at the mean of the extreme values of Pr in the particular problem under consideration.

Table 5(*b*) Reference-temperature relations for liquid mercury. (For use with Figs. 4*a* and 4*b*.)

	$(T_r - T_w)/(T_\infty - T_w)$
h	0.30
$\delta_{0.05}$	0.60
u_{max}	0
y_u	0
c_f	0.10
W	0.40

Conclusion

From the findings reported here, it appears that free-convection heat-transfer under variable-property conditions can be computed quickly and accurately by using the constant-property results in conjunction with reference-temperature relations. For gases and liquid mercury, the reference-temperature relations are given by Equations [20] and [23], respectively. Further, it may be observed that the film temperature appears to serve as an adequate reference temperature (with $\beta = 1/T_\infty$ for gases) for most engineering purposes.

Reference-temperature relations for use in computing boundary-layer thickness and velocity parameters for variable-property conditions are given in Tables 5(*a*) and 5(*b*).

The results obtained here are based on laminar boundary-layer theory and are expected to be applicable within the range of validity of this model. The extension of the results to situations outside this range is uncertain.

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Discussion

J. R. Moszynski.[9] The authors are to be complimented on their attempt to tackle a very laborious, albeit important, problem and their results are particularly gratifying inasmuch as they show that the comparatively simple constant-property analysis may, with small modifications, yield accurate heat transfer predictions under conditions where the variations of thermodynamic properties are of importance.

While it is perhaps surprising that an exact solution for gas A can be approximated so well by the use of a single reference temperature over a wide range of T_w/T_∞, it may be expected that in the particular cases presented in Table 3 this same reference temperature used in conjunction with the constant-property solutions will give fairly accurate results. For the case of $T_w/T_\infty = 3$ a comparatively simple computation will show that the expression $\overline{\text{Nu}}_{L,w}\Big/\frac{4}{3}\,\text{Gr}_{L,w}{}^{1/4}$ is rather insensitive to the exponent n of the power law used to represent the variation of thermal conductivity and viscosity. Thus for example if $k \sim T^{1/2}$; $\mu \sim T^{1/2}$

$$\overline{\text{Nu}}_{L,w}\Big/\frac{4}{3}\,\text{G}_{L,w}{}^{1/4} = 0.379$$

and from Table 2 we obtain 0.368 for $n = 3/4$.

Furthermore it may be shown that for the cases of $n = {}^1/_2$ and $n = {}^2/_3$ the application of the usual film temperature $T_f = T_w - 0.5(T_w - T_\infty)$ with $\beta = 1/T_\infty$ yields values not too widely divergent from those obtained with the authors' reference temperature rule.

For the region of high absolute temperatures, investigated in the paper, the Sutherland-type equations of gases D and E differ but little from Maxwell's equations for viscosity and thermal conductivity and, in the light of the foregoing, the good agreement shown in Table 3 is to be expected.

It should be worth while, however, to investigate the accuracy of the reference temperature rule for similarly high values of T_w/T_∞, but at lower temperatures where the Sutherland equations diverge more from the simple approximation $\mu \sim T^n$; $k \sim T^n$.

The writer would also like to know whether the reference temperature relations of Table 5*a* have been tested for several types of property variation similarly to the heat transfer results.

Authors' Closure

The authors wish to extend their thanks to Dr. Moszynski for his interest and comments. The further tests of the reference temperature procedure for the very low temperature range, as suggested by him, would be interesting; but such a study is not feasible at present. With regard to the question about the reference temperatures of Table 5(*a*), all were derived and tested in a manner similar to that described for the heat transfer. Further details may be found in chapter 4 of reference (1).

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