paper is complete in that it is presented in terms of loads and arch displacements. The reader should note that the resulting exact solution also exhibits equilibrium branches not discussed in the previously stated papers. These branches are discussed in reference [20].

Thanks are extended to the discussers, Professors Koiter, Masur, and Smith, for their comments. It is hoped that their comments and the foregoing reply will contribute to the clarification of a fundamental question in the theory of stability of conservative elastic solids.

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An Asymptotic Solution for Laminar Flow of an Incompressible Fluid Between Rotating Disks[1]

M. DeSANTIS,[2] L. GALOWIN,[2] and E. RAKOWSKY.[2] The authors' solution for this problem illustrates the viscous dissipation of angular momentum in the radial inflow between parallel disks and is of special interest since we are currently investigating a similar phenomenon.[3] Results which display very similar behavior to the theoretical values of the radial velocity profiles were obtained in our experiments for $\Omega = 0$ with $h/r_0 = 0.031$. Development of nonsimilar velocity profiles from "fully developed" is anticipated in the presence of a varying strong favorable pressure gradient, e.g., see comments by Savage, reference [1].[4]

The flow field in a sink vortex device was investigated with application to a flueric angular rate sensor and was reported in reference [2]. In that study the development of the radial inflow of the fluid between parallel coaxial disks into an axisymmetric outlet was determined. Experimental velocity profiles were measured by hot-wire anemometry, and numerical solution of the axisymmetric Euler equations was obtained from a computer program. Comparisons of the radial velocity component, taken from Fig. 3,[1] with our experimentally measured values indicate substantially the same development with decreasing r, as shown in Fig. 1. Differences are attributed to the zero angular rate in the experiments.

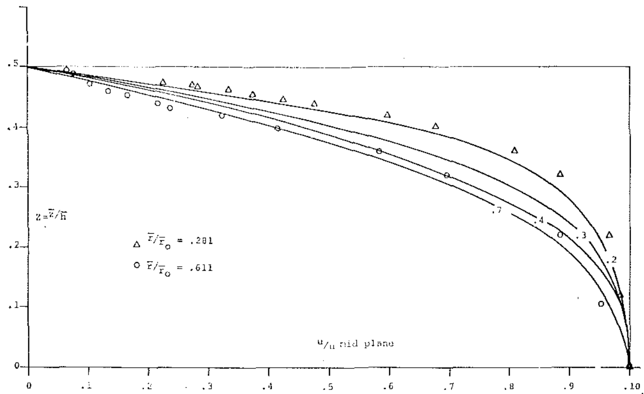

Fig. 1 Velocity profiles

Numerical results obtained from the inviscid analysis for $r \to 0$ indicate local maxima in the radial velocity component, i.e., inflections do occur in the profiles approaching the axis of symmetry.

In our case the pressure field is a function of r and z so that the adverse pressure gradient in the vicinity of the stagnation point decelerates the flow. Outside that region the flow is turned and accelerated into the outlet. In the analysis presented in the subject paper, $p(r)$ only is assumed, but with the varying centrifugal field (due to the z distribution of the v^2/r term becoming increasingly large at small z and decreasing r) the radial inflow is then subjected to an effective $p(r, z)$. A mechanism then becomes apparent for radial decelerations about $z = 0$ for decreasing r and inflections develop in the profile. A mechanism for acceptance of profile solutions with velocity overshoot is postulated and discussed in a Note on the Falkner-Skan equation by Libby and Liu, see reference [3]. Consequently, the interpretation that inflections in the radial velocity distributions are necessarily associated with the onset of transition to turbulence is speculative and unwarranted.

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Authors' Closure

The authors appreciate the references to experimental work in the subject area dealt with analytically in the paper. The discussion concerning the meaning of inflected profiles is welcomed by the authors; in particular, the conclusion reached by the discussers is also held by the authors at the time of preparation of this closure.

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[2] Aerospace Research Center, Fluidics Department, General Precision Inc., Little Falls, N. J.

[3] Study of Vortex Phenomena performed under Contract DA-01-021-AMC-11203(Z), Guidance and Control Lab., USAMC, Redstone Arsenal, Huntsville, Ala.

[4] Numbers in brackets designate References at end of Discussion.