A Static Model for the Continuum-Lowering of a Hydrogen-Like Ion as an Ultra-Dense Plasma Diagnostics

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A simple formula for the density diagnostics of a plasma is proposed in the present work. The idea comes from the level-crossing feature of a hydrogen-like ion immersed in the plasma. Viewing the lowest electronic states to which the crossing between all neighboring levels takes place, the density is shown to be expressed in a tractable form as a function of the ionization-potential lowering of the emitter. The calculation performed for the ion-sphere model with the Hartree-potential turns out to be as accurate as those for the Stewart-Pyatt model. The edge-shift is discussed in connection with the ionization-potential lowering which is caused from the screening effect in the plasma.

§ 1. Introduction

A practical scheme of taking account of the high-density plasma perturbations was proposed to explain the continuum-lowering phenomena for the recombining process: \( \text{Ne}^{10+} + e \rightarrow \text{Ne}^{19+} + h\nu \) on the basis of the ion-sphere model along with the Stark broadening theory. The ion-sphere model is generally considered to be useful and is commonly used in the plasma diagnostics. On the other hand, the X-ray spectroscopic method has been often used to measure the density of laser-irradiated plasmas. The edge-shift of free-bound continuum from fully stripped neon ions observed just after the maximum compression was analyzed by using the Stewart-Pyatt model, where the electron temperature and the density were arranged by computer simulation. That was also studied by the density-of-state approach based on the band-theory of a random system, resulting in a qualitative agreement with our spectroscopic investigation of the plasma. It was shown that the uniform-electron assumption is effective even within the first-order perturbation theory to obtain a fairly good result for the frequency shift of the \( \text{Ne}(X)\text{Ly}-\alpha \) line of an ultra-dense and moderate temperature plasma. A detailed analysis for plasma electrons was able to present a self-consistent potential for the bound electron, but it only proved on the contrary that the Stewart-Pyatt-model calculation has an accuracy of a passing mark by itself in comparison with the elaborate theory.

In this paper, the edge-shift of free-bound continuum from fully-stripped ions is explained as a result of the screening effect in the plasma with inclination of strong-coupling by the first-order perturbation theory on the uniform-electron model without simulation. We do not take into any account the ion's effect or the electron's dynamical effect such as an impact broadening for the sake of simplicity. Instead, the static plasma-electrons are considered to play the most important part of the vast line-shifting that is indispensable for the investigation of a near-strongly coupled plasma. We find that the degree of cross-over or overlap among the electronic states
of a hydrogen-like ion depends very strongly on the electron density of the plasma.

In the following sections, the lowest electronic-state to which the crossing between all neighboring levels takes place in the proposed model is found to be just the same energy position where the X-ray-edge-shift corresponding to the continuum-lowering was found in the experiment. An analytical formalism for finding the lowest or critical state is so suggested as to play an important role of the spectroscopic investigation of a laser-imploded ultra-dense plasma. The comparison is made between the theory and an experiment in detail, and the discussion is presented as well.

§ 2. The level-crossing model

Consider a hydrogen-like ion of nuclear charge $Z$ for an emitter immersed in the uniform electron gas of density $N_e$. The energy level of the emitter characterized by the principal quantum number $n$ can be written as

$$E_n = E_n^{(0)} + \Delta E_n,$$

where $E_n^{(0)}$ is the free-state energy, i.e.,

$$E_n^{(0)} = -\frac{e^2}{2a_0} \frac{Z^2}{n^2},$$

and $\Delta E_n$ is the level shift produced by surrounding electrons. Define the radius of a sphere containing enough electrons to neutralize the ion as

$$R_0 = \left( \frac{Z-1}{4\pi N_e} \right)^{1/3}.$$  

We consider that the emitter is only an impurity and the probability for finding a neighboring ion is very small. The plasma under consideration is so extremely compressed that it is actually composed of fully stripped ions and a small number of one-electron ions together with enough free electrons distributed around these ions.

Now let us assume that the distribution of $Z-1$ free electrons is limited inside the sphere of the radius given by Eq. (2). The electrostatic perturbation potential felt by the optical electron is given by the Hartree-potential:

$$\Delta V(r) = \frac{Z-1}{2R_0} e^2 \left[ 3 - \left( \frac{r}{R_0} \right)^2 \right]$$  

for $r \leq R_0$, and

$$\Delta V(r) = \frac{(Z-1)e^2}{r}$$  

for $r > R_0$, where $r$ is the radial coordinate of the optical electron. The first term on the right-hand side of Eq. (3a) just yields the expression in the high-density limit for the edge-shift which depends on the density as $N_e^{1/3}$. We also consider the contribu-
tion from the second term of Eq. (3a).

The purpose of the present section is to find the lowest effective state $n_{\text{eff}}$ to which the crossing or overlapping in each pair of neighboring levels with quantum numbers $n$ ($n_{\text{eff}}$) and $n+1$ takes place,

$$E_n = E_{n+1},$$

because of the perturbation given by the Hartree-potential, Eq. (3). According to Eq. (1), we have

$$E_{n+1} - E_n = \frac{n^2 e^2}{2a_0} + \Delta E_{n+1} - \Delta E_n.$$  (5)

Within the first-order perturbation theory, the level shift is evaluated as

$$\Delta E_n = \frac{1}{\sum_{l=0}^{n-1} (2l+1)} \sum_{l=0}^{n-1} (2l+1) \frac{\langle nl | \Delta V(r) | nl \rangle}{\langle nl | nl \rangle},$$  (6)

where $|nl\rangle$ is the emitter wave function, with $\Delta V(r)$ given by Eq. (3). Since the contribution to $\Delta E_n$ from the outer region is very small at the density under consideration, we use the potential Eq. (3a) over the entire region of space for the sake of convenience. In effect, the density range under consideration corresponds to the situation where the coupling-coefficient $I_n$, i.e., a ratio of the Coulomb potential to the kinetic energy of charged particles, is not so small in comparison with unity. The radii of highly excited states exceed the ion sphere as the plasma density increases, and they can no more be well-defined bound states. In the present case, we deal with the extrapolated bound-states which are assumed to remain unchanged in the first-order theory.

By inserting Eq. (6) into Eq. (5), we readily find

$$E_{n+1} - E_n = \frac{2n+1}{n^2(n+1)^2} \frac{Z^2 e^2}{2a_0}$$

$$- (14n^3 + 21n^2 + 19n + 6)$$

$$\times \frac{e^2}{2Z^2 a_0^2} N_e,$$  (7)

where use has been made of Eq. (2). Figure 1 shows the density dependence of the energy levels of the emitter Ne$^{9+}$. Note that only the $n$-dependent terms are illustrated, because the constant term contributes to raise all the discrete levels by the same amount towards the continuum. The curve linking the cross points becomes a straight line which falls off in proportion nearly to $N_e^{1/3}$.

Figure 1. Density dependence of hydrogenic energy levels for $Z=10$ in atomic unit. Only the $n$-dependent terms, i.e., $E_n - (3e^2/2)(\pi N_e/2)^{1/3}$ are illustrated. The continuum-lowering is represented by a curve which links the cross points of neighboring levels.
This is interpreted as the continuum-lowering that was observed\(^3\) and explained by the complex level shift theory.\(^1\) The Stewart-Pyatt model gives\(^4\) the same density dependence of the edge-shift but it originates in contrast to the present theory from the constant term of Eq. (3a) and depends scarcely on the temperature. The condition of Eq. (4) is only fulfilled if the electron density is as large as

\[
N_e = \frac{2n+1}{n^2(n+1)^2(14n^3+21n^2+19n+6)} \frac{3Z^4}{2\alpha_0^3}.
\]  

(8)

The experimental result\(^3\) or Fig. 12 in the literature,\(^4\) for example, shows that the free-bound continuum spectrum lowers down to just below the Ne(X)Ly-\(\gamma\) line, and a sign of spread-over in the Ly-\(\beta\) line profile is visible at the same time. We can expect that the level-crossing of all neighboring pairs among the states with principal quantum numbers \(n=4, 5, 6, \ldots\) have already completed at this time of compression, and the level with \(n=4\) has partially overlapped with the next lower level with \(n=3\). Even if the lines from lower levels, such as the Ly-\(\alpha\) and \(\beta\) lines, were emitted at the earlier or later stage of compression, other higher levels in most of neon ions had completely overlapped with their neighbors when the edge-shift appeared to occur and various recombination processes began to prevail. This is confirmed by the aspect stated in the footnote of Ref. 1). Hence, it is reasonable to put \(n=3\) and 4 into the right-hand side of Eq. (8) to obtain the upper and lower bounds on the electron density at the time of the maximum compression by

\[
5.5 \times 10^{23} < N_e < 2.5 \times 10^{24} \text{[cm}^{-3}\text{]}.
\]  

(9)

Neglecting the other ionization channels from impurities than those from neon, we can estimate the ion density of the plasma by using the relation \(N_i = N_e/Z\). The mass density then covers the range of \(2 < \rho < 8\) [g·cm\(^{-3}\)], which is in accordance with the value 5 g·cm\(^{-3}\) obtained by the simulation.\(^4\) Since the critical density given by Eq. (8) is nearly proportional to \(Z^4/n^6\), the higher-\(Z\) emitter is useful to get the more restricted range of plasma density.

Let us formally define an energy level corresponding to the normal hydrogen-like level with an effective principal quantum number \(n_{\text{eff}}\) as

\[
E_{n_{\text{eff}}}^{(0)} = -\frac{Z^2}{n_{\text{eff}}^2} \frac{e^2}{\alpha_0}.
\]  

(10)

As for a full comparison with the experiment,\(^3\) inserting \(E_{n_{\text{eff}}}^{(0)} = -127\) eV and \(Z = 10\) into Eq. (10), we find \(n_{\text{eff}} = 3.27\) which can be used as the critical value for the principal quantum number in Eq. (8). Then the electron and mass densities at the maximum compression are found to be \(N_e = 1.6 \times 10^{24}\) cm\(^{-3}\) and \(\rho = 5.4\) g·cm\(^{-3}\), respectively. These values are about the same as those obtained by the simulation.\(^4\)

\section{Discussion}

The level-crossing that gets down to \(n_{\text{eff}} = 3.27\) corresponding to the edge-shift of 127 eV seems to explain the observed continuum-spectrum\(^3\) quite well. The edge-shift is considered to be the lowering of the ionization potential for the ground state.
The upper state of the Ly-β line with $n = 3$ exceeds the ionization threshold, when the estimate for the electron density amounts to $1.4 \times 10^{24} \text{ cm}^{-3}$. This can be seen in Fig. 2, where the interpretation about the continuum-lowering is illustrated schematically in connection with the edge-shift. Remember a small time-lag for the emission of a recombination - continuum spectrum from those of the discrete-spectra in one shot of observation.\(^3\),\(^4\) It should be noted that the Lyman-series lines of Ne\(^{9+}\) were observed at a little bit lower density of $N_\text{e} \approx 2.1 \times 10^{22} \text{ cm}^{-3}$. Therefore, at this stage of compression, the shift of the ground state which amounts to 41.6 eV should also be taken into account. This implies $N_\text{e} \approx 1.4 \times 10^{24} \text{ cm}^{-3}$ and 4.8 g cm\(^{-3}\) for the edge-shift of 127 eV at the maximum compression in good agreement with the result of the present level-crossing approach.

The present approach seems to succeed in explaining the edge-shift of free-bound continuum as a phenomenon arising from the overlap of the states in the emitting ions. Besides the complex level shift theory,\(^1\) this model independently proposes an exceptionally good explanation about the phenomenon. Of course the perturbation theory breaks down if the splitting is large enough to disturb the unperturbed atomic states. In that case, a potential that satisfies the boundary condition for the bound electron is necessarily prepared. But the conclusion of the present research is consistent with the assumption stated just below Eq. (6) and also with an indication of the example represented in Fig. 2. The theoretical idea of the present model is not only valuable but also interesting in this respect, because the coupling coefficient among the corresponding experimental plasma conditions becomes at least of the order of $\sim 0.1$ at the maximum compression.

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**References**