The meson-deuterium reaction and nucleon isobars
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The meson-deintegration of deuterons and the scattering of \( \pi \)-mesons by deuterons are discussed introducing the nucleon isobars rather phenomenologically. Agreements with experiments are satisfactory in the case of meson-deintegration, and the deviations from impulse approximation in the case of the scattering are in right direction.

§ 1. Introduction

The existence of nucleon isobars was formerly anticipated from the strong coupling meson theory, and seemed to be argued by the experiments on the photo-production of neutral \( \pi \)-mesons from nuclei for which the cross sections were comparable with those for charged \( \pi \)-mesons.\(^1\) Experiments on the scattering of \( \pi \)-mesons by hydrogen also seem to suggest the existence of nucleon isobars.\(^2\) To clarify the role of the nucleon isobars, it may be worthwhile to investigate the interaction of \( \pi \)-mesons with two nucleon systems, i.e., the deuterons.

The strong coupling theory is the only one at present that can predict the existence of the nucleon isobars consistently, but it is in many aspects ambiguous and imperfect (such as the lack of the relativistic invariance, extremely large coupling constant, etc.) and its form is so complicated that it is very difficult to carry out the calculations in a good approximation. We therefore do not refer to the strong coupling theory in detail, but start with some assumptions which can be deduced from the strong coupling theory. They are:

\( a \) The existence of the nucleon isobars with spin and isotopic spin both equal to 3, 2. Their energies and lives are to be suitably chosen to fit the experimental data.

\( b \) A selection rule. When a nucleon absorbs (or emits) a meson, it cannot stay in an ordinary nucleon state, but must be excited to an isobar state, or inversely, an isobar must fall into an ordinary nucleon state when it emits (or absorbs) a meson.**

\( c \) The predominant \( p \)-state interaction of the \( \pi \)-mesons with nucleons.\(^3\)

\( d \) The potential of the force between a nucleon and its isobar, which is, according to Pauli and Kusaka,\(^4\) of the form

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\(**\) In the strong coupling theory, the interaction Hamiltonian of nucleons and unbound mesons is proportional to \( \langle \epsilon_a \frac{1}{2} I_a \rangle_H \) (written in Pauli and Dancoff's notation), from which this selection rule is derived.
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\[ V(r) = f(r)(e_i^* e_{ii}^*) + g(r)\left(\frac{(e_i^* r_i)(e_{ii}^* r_i)}{r^2} - \frac{1}{3} (e_i^* e_{ii}^*)\right). \] (1)

e) The charge independence of the whole theory; i.e., the invariance under the rotation in the isotopic space. (see ref. 2 Anderson et al.)
f) The energy dependence of the matrix elements of the meson-nucleon interaction proportional to \( \frac{p}{\omega^3} \). (\( p \) : momentum of incident mesons, \( \omega \) : their energy) (This assumption is not very essential in our discussions.)

With these assumptions, we discuss in Sec. II, the meso-disintegration of deuterons, and in Sec. III, the scattering of \( \pi \)-mesons by deuterons.

According to the assumptions (e) and (f), the meson-nucleon interaction Hamiltonian must be of the form

\[ H = \frac{g}{\omega} \Phi^{*-\frac{3}{2}} O \phi^{*-\frac{3}{2}} + \cdots \] (2)

where \( g \) is the coupling constant, \( \omega \) the energy of the incident mesons, \( \Phi^{*-\frac{3}{2}} \) the wave function of the isobars with charge \( 2e \) (the upper suffix means the charge number), \( O \) an operator which is linear in \( \text{grad} \) and contains the spin and isotopic spin matrices, \( \phi^{*-\frac{3}{2}} \) the wave function of the meson field, and \( \psi^{\frac{3}{2}} \) the wave function of the proton. Assumption (c), the hypothesis of charge independence and the hermitian property of the Hamiltonian completely determine the coefficients of the other terms such as \( \Phi^{*-\frac{3}{2}} O \phi^{\frac{3}{2}} \) etc. (except for sign). Namely we have

\[ H = \frac{g}{\omega} \left\{ \Phi^{*-\frac{3}{2}} O \phi^{*-\frac{3}{2}} + \frac{1}{\sqrt{3}} \Phi^{*-\frac{3}{2}} O (\phi \psi^{\frac{3}{2}} + \sqrt{2} \varphi_s \psi^{\frac{3}{2}}) + \frac{1}{\sqrt{3}} \Phi^{\frac{3}{2}} O \left[ \phi^{*-\frac{3}{2}} + \sqrt{2} \varphi_s \phi^{\frac{3}{2}} \right] \right\} + \text{herm. conj.} \] (2')

Nucleons are always treated as non-relativistic particles in our discussions.

§ 2. The Mesodisintegration of the Deuterons

Experiments were performed by Durbin, Loar and Steinberger\(^5\) using positive \( \pi \)-mesons with energies of 20 \( \sim \) 50 Mev. The striking features of their results were:

i) almost \( \cos^2 \theta \) angular distributions of two protons

ii) steep rise in cross sections with the energy of the incident mesons.

It has been shown that the usual weak coupling perturbation calculations\(^7\) entirely fail to account for both i) and ii). To remedy these defects, Chew et al.\(^7\) had to take the \( D \)-state of the deuteron into consideration that is far less than the \( S \)-state, and Matsuzaki and Sasaki\(^7\) introduced an extraordinarily strong interaction between the two nucleons in the final state. Our theory is, however, in a satisfactory agreement with experiment without referring to any such assumptions.
1. Angular distribution

Following to the perturbation procedure, we treat the problem in two steps. The incident meson is first absorbed by one of the nucleons and this nucleon goes into an isobar state, (rejection rule $b$) then under the influence of the nuclear force, the system of a nucleon plus an isobar goes into a two-proton state.

The two-nucleon state with total charge $2e$ interacting through nuclear forces (containing the tensor force), can be in either of the following states,

$$1S' + bD + \cdots, \ 3p + \cdots, \ 1D + bS + \cdots$$

and so on, where the black letters mean that one of the nucleons is excited to an isobar state. As the $\pi$-mesons can interact with nucleons only in $p$-state (assumption $c$), it is easily seen from the parity consideration that the $3P$ states are forbidden as the intermediate states. Since we are dealing with $\pi$-mesons with small momentum, (less than $\mu c^2$), and since the ground state of the deuteron consists mostly of the $S$-state, the intermediate states cannot be the $D$-state. Therefore, we finally see that the intermediate state must be $bS$. The orbital angular momentum, $L$, of the final state must be 0 or 2 since the nuclear force of the type (1) can change the orbital angular momentum only by 0 or 2. The total isotopic spin, $T$, of the final state is obviously 1, there fore must be equal to 1 also in the intermediate states. The total spin, $S$, of the final state must be zero by Pauli principle, and accordingly, the total angular momentum $J$ must be equal to 0 or 2. In the intermediate the value of $J$ was 1 or 2, and since the nuclear force of type (1) cannot change the total angular momentum, the value of $J$ in the final state must be 2. The values of $S$, $L$, $T$, and $J$ are summarized in Table 1.

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The orbital angular momentum, total spin angular momentum, total isotopic spin, and total angular momentum of our two-nucleon system.

The nuclear force cannot excite the deuteron in the ground state, a charge singlet state, into a state that contains an isobar, since it cannot change the value of $T$.

From the above considerations, we see the final state must be the $1D$ state.

The state with $J=2$ can be constructed from the $p$-wave of the $\pi$-meson and the spin wave function of the deuteron as follows:

$$J_z = 1, \quad \frac{1}{\sqrt{2}}(\pi_0 d_1 + \pi_1 d_0),$$

$$= 0, \quad \frac{1}{\sqrt{6}}(\pi_{-1} d_1 + 2\pi_0 d_0 + \pi_1 d_{-1}), \quad (3)$$

$$= -1, \quad \frac{1}{\sqrt{2}}(\pi_0 d_{-1} + \pi_{-1} d_0),$$

where $\pi$'s and $d$'s are the wave function of the $\pi$-mesons in $p$-state and the spin wave
function of the deuteron respectively, and the suffixes indicates the values of the \( z \)-component of the angular momenta. If we choose the direction of the incident mesons as the \( z \)-axis, the wave functions of the initial states are \( \psi_n d_m \), with \( m = 1, 0, \) or \(-1\), and we get from eqs. (3) the ratio of the final states with \( J_z = 1, 0, \) and \(-1\) equal to 1/2 : 2/3 : 1/2. We finally find the angular distribution proportional to

\[
\frac{1}{2} \cdot (Y_1^1)^2 + \frac{2}{3} \cdot (Y_0^0)^2 + \frac{1}{2} \cdot (Y_1^1)^2 \sim 3 \cos^2 \theta + 1,
\]

which is in good agreement with experimental data.

II. The excitation function

The excitation function can also get a better agreement with experimental data by introducing the nucleon-isobars owing to the following reasons:

i) The energy denominator in the perturbation calculation \( E - \omega \), where \( E \) is the energy of the isobars, decreases as the energy of the incident mesons increases.

ii) As our process occurs in two steps, the high momentum component of the wave function of the deuteron is not required to give a large contribution to the transition matrix elements.

Unfortunately, because of our ignorance of the detailed nature of the nuclear forces, especially the force between a nucleon and an isobar, we cannot draw any definite conclusion, but we can inspect some features of the excitation functions by assuming some suitable radial dependences of the nuclear force.

As the intermediate states and the final state have different values of orbital angular momentum, the central force does not contribute to our process. The tensor force, according to the meson theory (ps. pv.) of nuclear forces, has the well-known \( 1/r^3 \) singularity, and does not give any meaningful result for our problem. We, therefore, according to Araki and Morin, replace it by

\[
\frac{e^{-\mu r}}{r^3} \delta \left( \frac{e_1^e r}{r^3} - \frac{1}{3} (e_1^e e_1^e) \right),
\]

where \( \mu \) is the meson mass (we shall always refer to the unit system in which \( \hbar = c = \mu = 1 \)) and \( \delta \) is a constant which should be taken so as to give correct values for the magnetic moment and electric quadrupole moment of the deuteron. Araki and Morin gives \( \delta = 2.7 \sim 4.5 \).

Following the usual perturbation procedure, one can easily obtain the differential cross section

\[
d \sigma (\theta) = \frac{4}{9} \frac{\theta^2}{27\pi} g^2 \frac{M}{pk} \cdot 2\pi A^2 \cdot \frac{1}{\omega^2} \frac{p^2}{|\Delta E|^2} f(k) (3 \cos^2 \theta + 1),
\]

where \( p \) and \( \omega \) are the momentum and energy of the incident mesons respectively, \( k \) the relative momentum of the nucleons in the final state, \( M \) the mass of the nucleon, \( \Delta E = \)
$E - \omega$ and $Af(k) = \int_0^\infty j_2(kr) \varphi_n(r) e^{-\omega r} r^2 dr$, $\varphi_n$ is the wave function of the deuteron, which we have taken to be of Hulthen type, i.e., $\varphi_n = A \frac{e^{-\alpha r} - e^{-\beta r}}{r}$ with $\beta = 7\alpha$, $A$ the normalizing constant. Accordingly, we have

$$f(k) = \frac{1}{2} \left( 1 + 3 \left( \frac{\alpha + 1}{k} \right)^2 \right) \tan^{-1} \frac{k}{\alpha + 1} - \frac{3}{2} \frac{\alpha + 1}{k}$$

and

$$A = \frac{1}{\beta - \alpha} \sqrt{\frac{\alpha \beta (\alpha + \beta)}{2\pi}}.$$

To get the best agreement with the experimental data of the meson-nucleon scattering, we put the value of $E$ to be 2.0 with imaginary part (half width) equal to 50 Mev., corresponding to $\omega = 6.86$ (see Fig. 1). The best agreement is obtained by putting $\beta = 3.6$ which is slightly smaller than the values required by the problem of magnetic dipole and electric quadrupole moment of the deuteron. The total cross sections, both calculated and experimental are plotted in Fig. 2.

![Fig. 1](https://github.com/academic-cuppaper/ptp/article-abstract/1954921613822)

Meson-nucleon scattering cross sections calculated from our theory putting $E = 2\mu c^2$ and $\Gamma/2 = 50$ Mev at the resonance. $\times$ : $\pi^+ H$ given by ref. 2. $+$ : $\pi^- H$ given by ref. 2. $\times$ : $\pi^+ H$ given by ref. 10. $+$ : $\pi^- H$ given by ref. 10

![Fig. 2](https://github.com/academic-cuppaper/ptp/article-abstract/1954921613822)

Cross sections for the meso-disintegration of the deuteron, calculated and experimental. Experimental values are due to ref. 5.
§ 3. Scattering of $\pi$-mesons by deuterons

The experiment performed by Isaacs, Sachs and Steinberger have shown that for the kinetic energy of the incident mesons of 58 Mev., $\pi^+\cdot D$ and $\pi^-\cdot D$ cross sections are nearly the same in accord with the hypothesis of charge independence, and less than $\pi^+\cdot H$ plus $\pi^-\cdot H$ by about 15 percent. It was shown by themselves that the calculation based on the weak coupling theories could not account for these experimental results. On the other hand, according to Anderson et al.,31 the difference between the cross sections of $\pi^\pm\cdot D$ and $\pi^\pm\cdot H$ is not so large in the low energy region, and becomes striking only in the high energy region (higher than 115 Mev.). If this effect were due to interference, the smaller it must be the higher the energy of the incident meson becomes.

One of the most reasonable way to account for this effect is, therefore, to consider the role of the nuclear force between two nucleons during the interaction with $\pi$-mesons.

Treating the interaction Hamiltonian as a perturbation, we can immediately get the second order transition matrix elements, i.e.,

$$A = A' + A'' ,$$

where

$$A' = \sum_{j} \frac{\langle f | H | j \rangle \langle j | H | i \rangle}{E_i - E_j}$$

and

$$A'' = \sum_{j} \frac{\langle f | H | j \rangle \langle j | H | i \rangle}{E_i - E_j} .$$

$A'$ is the matrix element corresponding to the process that the deuteron first absorbs a meson and then emits a meson, and $A''$ corresponding to the process that the deuteron first emits a meson and then absorbs a meson. We have, accordingly,

$$E_i = \omega + \frac{\langle q^2 \rangle}{M} + V,$$

$$E_j = E + \frac{\rho^2}{2M} + \frac{\langle q^2 \rangle}{M} + V,$$

and

$$E_i - E_j = (\omega - E - \frac{\rho^2}{2M}) + \left\{ \langle \frac{q^2}{M} + V \rangle_i - \langle \frac{q^2}{M} + V \rangle_j + \frac{\rho^2}{4M} \right\} .$$

where $q$ is the relative momentum of the nucleons that constitute the deuteron. Expanding (8) with respect to the quantity in the second bracket of (10), and retaining only the first term we have

$$A' = \sum_{j} \frac{\langle f | H | j \rangle \langle j | H | i \rangle}{(\omega - E - \frac{\rho^2}{2M}) \cdot \langle \frac{q^2}{M} + V \rangle_j + \frac{\rho^2}{4M}} .$$

Since we finally make use of the closure approximation, and due to the even parity of the wave
function of the deuteron, an operator linear in \( q \) does not contribute to the matrix element, and therefore, \( \langle q^2 \rangle - \langle q^2 \rangle + \frac{\rho^2}{4} \) makes no contribution to the matrix element. We finally get

\[
A^I = \sum \frac{(f|H|j)(j|H|i)}{\omega - E - \rho^2/2M} - \sum \frac{(f|H'|j')(j'|H|i)}{\omega - E - \rho^2/2M}, \tag{12}
\]

\[
A^H = -\sum \frac{(f|H|j)(j|H|i)}{\omega + E + \rho^2/2M} - \sum \frac{(f|H'|j')(j'|H|i)}{\omega + E + \rho^2/2M}, \tag{12'}
\]

The first terms in (12) and (12') are the terms that can be obtained by the impulse approximation, and the second terms are the corrections due to the nuclear force in the intermediate states.

If there were no nucleon isobar, \( E=0 \), the denominators of the second terms are always very small compared with the first terms. In our treatment, however, assuming the existence of nucleon isobars, the second term in (12) becomes appreciably large when the energy of the incident meson is near the energy of the isobar.

Neglecting the nucleon recoil in the final states, we obtain the differential cross section,

\[
da = \frac{\alpha^2}{4\pi} \sum \frac{(f|H|j)(j|H|i)}{\omega - E - \rho^2/2M} - \sum \frac{(f|H'|j')(j'|H|i)}{\omega + E + \rho^2/2M} \]  

\[-\text{Re} \sum \frac{(i|H|j)(j|H|i)}{\omega - E - \rho^2/2M} \sum \frac{(f|H'|j')(j'|H|i)}{\omega - E - \rho^2/2M} \cdot \tag{13}
\]

In calculating (13), terms small compared with the second term in the curly bracket are neglected, since they are smaller than the second term by a factor 20 or more. The first term in the curly bracket is the term that can be obtained by the impulse approximation. As we are neglecting the effect of interference, for they are known to be very small in the high energy region, this term gives \( \sigma(\pi^-+H) \) plus \( \sigma(\pi^-H) \). From Hamiltonian (2'), we have

\[
\sigma = \sigma(\pi^-+H) + \sigma(\pi^-H)
\]

\[
\frac{1}{12\pi^4} \sum \frac{\rho^4}{\omega^4} \left\{ \frac{2}{\omega - E - \rho^2/2M^2} + \frac{2}{\omega + E + \rho^2/2M^2} \right. \\
\left. - \frac{4}{9} \text{Re} \left( \frac{1}{\omega - E - \rho^2/2M^2} \right) \frac{1}{\rho + \omega + \rho^2/2M^2} \right\}. \tag{14}
\]

In evaluating the second term, the correction due to the nuclear force, we must now evaluate the matrix elements of \( V \). The intermediate state consisting of a nucleon and an isobar must have the total spin \( S \) and total isotopic spin \( T=1 \) or 2. As the ground state of the deuteron is the charge singlet state, \( T \) in the intermediate state must be equal to 1. Therefore, we have only to evaluate the matrix elements of \( V \) for the states \( T=1 \), \( S=1 \) or 2.

The tensor force only appears in combination with the \( D \)-state of the deuteron which is far less than the \( S \)-state, and is very complicated to manipulate, so we neglect the
effect of the tensor force. The expectation value of \((e^*_f e^*_n)\) is \(-\frac{1}{3}\) for the state \(S=2\), \(\frac{5}{9}\) for the state \(S=1\), and \(-\frac{1}{3}\) for the ground state of the deuteron. Therefore the matrix element \([H, V]\) vanishes for \(j\) with \(S=2\), (strictly speaking, after summing up with respect to \(j\) and \(f\)), and we must calculate the ratio of the matrix elements corresponding to the processes that occur through 3-let and 5-let intermediate states. This ratio can easily be evaluated from the invariant property of the Hamiltonian under the rotation in space. If we take the direction of the incident mesons as \(\varepsilon\)-axis, 1/4 of the total processes occur through 3-let state when the \(s\)-component of the angular momentum of the deuteron is 1 or \(-1\), and zero when the \(s\)-component of the angular momentum of the deuteron is zero. We get finally, averaging with respect to initial spin,

\[
\sigma(D) = \{ \sigma(\pi^+ - H) + \sigma(\pi^- - H) \}
\]

\[
= -\frac{\delta^4}{4} \left(\frac{8}{3} \langle V \rangle_{\text{dip.}} \right) \frac{2}{3} \frac{4\pi}{3} \frac{\omega^2}{4\pi^2} \frac{\rho^4}{\omega^6} \cdot 2 \text{Re} \left( \frac{1}{(\omega - E - \rho^2/2M)^*} \times \frac{1}{(\omega - E - \rho^2/2M)^2} \right) \left\{ \frac{1}{4} \left( 1 + \frac{1}{9} + \frac{1}{2} + \frac{1}{18} \right) + \frac{1}{4} \left( \frac{9}{2} + \frac{1}{9} \right) \right\}.
\]

We have taken the wave function of the deuteron to be of Hulthen type as in Sec. II. This choice of the wave function gives the value of \(\langle V \rangle_{\text{dip.}}\) 17.9 Mev. Numerical values are plotted in Fig. 3. Agreement with experiment of Anderon et al. is fairly good in the region \(100 \sim 130\) Mev. of the incident meson energy.

§ 4. Summary and conclusion

We have discussed the meson-deuteron reaction by assuming \(a\) the existence of nucleon isobars with spin and isotopic spin both equal to \(2/3\), \(b\) a selection rule, \(c\) the \(\rho\)-state
interaction of $\pi$-mesons with nucleons, and a few other assumptions. Following the second order perturbation procedure, we have obtained the cross sections and the angular distributions of two protons for the meso-disintegration, and the difference of the scattering cross sections between $\pi^+H$ plus $\pi^-H$ and $\pi^\pm D$. Agreement with experiment is satisfactory in the case of the meso-disintegration, and right in sign and order of magnitude in the case of the scattering.

We believe that we have revealed some evidence of the existence of the nucleon isobars by extending our discussion from the one-nucleon systems to two-nucleon systems. More strong evidence will be got when the experiments on the meson scattering and photomeson production proceed into more high energy region. The authors thank to Professor Yamanouchi for his kind encouragement. They are also indebted to the Yukawa-Yomiuri Fellowship for the financial aid supplied to one of them (S.M.).

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