The Stress around a Fault according to a Photoelastic Model Experiment

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Summary

A report is given of preliminary photoelastic measurements of the two-dimensional stress field around a fault or crack in a plate. The measurements include the following cases: open slit with the uniaxial applied pressure field making an angle of 45° with the slit; closed slit (zone of weakness) with two different thicknesses of the weak zone and again 45° to the external pressure field; the measurements for the weakest slit were made also with an angle of 22.5° to the pressure field. The determined normal and shear stresses are represented in graphical form. The results provide explanations for some earthquake characteristics, for example distribution of shear stress and patterns of geographical extension of seismic activity during an aftershock sequence.

1. Introduction

A knowledge of the stress field around tectonic faults is of great tectonophysical importance. However, generally only indirect methods are at our disposal for its determination. These methods work with simplified models, either mathematical models or laboratory models. We shall only be considering two-dimensional models.

Inglis (1913) gave the analytical solution for the two-dimensional stress state around an elliptical hole in an infinite plate for different external stresses. Anderson (1951) applied these results to a degenerated ellipse, i.e. with one axis equal to zero, under the following boundary conditions:

1) External pressure and tension, mutually perpendicular, act under 45° to the fracture, so that a pure shear stress arises parallel to the fracture;
2) There is no transmission of shear stress across the fracture.

The principal stresses along the fracture were found to be zero in the middle, increasing to infinity towards its ends. This is a first approximation of a tectonical fault in a pure shear stress field and furnishes a good explanation of some observed earthquake patterns (Duda 1962), especially the accumulation of epicentres and even more of seismic energy towards the ends of a fault. However, Anderson’s model is time-independent, whereas an aftershock sequence continues for a considerable time. Therefore, only the time-integral of the aftershock pattern can be compared with Anderson’s model.

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The condition (2) above is a severe simplification of natural conditions. A real fault has to be understood as a zone of weakness, still able to transmit shear stresses from one side to the other. A generalization of Anderson’s model introducing friction at the fault would lead to considerable mathematical difficulties, and to our knowledge this problem has not been analytically solved.

In the present research we have tried to solve this problem by an analogue process, using the photoelastic method for determination of the stress field around a rectilinear, finite fracture in a transparent plate. Our boundary conditions are:

1. External homogeneous pressure field acting parallel to the plate under angles of 45° and 22.5° respectively to the fracture;
2. Transmission of shear stress across the fracture.

The application of a uniaxial, homogeneous pressure field on the plate represents a more general analogy to tectonic conditions than a pure shear stress field parallel to the fault. At any point in the orogenetic shell, the two principal stresses will in most cases be two pressures, generally unequal. Such a pressure field may be divided into a hydrostatic pressure and a residual pressure, of which only the latter is of importance for tectonic movements. Its action will depend on the angle it forms with the fault. If the pressure is perpendicular to the fault, this will be closed and no earthquake occurs. If they are parallel, the fault will be opened or bent, and no seismic activity will be observed either. Seismic activity will be expected for all other angles, being a maximum for an angle of 45°.

The photoelastic method has so far been used only exceptionally in seismology or general geophysics, as for example for studies of seismic wave propagation (Thomson 1963) or of stress distribution in models of wells and galleries (Schmidt 1961).

2. The photoelastic method

Extensive treatments of the photoelastic method have been given by Hiltscher (1958) and by Frocht (1960). The basic physical principle of the method is that some transparent materials, when loaded, show double refraction of light. The velocity of light changes in a loaded model from point to point and depends upon its direction of propagation. Thus a light beam leaving the loaded model will be polarized. In a two-dimensional model, like a plate, loaded parallel to its plane, a light beam will be plane-polarized at every point in the directions of the principal stresses. The two light components will have different velocities, proportional to the respective principal stress.

We introduce the following notation:

\[ \sigma_1, \sigma_2 = \text{principal stresses} \]
\[ R = \text{relative retardation or phase difference, expressed in wavelengths} \]
\[ C = \text{stress-optic coefficient (Frocht 1960)} \]
\[ d = \text{plate thickness} \]
\[ \Delta d = \text{change of plate thickness due to load} \]
\[ k = \text{lateral constant (Hiltscher 1963)} \]

Then, the following two equations are valid:

\[ R/d = C(\sigma_1 - \sigma_2), \]  
(1)

and

\[ \sigma_1 + \sigma_2 = k\Delta d/d. \]  
(2)
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The relative retardation and the directions of the principal stresses are measured in a polariscope (Figure 1). Equation (1), called the stress-optic law in two dimensions, provides a possibility to determine the difference of the principal stresses, if \( R \) is measured and \( d \) and \( C \) are known.

![Polariscope after Hiltscher (1958).](image)

**FIG. 1.—**Polariscope after Hiltscher (1958). \( a = \) analyser, \( p = \) polarizer with light source.

The sum of the principal stresses, expressed in equation (2), can be determined by different mechanical, optical or electrical methods. In our experiments we used the very efficient and accurate lateral extensometer after Hiltscher (Figure 2) for the measurement of the thickness change \( \Delta d \). Knowing \( k \) and \( d \), we can calculate the sum of the principal stresses from (2). Figure 3 shows schematically the complete apparatus used for the photoelastic measurements.

The described measurements and equations (1) and (2) reduce the problem to express stresses in any coordinate system to a matter of calculation.

3. Description of our experiments

Four different models, all being transparent plates made of Araldite D, were measured by the photoelastic procedure. The slits were in every case located in the centre of the plate and orientated along a diagonal. Model O (where O stands for “open”) had an open slit, the other models closed slits, consisting of cuts from either side, 0.4 cm deep in model A and 0.45 cm deep in models B and C. Profiles I and II were parallel to the slit and profile V perpendicular to it. Profiles III and IV, located symmetrically to I and II with respect to the slit, do not need to be measured for symmetry reasons (Table 1 and Figure 4).

Under load, the slit in model O is expected to close and friction between its walls to arise. This corresponds to a fault after a rupture of the material along the fault plane. As it is impossible to know, if the slit is first closed and then friction arises or if the walls are first displaced and then touch each other, the three other models were made. In these the slit corresponds to a zone of weakness, the strength being \( \frac{1}{2} \) of the surrounding medium in model A and \( \frac{1}{10} \) in models B and C.

In case of models A, B and C there is really a transmission of shear stress across the slit. These models correspond to pre-existing but blocked faults and the stress...
FIG. 2.—Hilitischer lateral extensometer (Hilitischer 1963) for measurement of plate thickness. pp = plate, mt = measuring tips, ms = micrometer screw, d = dial gauge, s = suspending device of the extensometer and its counter-weight c.

FIG. 3.—Loading and measuring device for photoelastic experiments, after Hilitischer (1958). Loading device to the left, front view. Measuring device (polariscope) to the right, side view. a = frame, b = measuring table, c = lever, d = loaded pan of the scales, f = clamping device for lever, g = automatic loading and unloading control, h = water-level, i = analyser and quarter plate, k = polarizer with source for monochromatic and white light.
Fig. 4.—Isochromatics for model A with closed slit, showing the profiles V, I, II and the measuring points in relation to the slit s.
Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>Plate dimensions</th>
<th>Slit dimensions</th>
<th>Profile I</th>
<th>Profile II</th>
<th>Profile V</th>
<th>Angle between slit and applied pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>side × side × thickness</td>
<td>length × width × depth</td>
<td>n</td>
<td>a</td>
<td>n</td>
<td>a</td>
</tr>
<tr>
<td>O</td>
<td>15 × 15 × 1</td>
<td>3 × 0.001 × 1</td>
<td>39</td>
<td>0.3</td>
<td>39</td>
<td>0.9</td>
</tr>
<tr>
<td>A</td>
<td>18 × 18 × 1</td>
<td>3.8 × 0.1 × 0.8</td>
<td>23</td>
<td>0.5</td>
<td>23</td>
<td>1.5</td>
</tr>
<tr>
<td>B</td>
<td>18 × 18 × 1</td>
<td>3.8 × 0.1 × 0.9</td>
<td>23</td>
<td>0.5</td>
<td>23</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>14 × 20 × 1</td>
<td>3.8 × 0.1 × 0.9</td>
<td>23</td>
<td>0.5</td>
<td>23</td>
<td>1.5</td>
</tr>
</tbody>
</table>

n = number of measured points. a = distance from slit.

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4. Results

We will be using the following notation:

x, y = rectangular coordinates in the plane of the plate, the y-direction coinciding with the direction of applied pressure (Figure 4);
α = angle between σ₁ and positive y-axis, counted positive counterclockwise (α < ±90°);
σ₀ = applied pressure, only absolute values used;
σₓ, σᵧ = x and y components of pressure (negative) or tension (positive);
τ₀ = reference shear stress, chosen = σ₀;
τₓᵧ = τᵧₓ = shear stress, positive for clockwise shear, negative for anticlockwise shear.

The following formulas are used in the computation:

\[
\begin{align*}
\sigma_x &= \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha \\
\sigma_y &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha \\
\tau_{xy} &= -\frac{\sigma_1 - \sigma_2}{2} \sin 2\alpha
\end{align*}
\]

The results are given in graphical form in Figures 5 to 8. All stress components are expressed in relation to the absolute value of the applied external field, which renders all curves immediately comparable. As the slit has a finite width in models A, B and C, stress values near the slit are less reliable because of three-dimensional effects. Some of the results need to be specially emphasized and the corresponding remarks are collected in what follows.

(a) Profile V. Models O, A, B (Figure 5)

σₓ exhibits a maximum value at some distance from the slit, decreasing both towards and away from the slit. It is a tension, except for model O, where it is a
pressure near the slit. The maximum value of $\sigma_x$ increases from model A (far from yielding) to model B (nearer yielding) and is highest for the model O (open slit).

$\sigma_y$ is always a pressure with a maximum value at some distance from the slit, decreasing rapidly towards the slit and approaching $\sigma_0$ away from the slit.

$\tau_{xy}$ has a high positive value far away from the slit, and a high negative value when approaching the slit.

(b) Profile I. Models O, A, B (Figure 6)

There is no more any symmetry of $\sigma_x$, but the extremities exhibit besides double maxima a high negative value near the left-hand edge of the slit and a high positive value near its right-hand edge. This stress distribution corresponds to a tendency
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Fig. 6.—Stress components along profile I, models O, A, B. $s = \text{slit.}$
of the fault to rotate in the considered external pressure field. This tendency is naturally most pronounced in the case of an open slit.

Also \( \sigma_y \) exhibits asymmetry and decreases near the middle of the open slit far below \( \sigma_0 \). There is no axial symmetry because of the fact that the profile runs at some distance from the slit. A parallel profile on the other side of the slit and at the same distance from it would provide reversed mirror-image curves to those given here. This is true in all cases considered.

The asymmetry of \( z_{xxx} \) with bigger absolute values to the right in Figure 6 than to the left for models A, B is also clear from Figure 4, showing the isochromatics for model A. Curves of equal colour connect points with equal shear stress, increasing from red to green. A high density of isochromatics corresponds to a rapid increase of the shear stress. In Figure 4, profile I between points 15 and 21, there is a region of high density of isochromatics, corresponding to the large shear stress in Figure 6. For the open slit, the largest negative shear stress appears on the left side of the profile.

In all models (O, A, B) there are positive as well as negative shear stresses on the left part of profile I but only negative stresses on the right part. This result may be of importance in the interpretation of fault plane solutions in aftershock sequences.

(c) Profile II. Models O, A, B (Figure 7)

The fact that profile II is at a greater distance from the slit has the result that the stress curves, although similar to those for profile I, are more smoothed. For instance, the double maxima have disappeared.

(d) Profiles V, I, II. Model C (Figure 8)

Model C differs from B only in the angle between applied pressure and the slit. Whereas in profile V, \( \sigma_x \) shows a shape similar to models O, A, B, the \( \sigma_y \)-curves follow opposite tendencies towards the ends of the profile. For the shear stress along profile V, the main difference is the negative value of \( \tau_{xy} \) at the ends of the profile in model C as compared to models O, A, B.

The stress curves for profile II are more smooth as compared with profile I, just as the case is for the other models. For these two profiles, all stress values are smaller for model C than for models O, A, B.

(e) General remarks

As we have measurements only along a few profiles, these are naturally insufficient to give a complete two-dimensional picture of the stress distribution around the fault, for example, in the form of stress isolines. On the other hand, the measured profiles are sufficient to give a clear idea of the main features of the stress distribution.

We have started from a given external pressure field and measured the resulting stress distribution around a fault. Applied to nature, the problem may arise to reverse the procedure, i.e. to start from a number of stress measurements around a fault and to deduce the external pressure field. This is an enormously more difficult problem, and it is not even certain if it has a unique solution.

5. Discussion

The purpose of this report is (1) to present an efficient analogue processing method for investigation of stress distributions around model faults, and (2) to
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FIG. 7.—Stress components along profile II, models O, A, B.
present these distributions in a few cases resembling tectonic faults. The ultimate goal is to explain the behaviour of earthquakes. In this preliminary work we should like to point out a few results, which already seem to provide explanations for some phenomena found in nature.

(1) In the photoelastic model experiments, with improved boundary conditions as compared with earlier mathematical models, the increase of shear stress towards the ends of a fault is confirmed. There is also no longer an infinite shear stress at the fault ends.

(2) The shear stress reaches a much higher absolute value at the left end of the slit in profile I, models A, B (Figure 6) than on its right end. For a profile on the opposite side of the slit, the behaviour is opposite. The strain release density for the Desert Hot Springs aftershocks (Richter & others 1958, Figure 6) exhibits a similar pattern in relation to the Mission Creek fault, which may be explained by our finding. This would mean that our two-dimensional stress distribution is a
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good approximation to the stress field around Mission Creek fault, and also that
the hypocentres do not vary much in depth. Of course, the aftershocks occur in a
material with elastic afterworking, and the strain release density map shows a
summary effect in time, whereas the photoelastic material used by us had to be
nearly perfectly elastic.

(3) In Figure 4, the isochromatics at both ends of the slit have a shape resembling
the wings of a butterfly. It is well known from laboratory experiments that the
fracture will propagate preferably in the direction of the “butterfly’s body” (oral
communication by R. Hiltscher). This direction depends strongly on the external
pressure field. In an investigation by the present author of the Mongolian aftershock
sequence starting on 1957 December 4 (03h 37m 50s GMT, 45.1° N, 99.4° E) it was
found that the aftershocks were distributed along the E–W striking fault of the main
shock up to a shock on 1960 December 3 (04h 24m 18.9s GMT, 42.9° N, 104.4° E).
With this shock seismic activity started along a fault extending in SE direction
from the eastern end of the old fault, making an angle of 40° with this fault. As
explanation for this secondary activity we propose that the faulting propagated in
the direction of the body of the stress “butterfly”, corresponding to the primary
fault.

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References

Anderson, E. M., 1951. The dynamics of faulting, Oliver & Boyd, Edinburgh and
London.

Gebiet der Aléuten-Inseln. Freiberger Forschungshefte, C 132, 7–90.