# Closure to "Discussion of 'A Paradox in Sliding Contact Problems With Friction' " (2006, ASME J. Appl. Mech., 73, pp. 884–886)

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The purpose of our technical brief was to first demonstrate that a paradox involving slip reversal exists in a class of sliding contact problems with friction. We were then motivated by the desire to determine a mechanics formulation of these problems which is self-consistent, in which case we do not have to resort to making arguments that depend on the actual dimensions and material properties of a particular situation. We also stated "The finite strain kinematics analysis shows that the paradox disappears when the correct kinematics is used. In this case it is a reasonable engineering solution to use the infinitesimal theory with the assumption that slip is always in the original direction of sliding, because the paradox occurs only in very small regions in which the infinitesimal theory is unrealistic."

We thank the authors of the Discussion for detailing some of the issues with plasticity and rounding as related to *traditional* engineering materials. The statement is made in the Discussion that a radius of greater than 120 nm is necessary to avoid the paradox. While such a tiny radius may be unrealistic in many applications, it is not at all unusual in, for example, silicon MEMS devices where radii can be nearly atomically sharp as a consequence of an anisotropic etch.

The plasticity analysis used in the Discussion assumed a ratio of yield stress to shear modulus of 0.003. However, in rubber and biological materials the "yield" stress can be a significantly higher fraction of the modulus. Even for traditional engineering materials, the elastodynamic problem behaves as the corresponding quasi-static problem with a reduced modulus. As pointed out in the original technical brief, this reduced modulus approaches zero as the speed of the punch approaches the Rayleigh wave speed.

For the reasons outlined above we disagree with the statement that "…it is unlikely that the paradox described, though interesting, will have any bearing in a real contact." We also reiterate that the purpose of our technical brief was to develop a mechanics formulation which is self-consistent.



  