Table 1
Geometric Configuration

γ_ρ	$= 0.489$	b/ℓ	$= 0.207$
γ_c	$= 0.0978$	a/b	$= 0.25$
ρ	$= 0.0000654$	μ	$= 0.957$

Table 2
Selected Data

Figure	c/ℓ	μ	λ_s	$(\beta\ell)_s$ 1	2	3	4
2	0.5	0.1	0	2.585	3.391	6.296	8.416
2	0.5	1.00	0	2.585	5.171	7.005	8.416
2	0.5	100.	0	2.585	5.630	8.416	9.180
3	0.25	0.1	0	2.748	3.459	5.464	8.719
3	0.25	1.00	0	2.761	5.365	5.816	9.143
3	0.25	100.	0	2.762	5.431	7.495	9.915
4	0.5	.957	1.0	2.585	6.642	8.416	
4	0.5	.957	0.0625	2.584	5.330	7.071	8.416
4	0.5	.957	−1.0	2.585	4.132	6.957	8.416
5	0.25	.957	1.0	2.806	5.477	9.251	
5	0.25	.957	0.0625	2.765	5.403	5.996	9.152
5	0.25	.957	−1.0	2.713	4.492	5.519	

(b) Natural frequencies may be significantly affected by disk flexibility.

Both general conclusions are based upon the first three natural frequencies or critical speeds and are supported by results covering a wide range of parameters. The differences between the solid and dashed lines on Fig. 4 and 5 illustrate the effect of disk flexibility on shaft natural frequencies. For $\lambda_s = 0.5$ there is no effect, and the effects are not large for λ_s greater than 0.5 (encompassing $\lambda_s = 1$, critical speeds). The value of λ_s for which disk flexibility has the largest effect occurs between approximately $\lambda_s = 0.1$ and $\lambda_s = -1.0$. The effect of disk flexibility on non-rotating natural frequencies ($\lambda_s = 0$) is seen to be indicative of the effect of disk flexibility on rotating natural frequencies for $\lambda_s < 0.5$. This is particularly convenient since it therefore appears reasonable to establish the significance of disk flexibility for rotating systems from the nonrotating results (i.e. $\lambda_s = 0$).

The body of non-rotating results that have been produced show that the influence of disk flexibility depends upon all of the dimensionless geometric parameters, not only the flexibility parameter μ. In no case, however, was the effect of disk flexibility found to be of practical significance in the calculation of the first three natural frequencies when μ is larger than 10. For $\mu < 10$, the effects of disk flexibility may be quite pronounced as Figs. 2 and 3 illustrate.

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DISCUSSION

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The authors' conclusion that critical speeds are not significantly affected by disk flexibility may be misleading because it does not account for asymmetry of the support structure typical in most applications, and especially in aircraft engines where flexible disks are common. In such applications, the shaft motion is elliptical rather than circular as in the axisymmetric problem treated here. Circular motion of the shaft forces the disk to flex into a stationary wave during synchronous whirl; elliptical motion forces a traveling wave into the disk. Possible high dynamic response of the traveling wave may produce powerful interaction between the shaft and the disk.

The remarks regarding previous papers indicate that the authors were not yet aware of a recent paper which deals with the general system encountered in practice. This discusser has presented, in ASME Paper 74-GT-159, "Theory of Rotor Dynamics with Coupling of Disk and Blade Flexibility and Support Structure Asymmetry," a method for analyzing practical systems involving elliptical shaft motion. This method lumps the dynamic moment at each disk, of which the pertinent response characteristics are assumed known, into an equivalent gyroscopic moment. The complete system is then analyzed by the familiar Prohl method. This traditional method is extended to solve simultaneously for two mode shapes of shaft deflection needed to define the elliptical motion.

Engine designers may find possible generation of traveling waves in bladed disks more serious than changes in critical speeds due to disk flexibility. Nevertheless, insurance against such waves requires analysis of the complete system. Therefore, the authors have broached a problem of current significance to engine design. However, the scope of their investigation falls short of yielding new knowledge of value to designers. In fact, their conclusion might be misleading.

Authors' Closure

The authors acknowledge that the results and conclusions presented do not include the effects of support asymmetry, and that these effects are of practical interest. The geometric configuration and its modeling are described in the introduction. We do not, however, agree with Mr. Klompas that the conclusions may be misleading. The statement that "possible high dynamic response of the traveling wave may produce powerful interaction between the shaft and the disk" relates not to critical speeds but to a phenomenon similar to the excitation of a general natural frequency. Figs. 2–5 illustrate that disk flexibility can have a profound effect on natural frequencies, and this is presented as general conclusion *(b)*.

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