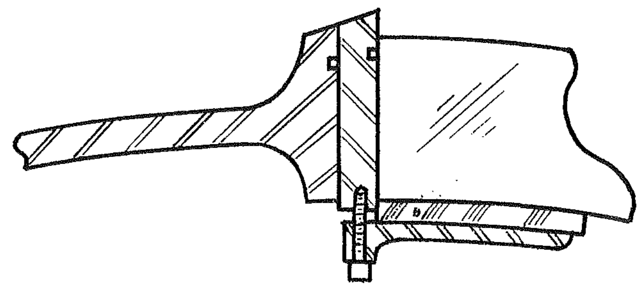

**Fig. 15 Recommended design changes for top hatch (compare with Fig. 2*a*)**

the acrylic may extrude into the O-ring groove causing severe cracks as the pressure is released. It is therefore recommended that the O-ring groove be strategically relocated near the external surface of the hull as shown in Fig. 15. It was extremely encouraging to the authors to see the O-rings in the JOHNSON-SEA-LINK hull located near the external surface [27].

Two advantages are gained by moving the O-ring groove out to within $^1/_2$-in. of the external surface: (*a*) Moving the groove will lower the stresses in the acrylic around the O-ring and (*b*) it will minimize seawater contact with the steel surface. It is also important to periodically (e.g. every 100 dives) replenish the lubricating grease at the acrylic-steel interface to reduce the frictional shear force.

As further evidence of the hull's potential, a full-scale hull of this design (O-ring location as in Fig. 2) was cycled 815 times to 1238 ft (550 psi) with no sign of crazing or cracks anywhere [20]. The test was run at 70 deg F with a one-hour dwell period for both load-on and load-off conditions.

3 Some fatigue data is available for acrylic, although it is by no means extensive [15, 28]. Since the fatigue curves are from uniaxial tests, no exact correlation is possible with the multiaxial stress states in the acrylic. Thus it is difficult to do a quantitative fatigue analysis. However, a qualitative judgment is possible. It was found that acrylic plastic has a well-defined $S$-$N$ curve with a knee at 6500 psi tension and 10,000 cycles. The maximum effective stress in the acrylic hull is, on the other hand, only 4050 psi in the all-compressive octant at 1000 ft.

Due to page limitations the authors did not, of course, discuss every detail of acrylic hulls in their original paper. While Dr. Stachiw's discussion permitted additional information on some of the aspects relating to operational depth and length of service, there are still other important items such as impact strength, weathering, and sea water degradation to consider. These items, as well as the integration of the acrylic hull with the rest of the submersible, i.e., connection points, are covered in more detail in reference [10].

The authors made no mention of tolerances because it was felt that with (*a*) the excellent correlation between experimental strain gage data and finite element results, and (*b*) the 1.5 safety factor based on yield and the 2.6 safety factor based on collapse, tolerances equal to or better than those used in the tested hulls would be satisfactory.

The authors stand corrected on the title of Fig. 13. However, the correct history of the hatch before the plastic hinge completely formed at a 2240 ft depth was 815 cycles to 1238 ft, 178 cycles to 1500 ft, and 257 cycles to 2000 ft [20].

The statement Dr. Stachiw makes about the full-scale hull being certified to 600 ft is correct. He neglects to point out, however, that no matter what depth the highly competent Navy and American Bureau of Shipping Certification Boards feel is possible, they cannot certify beyond the depth requested by the operating agency. It is important that future designers not be constrained by a 600-ft depth limit because, as shown, this acrylic hull can and should be certified to 1000 ft.

In conclusion, utilizing a thorough finite element stress analysis, the authors have shown that the 66 in. OD $\times$ 61 in. ID acrylic hull can realize its full-depth potential of 1000 ft, including fatigue and tolerance considerations, with a properly designed, hatch. Also the paper points out the urgent need for more data on material properties of acrylic plastic (e.g., uniaxial and multiaxial fatigue relationships) to accompany the sophisticated stress analysis tools available today. This same conclusion was also reached in the excellent paper by Maison and Ottsen after their stress analysis of the JOHNSON-SEA-LINK hull [27].

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