Discussion

Skin Friction and Heat Transfer for Laminar Boundary-Layer Flow With Variable Properties and Variable Free-Stream Velocity[1]

W. B. Brown.[2] This paper pr esents a skillful method of extrapolating from a known region, constant fluid properties, into an unknown region, variable properties combined with variable free-stream velocities. The assumptions are made to give a good fit in the constant-property case. Within the limits stated in the paper, namely, $0.5 < T_w/T_1 < 2$ and for $0 < \beta < 1$, the skin-friction and heat-transfer coefficients are not seriously in error. The boundary-layer thicknesses deviate considerably from the exact values, even within these limits.

The reason for the increasing errors is principally the inadequacy of Assumption [8]

$$\frac{u}{u_1} = 1 - e^{a\eta + \frac{b}{2!}\eta^2 + \frac{c}{3!}\eta^3 + \frac{d}{4!}\eta^4}$$

in the region mentioned, $\beta > 0$ and $T_w/T_1 > 1$. The type of profile required in this region is shown in Fig. 1,[3] herewith, where for

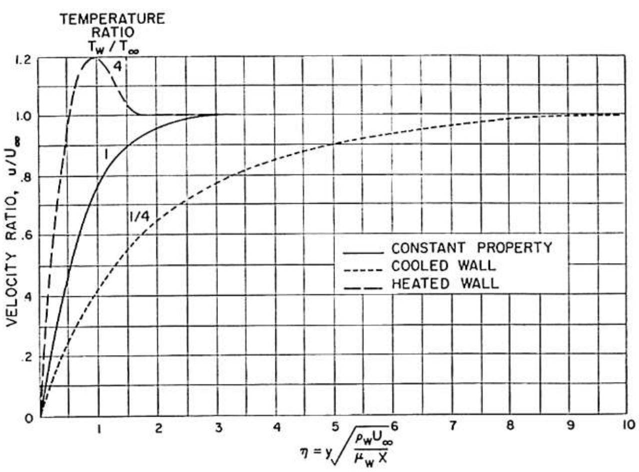

Fig. 1 Velocity Distribution in Boundary Layer for Impermeable Wall and $\beta = 1$

$\beta = 1$, velocity profiles are drawn for the constant-property case, a case where the wall is strongly cooled, and a case where the wall is strongly heated. In the latter case the peak of the velocity profile is 20 per cent above the free-stream value, a fact which cannot be expressed by Equation [8] of the paper. The result is an increasing discrepancy between the exact and approximate values for skin friction, heat transfer, and boundary-layer thickness as β increases above zero and T_w/T_1 increases above 1.

[1] By S. Levy and R. A. Seban, published in the September, 1953, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 75, pp. 415–421.

[2] Address: Santa Monica, Calif.

[3] "Tables of Exact Laminar-Boundary-Layer Solutions When the Wall Is Porous and Fluid Properties Are Variable," by W. B. Brown and P. L. Donoughe, NACA TN 2479, 1951.

Fig. 2 of this discussion shows this effect. When $T_w/T_1 = 3$, the discrepancy is 28 per cent for skin friction based on free-stream properties, and 8.5 per cent based on wall-temperature properties. Apparently the wall-temperature properties are more accurate in this case. The heat-transfer discrepancy is 7.2 per cent, but the difference from the constant-property value is 12 per cent (not the 6 per cent mentioned in the paper).

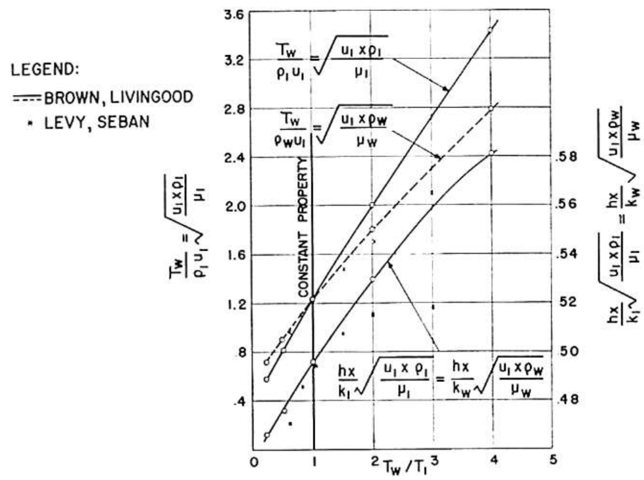

Fig. 2 Skin Friction and Heat Transfer for Variable Property Wedge Flow ($\beta = 1$)

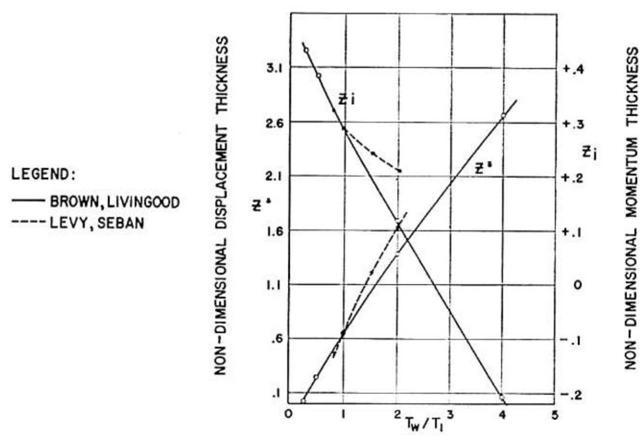

Fig. 3 Nondimensional Displacement and Momentum Thickness for Variable-Property Wedge Flow ($\beta = 1$)

The exact values for $\beta = 1.6$ have not been calculated, in so far as the writer knows, but the discrepancies there may very well be larger than for $\beta = 1$. Fig. 3,[4] herewith, shows the boundary-layer thicknesses when $\beta = 1$. The percentage discrepancies here are larger than in the skin-friction or the heat-transfer coefficients. For a temperature ratio of 2, the discrepancy in the displacement thickness is 18 per cent; in the momentum thickness, 82 per cent.

Finally, the case where the fluid properties, free-stream veloc-

[4] "Solutions of Laminar Boundary-Layer Equations Which Result in Specific Weights-Flow Profiles Locally Exceeding Free-Stream Values," by W. B. Brown and H. B. Livingood, NACA TN 2800, 1952.

ity, and wall temperature all vary simultaneously is not quite as difficult as the last sentence of the discussion would imply. It is now in process of numerical solution at the NACA Laboratory in Cleveland.

E. R. G. Eckert.[5] The paper presents an interesting approach to obtain an approximate solution for skin friction and heat transfer in a laminar boundary layer under the condition that the pressure varies along the surface. The flowing fluid is assumed to have its heat conductivity and the viscosity vary proportionally to the absolute temperature and the density to vary inversely proportionally to the temperature; specific heat and Prandtl number are assumed constant. (Unfortunately, in the paper it is in two places stated erroneously that the product of thermal conductivity and viscosity is considered to be constant.) Numerical solutions have been obtained for a wide range of accelerated flow conditions and a ratio of wall temperature to stream temperature varying between 0.6 and 3. These solutions extend the information obtained by Brown and Donoughe through a numerical integration of the differential equations describing flow and heat transfer in laminar boundary layers.[3]

An interesting fact apparent from the results of the present calculations is a significant variation of the skin-friction coefficient for accelerated flow conditions with the ratio wall temperature to stream temperature. This variation cannot be accounted for by the usual procedure to use the friction coefficient as derived for constant-property values and to adapt it to variable properties by introducing the properties at an appropriately defined reference temperature. The Nusselt number describing the heat transfer depends to a much smaller degree on the foregoing temperature ratio.

It is mentioned by the authors that their method of solution breaks down for values of the temperature ratio larger than 3. Brown and Donoughe have observed that for large values of the mentioned temperature ratio, the velocity profile within the boundary layer does not as usual increase monotonously from the value 0 at the wall toward the stream velocity, but reaches a maximum within the boundary layer larger than the stream velocity.[3,4] It may be that a connection exists between this peculiar behavior of the velocity profile and the breakdown of the method as presented in this paper for certain values of the temperature ratio.

J. N. B. Livingood.[6] The paper presents a very interesting and useful method for allowing for property variations in the calculation of laminar skin friction and heat transfer from an isothermal surface for a variable free-stream velocity. Employment of the assumption $\rho\mu$ = const greatly simplifies the solutions of the laminar boundary-layer equations for wedge flows, as compared to the method of Brown and Donoughe,[3] where each property varied as a power of temperature. For the common cases solved by both methods, results are in excellent agreement and it appears that future solutions for isothermal surfaces that may be required might well be obtained by the simpler method of this paper.

Unfortunately, however, the method of this paper is not applicable for accounting for a variable wall temperature. Such effects possibly can be included if the more difficult method of the paper mentioned[3] is applied. Moreover, if wedge-flow solutions are obtained for the variable wall-temperature case, laminar heat transfer around cylinders of arbitrary cross section can be determined by an extension of the method by E. R. G. Eckert and J. N. B. Livingood.[7]

Authors' Closure

The authors wish to thank the discussers for their most interesting and welcome comments. As Dr. Brown points out, the assumed velocity profile is inadequate whenever the velocity within the boundary layer exceeds the free-stream velocity. In particular, the displacement and momentum thickness will be in error. However, the simplified calculation method will not exhibit this error exclusively. In the forward integration process used by Brown and Donoughe[3] and Brown and Livingood[4], a very small change (in the third significant figure) in the starting value of the temperature gradient at the wall will produce a large departure in the velocity profile for large values of T_w/T_1; and the values presented by Brown, even though more accurate, are still uncertain.[8]

The failure of the simplified method to account for a variable wall temperature has been noted. Because the momentum equation is dependent upon the temperature ratio T/T_1, the authors believe still that solutions will have to be computed for each specified wall-temperature variation and that extension of the method of Eckert and Livingood to include surface-temperature variation may be much more complicated than in the incompressible case.

The authors wish to thank Dr. Eckert for calling attention to the fact that the preprint stated erroneously that the product of thermal conductivity and viscosity is assumed constant. His comment enabled the authors to correct the error before publication of the paper.

Determination of Stresses in Cemented Lap Joints[1]

C. D. Coxe.[2] The writer wishes to congratulate the author for this excellent contribution which we believe is of considerable practical importance.

In our rather broad experience with the joining of structural-metal parts by brazing we find among many designers and industrial users a singular lack of appreciation of stress-analysis concepts and of the importance of nonuniform distribution of stresses in brazed joints. The all-too-usual approach in joint design is to calculate the total load, divide by the unit strength of the brazing alloy, and multiply by some empirical factor of safety to obtain the area of joint required. In many cases, of course, this suffices for noncritical applications such as ordinary piping, or where the yield strength of the material being joined is less than the tensile strength (or shear strength) of the brazing alloy. Thus, where copper, brass, or soft steels are joined with a strong silver-brazing alloy, the structural metal will deform before it can transmit any damaging stress to the brazing layer.

On the other hand, where the strength of the metal joined

[5] Professor, Mechanical Engineering Department, Institute of Technology, University of Minnesota, Minneapolis, Minn. Mem. ASME.

[6] National Advisory Committee for Aeronautics, Lewis Flight Propulsion Laboratory, Cleveland, Ohio.

[7] "Method for Calculation of Heat Transfer in Laminar Region of Air Flow Around Cylinders of Arbitrary Cross Section, Including Large Temperature Differences and Transpiration Cooling," by E. R. G. Eckert and J. N. B. Livingood, NACA TN 2733.

[8] "Effect of Large Temperature Changes (Including Viscous Heating) Upon Laminar Boundary Layers With Variable Free Stream Velocity," by S. Levy, to be published in the *Journal of Aeronautical Sciences*, 1954.

[1] By R. W. Cornell, published in the September, 1953, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 75, pp. 355–364.

[2] Chief Metallurgist, Handy and Harman, Bridgeport, Conn.