Love Waves and the Structure of the Upper Mantle

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Summary

An investigation is made as to what information on the upper mantle can be obtained from Love-wave dispersion.

The method of calculation of the dispersion and intensity is based on the spectral theory of operators. The usual technique of interpretation of dispersion curves gives only a random sample from the great number of velocity-depth curves fitting the experimental data. Some methods of restricting this number are indicated. In particular, some peculiarities of higher-mode dispersion could be very important. An attempt at the joint interpretation of body (S) and surface (Love) wave data from the central part of U.S.A. is described.

1. Surface waves give, in principle, many new possibilities for investigating the upper mantle, in particular the low-velocity layer within it. However, their study has only recently been introduced into this sophisticated problem and experience and intuition are practically absent. Therefore, correct formulation of the inverse problem in the interpretation of surface waves is even more necessary than for body waves (Yanovskaya 1963). It is less clear which particular observations can give the desired information, and what information can be obtained from any given observations.

2. An effective method for the calculation of surface waves was necessary to clear up these questions and to obtain a basis for the solution of the inverse problem of interpretation. This method was worked out using the theory of linear differential operators. The corresponding programmes for the calculation of Rayleigh and Love waves by an electronic computer have been written. These programmes allow us to calculate not only phase and group velocities, but also the intensity for any modes and periods. The medium is not approximated as usual with a system of homogeneous layers. This raises the precision and speed of computing. In this paper only Love waves are investigated.

3. First of all we wish to find out what information on the velocity–depth curve (referred to as v–dc) of the upper mantle can be obtained from the dispersion of the fundamental Love wave mode. For this purpose dispersion curves for 22 v–dc of the upper mantle were computed (2 v–dc for each of 11 types of v–dc corresponding (as in Yanovskaya 1963) to travel times of the P-phase in Europe; the total number of v–dc, found by Yanovskaya (1963), is 115). The shear velocity $V_s$ was computed from the longitudinal velocity $V_p$ according to Gutenberg’s data on $V_s/V_p$ (Gutenberg 1953) and also for the case $V_p^2 = 3V_s^2$. Some results of the
computations are shown in Figure 1 together with observational data from Brune & others (1961), Ewing & others (1957) and Brune & others (1960).

Only one practical conclusion follows from the comparison of calculations and observations: the true $V_p$ for depths between 100 and 300 km are at least 0.2–0.25 km/s higher than all the calculated $v_{dc}$, including Gutenberg's. It follows from the fact that, for the period range 100–250 s, calculated phase velocities are less than observed ones. The difference of 0.2–0.25 km/s indicated is obtained from comparison with the case $V_p/V_s = \sqrt{3}$, when the calculated phase velocities are greater than the observed.

![Figure 1](https://academic.oup.com/gji/article-abstract/9/1/1/636840)

**FIG. 1.**—On the left side—some models of the upper mantle from Yanovskaya (1963). On the right side—corresponding phase ($V_1$) and group ($C_1$) velocities of the fundamental mode. The ratio $V_p/V_s$ (longitudinal velocity/shear velocity) is taken from Yanovskaya (1963). Densities after model A of Bullen. Observational data are shown by points ($V_1$) and crosses ($C_1$) (Brune & others 1961, Ewing & others 1957).

Methodical conclusions seem to be more complicated. The difference in dispersion for different $v_{dc}$ is not greater than the scatter of the observational data. We find the greatest difference for periods 60–100 s. However, the greatest scatter of the experimental data because of crustal structure variations is possible at the same periods.

The main features of $v_{dc}$ such as low-velocity layers, zones of small and large gradients cannot be recognized using the fundamental mode. For example, practically identical dispersion corresponds to all the $v_{dc}$ considered. On the other hand, these $v_{dc}$ give identical average velocities. Thus the dispersion of the fundamental mode is determined by some integral effect of averaging the velocity distribution.
Similar calculations with quite analogous conclusions were obtained for the Earth's crust. The only result which is important for the upper mantle is that the fundamental mode, for periods of more than 100 s, is determined by the upper mantle only and is not influenced by any reasonable variations in the crust.

Fig. 2.—Velocity-depth curves, corresponding to observed S-wave travel times (1); and the dispersion of Love waves (2) for central part of U.S.A.; the common curve from both sets (3); assumed limits of possible shear velocity in the upper mantle (4).
4. A set of possible v–dc corresponds to the observed dispersion of the fundamental mode, as well as to the observed travel time of an S-wave.

The question arises, would the intersection of these sets (i.e. the set of curves, corresponding both to S-wave travel times and Love waves dispersion) be more restricted. The programme for solving the inverse problem, that of determining the set of v–dc fitting the given phase and group velocities, has been written. Examination of the v–dc was done just as in the case of body waves (Yanovskaya 1963). The results of joint interpretation of S- and Love waves are shown in Figure 2. The S-wave travel times were taken from Lehman (1955) and the dispersion curves from Ewing & others (1957) and Brune & others (1960). Both were for the U.S.A.

We see that this joint interpretation greatly reduces the ambiguity.

5. High modes open a promising pathway for more detailed and unambiguous interpretation. They have the following advantages:

5.1 Greater depth of penetration for a given period. For instance to study the v–dc up to a depth of 300 km the following periods are necessary: up to 150 s for the first mode, up to 20 s for the third mode.

5.2 More detailed correlation with v–dc. In Figures 3 and 4 the computed group-velocity curves are shown for the second and third modes for the same v–dc as in Figure 1. Different v–dc are here far more distinct than in Figure 1. Many elements of the v–dc are almost repeated on group-velocity curves. In particular the structures of the crust and of the mantle are characterized for the second mode by separate parts of these curves (periods less and more than 10–12 s respectively).

6. High modes are especially promising for investigation of low-velocity layers.

In Figure 5 the dispersion curves of group velocity for the third mode for the upper mantle, either with such a layer or without it, are shown. The crust is continental (thickness 30 km, average velocity in both cases 3·6 km/s). The low-velocity layer gives an additional sharp minimum for the group velocity of the third mode; this minimum depends little upon other features of v–dc:

A wave with the following properties must correspond to this minimum on seismograms: sharp arrival with velocity slightly less than for the Sn phase; great duration of the wave-train; slow attenuation in time; an almost constant period T (Figure 5). The period depends upon both the thickness of the crust H and
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Fig. 4.—Group velocities of the third mode for the same models which are shown in Figure 1.

Fig. 5.—The dispersion of the third mode in models of the upper mantle with, or without, a low velocity layer.
the depth of the low-velocity layer \( h \). If the low-velocity layer is as in Figure 5 and \( H \) changes from 30 to 50 km, then \( T \) changes from 8 to 12 s.

\( T \) increases and duration decreases, when \( h \) increases. For instance, if \( H = 30 \) km and \( h \) changes from 150 to 300 km, \( T \) increases from 8 to 12 s and the duration of the wave-train for the distance of 3 000 km decreases from 120 to 45 s. If the minimum of the \( v \)-dc in the wave-guide becomes sharper, the duration of the train increases and the range of periods narrows. The shape of the \( v \)-dc in the wave-guide is less important.

The existence of this wave can be explained physically by a resonance. The frequency response of the medium for the third mode shows a very sharp resonance at this range of period, quite as sharp as in the crust (Figure 6).

The greater part of the energy of this wave at the resonance period is propagated, as is shown in Figure 6, above the wave-guide axis.

Thus it is not the low-velocity layer itself that appears to be a resonator but all the medium from the surface to the bottom of the layer.

\[ \text{Fig. 6.—Variations of amplitude } V_k \text{ with period } T \text{ and depth of source } h \text{—in the mantle with low-velocity layer (Gutenberg's model (Gutenberg (1963)) with the continental crust). The source is a point horizontal force, the point of observation is situated on a surface. } k \text{—number of mode (1, 2, ...). (a) The frequency response } V_k(T) \text{ for a surface source } (h = 0, \ k = 2:3). (b) The scheme of amplitude-depth variation. Horizontal broken lines are } M \text{-boundary and the axis of low-velocity layer. Vertical broken lines indicate the corresponding periods on Figure 6(a). The scales of the first and third curves are the same. The amplitudes of the second curve are multiplied by 100.} \]
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Depths of focus about 150–200 km are most favourable for separation of the resonant wave from other wave-trains.

One must not mix this wave with so called “channel waves”. Figure 6 shows that channel waves SH really exist (they correspond to a constant group velocity). However, they are wholly concentrated within the low-velocity layer and could not be detected on the surface (at any rate on continents).

References


