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TABLE 13 EQUATIONS FOR STRESS RATIOS

Stress ratios in cylinder at the junction:

Shear

$$I_{sj} = 3 \left| \frac{Q_0}{pd} \right|$$

Axial

$$I_{aj} = \frac{1}{2} + 12 \frac{d}{t} \left| \frac{M_0}{pd^2} \right|$$

Circumferential

$$I_{cj} = \left| 1 + 2\beta d \left(\frac{Q_0}{pd} \right) (1 + \alpha_0) \right| + 12\mu \frac{d}{t} \left| \frac{M_0}{pd^2} \right|$$

Maximum stress ratios in cylinder (other than at junction):

Shear

$$I_{sm} = \frac{I_{sj} \sqrt{1 + 2\alpha_0 + 2\alpha_0^2}}{e^{\beta x_s}} \qquad \frac{x_s}{d} = \frac{\tan^{-1}\left(\frac{1 + \alpha_0}{\alpha_0} \right)}{\beta d}$$

Axial

$$I_{am} = \frac{1}{2} + 3.4126 \sqrt{\frac{d}{t}}\, I_{sm} \qquad \frac{x_a}{d} = \frac{x_s}{d} - \frac{\pi}{4} \frac{1}{\beta d}$$

Circumferential

$$I_{cm} = 1 + 0.7326 \sqrt{\frac{d}{t}}\, I_{sm} \qquad \frac{x_c}{d} = \frac{x_a}{d} + \frac{1.0720}{\beta d}$$

Stress ratios in hemispherical head at junction:

Shear

$$I_{sj} = 3 \frac{t}{T} \left| \frac{Q_0}{pd} \right|$$

Axial

$$I_{aj} = \frac{1}{2} \frac{t}{T} + 12 \frac{d}{t} \frac{t^2}{T^2} \left| \frac{M_0}{pd^2} \right|$$

Circumferential

$$I_{cj} = \frac{1}{2} \frac{t}{T} + 12\mu \frac{d}{t} \frac{t^2}{T^2} \left| \frac{M_0}{pd^2} \right| + \frac{4t}{T} \left(\frac{a_1 M_0}{pd^2} + \frac{a_2 Q_0}{pd} \right)$$

TABLE 14 VARIATIONS IN STRESS RATIOS AS d/T AND T/t INCREASE

Numbering system	Location	Quantity	T/t constant d/T increasing	d/t constant T/t increasing
1	Cylinder	I_{sj}	Decreasing	Increasing
2	Cylinder	I_{aj}	Slight increase if $T > t$ Slight decrease if $T \leq t$	Decreasing if $T < t$ Increasing if $T > t$
3	Cylinder	I_{cj}	Slight increase	Decreasing
4	Sphere	I_{sj}	Decreasing	Increasing if $T < t$ Decreasing if $T > t$
5	Sphere	I_{aj}	Slight decrease if $T \leq t$ Slight increase if $T > t$	Decreasing
6	Sphere	I_{cj}	Slight decrease	Decreasing
7	Cylinder	I_{sm}	Decreasing	Increasing
8	Cylinder	I_{am}	Decreasing	Increasing
9	Cylinder	I_{cm}	Slight decrease	Slight increase
	Cylinder	x_s/d	Decreasing	Decreasing
	Cylinder	x_a/d	Decreasing	Increasing
	Cylinder	x_c/d	Decreasing	Decreasing

NOTE: Using numbering system of table, stress ratios may be arranged in descending order of magnitude for each value of ratio T/t, as follows:

T/t	
0.8	9 > 3 > 5 > 8 > 2 > 4 > 1 > 7
1.0	9 > 3 > 8 > 2 = 5 > 1 = 4 > 7
1.2	9 > 3 > 8 > 2 > 5 > 1 > 4 > 7
1.6	9 > 3 > 2 > 8 > 5 > 1 > 4 > 7
2.0	9 > 2 > 3 > 8 > 5 > 1 > 4 > 7

The three location ratios, arranged in order of decreasing distance from junction, are $x_c > x_s > x_a$ for all ratios T/t.

2 "The Stresses in a Pressure Vessel With a Flat Head," by G. W. Watts and H. A. Lang, Trans. ASME, vol. 74, 1952, pp. 1083–1091.

3 "Theory of Plates and Shells," by S. Timoshenko, first edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1940.

4 "The Basic Elastic Theory of Vessel Heads Under Internal Pressure," by G. W. Watts and W. R. Burrows, *Journal of Applied Mechanics*, Trans. ASME, vol. 71, 1949, pp. 55–73.

Report of the Engineering Research Department, ERR-A6-2, "The Stresses in a Pressure Vessel With a Conical Head," Standard Oil Company (Indiana), July 26, 1949.

5 "The Maximum Stresses in the Cylindrical Body of Pressure Vessels," by H. A. Lang, Property of Engineering Research Department, Standard Oil Company (Indiana).

6 API-ASME Code Unfired Pressure Vessels for Petroleum Liquids and Gases (1943 edition).

Discussion

T. McLEAN JASPER.[13] This paper gives a theoretical approach to a pressure-vessel problem which applies essentially to a totally elastic material. In steel, for instance, we have an excellent material for the adjustments of slight imperfections in fabrication, providing we have the material mass available in approximately the right location. Steel used in pressure vessels, in general, has great ductility in comparison to its natural elastic strain so that it can make adjustments to slightly imperfect shapes or designs. That is why steel is a most useful material for fabricated structures.

It is doubtful if before this there has been a paper in which so much sound thinking has been given to the theoretical details of an elastic problem of this kind. We are fast approaching a condition in the use of materials where the elastic details are of extreme importance because demands for high-strength materials at temperatures of 1500 to 2000 F are close to the engineer's present requirements. In general, such materials are very brittle even at those high temperatures.

This need also applies to pressure vessels of thick solid sections in which adjustments are not accommodated so easily. The stress-calculation system that the authors have used involves the simple formula for cylinders. The stresses due to internal pressure vary in a thick-wall high-pressure cylinder so that the difference between the inside and outside surface stresses is equivalent to the pressure P. In strain measurements on the outside surface of cylinders under pressure the maximum stress on the outside surface should have the pressure value P added if the stress value on the inside is to be obtained. In the case of precompressed cylinder adjustments, knowing the value of precompression will be required for the actual stresses in a cylinder. In stress measurements at the junction between cylinder and heads, strain gages are needed on both of the steel surfaces if a check on this analysis is to be made.

This paper should be appreciated greatly because it helps the strain-measuring setup for obtaining test results. Our factors of so-called safety should accommodate the designs and the slight fabrication difficulties in steel.

AUTHORS' CLOSURE

The authors wish to thank Mr. Jasper for his discussion. He points out that high local stresses in steel vessels are frequently relieved by plastic flow. Unfortunately, a designer cannot always rely on ductility as insurance against the bad effects of sharp corners and other stress raisers. Numerous failures have occurred because of brittleness at low temperatures; and as Mr. Jasper points out, many steels also become brittle at high temperatures.

[13] A. O. Smith Corporation, Milwaukee, Wis. Mem. ASME.

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The authors believe that current design requirements will tend, in many cases, to reduce the available ductility. One frequently heard proposal is to take advantage of the higher yield strengths of low-alloy steels. It is well known that these higher yield strengths are inescapably linked to lower ductility values. The only other alternatives are higher working stresses or thicker walls. Although the problem of size effect is not yet entirely clarified, it is generally agreed that the thicker the plate, the greater the tendency toward brittle failures, because of welding difficulties, greater triaxial tensile stresses, greater temperature differences through the plate, and so on.

One of the main purposes of the present paper is to point out the superiority of hemispherical heads, compared to flat or conical heads, as a means of avoiding high local stresses. If stresses throughout a pressure vessel can be held at fairly uniform levels by using the best available shape, it is unnecessary to rely on ductility to relieve sore spots.

Mr. Jasper also points out that the authors' analysis is reasonably accurate only when applied to vessels with rather thin walls. Even so, it represents a considerable advance over present textbooks in making the results of advanced mathematical analysis more available to engineering designers. Although a limited amount of elasticity theory exists for thick-walled cylinders and spheres, it is not yet in a sufficiently manageable form to permit satisfaction of the boundary conditions at a sphere-cylinder junction. This problem may yield more easily to approximations of the lattice type, rather than the formal mathematics of elastic theory.

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