Finite-Range DWBA Analysis of \( (p, \alpha) \) Reaction below the Coulomb Barrier

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The experimental angular distributions for the reaction \(^{19}\text{F}(p, \alpha)^{16}\text{O}(\text{g.s.})\) at incident energies of 250, 350 and 450 keV below the Coulomb barrier are analyzed with the finite-range distorted wave Born approximation formalism including both the direct and exchange processes. In order to treat in a consistent way the reaction mechanism of the reaction of \(^{19}\text{F}(p, \alpha)^{16}\text{O}(\text{g.s.})\) at keV and MeV energy ranges, we adopt the value of the spectroscopic amplitude obtained by the analysis of the experimental angular distribution for the same reaction at \(E_p=18\) MeV. Good agreements between theoretical calculations and experiments are obtained. Especially, the energy dependence of the differential cross section is nicely fitted to the data. Numerical calculations show that the direct reaction mechanism is dominant for this reaction, in particular the exchange process plays an important role for reproducing the experimental angular distributions in keV energy ranges.

§ 1. Introduction

Recently, the \(^7\text{Li}(p, \alpha)^4\text{He} \) and \(^{19}\text{F}(p, \alpha)^{16}\text{O}\) reactions\(^{11,12}\) at the sub-Coulomb energy ranges have been studied in terms of direct reaction mechanism. The former reaction is involved in the \(p-p\) chain of the Hydrogen burning in primordial and stellar nucleosynthesis at thermonuclear energies, the later reaction is responsible for the burning of \(^{19}\text{F}\) in the Carbon-Nitrogen-Oxygen (CNO) cycle. The investigation of the reaction mechanism of these reactions below the Coulomb barrier leads to the prediction of the creation of the elements in the early universe. No resonant behavior is observed for the ground state transition for the \(^{19}\text{F}(p, \alpha)^{16}\text{O}\) reaction up to about 500 keV, while the transitions to the excited states of \(^{16}\text{O}\) show marked resonance peaks. This indicates that the excited \(^{20}\text{Ne}\) states in this energy range have only a small \(\alpha + ^{16}\text{O}(\text{g.s.})\) component, meaning that the decay of the compound nucleus into the ground state of the exit channel is suppressed. Furthermore, in the reactions with a high \(Q\)-value such as the \(^{19}\text{F}(p, \alpha)^{16}\text{O}(\text{g.s.})\) reaction, the energy of the emitted particle is well above the Coulomb barrier in the exit channel. As a consequence, the ground state transition may be dominated by a direct reaction mechanism. Herndl et al.\(^2\) have studied the reaction \(^{19}\text{F}(p, \alpha)^{16}\text{O}(\text{g.s.})\) in the framework of triton pick up as the reaction mechanism and in terms of the zero-range distorted wave Born approximation (ZR-DWBA) at energies of 250, 350 and 450 keV below the Coulomb barrier. They pointed out that the direct reaction mechanism was dominant for this reaction in these energy ranges.

In this paper, we analyze the reaction \(^{19}\text{F}(p, \alpha)^{16}\text{O}(\text{g.s.})\) in the framework of the finite-range DWBA (FR-DWBA) instead of ZR-DWBA, considering the contributions to the exchange process as well as the direct process\(^4\) and examine more precisely the direct reaction mechanism in thermonuclear energy regions. Until now disagree-
ments between theoretical calculations, even though in terms of microscopic formalism with high accuracy, and experimental results for the cross sections with the three-nucleon transfer processes such as the \((p, a)\) reaction have been reported to be remarkable.\(^7\) In order to investigate in a consistent way the reaction mechanism of the \(^{19}\)F\((p, a)^{16}\)O\(\text{g.s.}\) reaction at keV and MeV energy ranges, we adopted the spectroscopic amplitude determined phenomenologically by the analysis of the experimental angular distributions for the same reaction at \(E_p=18\) MeV.\(^8\) A brief description of the transition amplitudes and the cross section are in § 2. In § 3, the results of the numerical calculation are given and some discussion is made in § 4.

\[ \text{§ 2. Formalism} \]

In this section, we discuss briefly the FR-DWBA differential cross section including both the direct and exchange processes in the \(T(p, a)R\) reaction.\(^9\)\(^-\)\(^11\) The differential cross section is given by

\[
\frac{d\sigma}{d\Omega} = \frac{\mu_p\mu_a}{(2\pi\hbar^2)^2} \frac{k_a}{k_p} \frac{1}{(2I_p+1)(2I_a+1)} \sum_{M_pM_a} |M|^2,
\]

where \(M\) is the total transition amplitude which is the sum of the direct transition amplitude \(M^D\) and the exchange one \(M^E\). The reduced mass and wave number in the initial (final) state are \(\mu_p\) (\(\mu_a\)) and \(k_p\) (\(k_a\)), respectively. The direct process (pickup and heavy-particle knockout) for the reaction \(T(p, a)R\) is described as

\[
p + T(=t + R) \rightarrow R + a (=p + t).\]

Similarly, the exchange process (knockout and heavy-particle pickup) is given by

\[
p + T(=a + C) \rightarrow R (=p + C) + a.
\]

These relations are schematically shown in Fig. 1.

In the expression (2), the target nucleus \(T\) is represented as bound states of a cluster \(t\) and a residual nucleus \(R\). The emitted particle \(a\) is a composite of the transferred cluster \(t\) and an incident particle \(p\). For the exchange process in the expression (3), a core nucleus \(C\) is transferred and the emitted particle \(a\) comes from the target nucleus \(T\). The total transition amplitude \(M\) for the reaction is

\[
M = N_D \cdot M^D + N_E \cdot M^E
\]

with the statistical factors \(N_D\) and \(N_E\) which depend on the number of transferable nucleons in each process.\(^{12}\)\(^,\)\(^13\) The

\[ \text{Fig. 1. The concept of } T(p, a)R \text{ for the direct and exchange processes.} \]
Finite-Range DWBA Analysis of (p, a) Reaction

FR-DWBA transition amplitudes $M^D$ and $M^E$ in the prior form are, respectively,

$$M^D = \langle \chi_f^D | V_{pt} + V_{pr} - U_{pt} | \chi_i^D \rangle$$  \hspace{1cm} (5)

and

$$M^E = \langle \chi_f^E | V_{pa} + V_{pc} - U_{pt} | \chi_i^E \rangle,$$  \hspace{1cm} (6)

where the interaction between the cluster $X$ and $Y$ is $V_{XY}$. The $U_{pt}$ is the optical model potential from which the entrance channel distorted waves are generated. The entrance and exit channel wave functions $\chi_i^D$ and $\chi_i^E$ in the direct process are given by, respectively,

$$\chi_i^D = \Psi_{I_{\phi M_{\phi}}(\xi_t, \xi_R, r_{\phi})} \Psi_{I_{\phi M_{\phi}}(\xi_p) \varphi_p^{(+)}(k_p, r_{\phi})}$$ \hspace{1cm} (7)

and

$$\chi_i^E = \Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_{\phi})} \Psi_{I_{\phi M_{\phi}}(\xi_a) \varphi_a^{(-)*}(k_a, r_{\phi})}.$$ \hspace{1cm} (8)

Similarly, the entrance and exit channel wave functions $\chi_f^D$ and $\chi_f^E$ in the exchange process are given by, respectively,

$$\chi_f^D = \Psi_{I_{\phi M_{\phi}}(\xi_t, \xi_c, r_{\phi})} \Psi_{I_{\phi M_{\phi}}(\xi_p) \varphi_p^{(+)}(k_p, r_{\phi})}$$ \hspace{1cm} (9)

and

$$\chi_f^E = \Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_{\phi})} \Psi_{I_{\phi M_{\phi}}(\xi_a) \varphi_a^{(-)*}(k_a, r_{\phi})}.$$ \hspace{1cm} (10)

where $\varphi_p^{(+)}(k_p, r_{\phi})$ and $\varphi_a^{(-)*}(k_a, r_{\phi})$ are the distorted waves for $p + T$ and $a + R$ elastic scattering, respectively. The functions $\Psi_{I_{\phi M_{\phi}}(\xi_t, \xi_R, r_{\phi})}$, $\Psi_{I_{\phi M_{\phi}}(\xi_p)}$, $\Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_{\phi})}$ and $\Psi_{I_{\phi M_{\phi}}(\xi_a)}$ are the internal or bound state wave functions for $T$, $p$ and $R$, respectively, in the direct process. Similarly, the internal or bound state wave functions in the exchange process are $\Psi_{I_{\phi M_{\phi}}(\xi_t, \xi_c, r_{\phi})}$, $\Psi_{I_{\phi M_{\phi}}(\xi_p)}$, $\Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_{\phi})}$ and $\Psi_{I_{\phi M_{\phi}}(\xi_a)}$ for $T$, $p$, $a$ and $R$, respectively. The spin (its $z$ component) and the internal coordinate of a particle $X$ are $l_X$ ($M_X$) and $\xi_X$, respectively. The cluster expansions for the target nucleus $T(=t + R)$ and the emitted particle $a(=p + t)$ are given by

$$\Psi_{I_{\phi M_{\phi}}(\xi_t, \xi_R, r_1)} = \sum_{[\delta_t]} \theta_{T(t,R)}(I_{1l_1 M_{1m_1} j_{1l_1 t_{1M_1}} j_{1l_1 t_{1M_1}}}) (j_{1l_1 t_{1M_1}} M_{1R} M_{T}) \times \Psi_{I_{\phi M_{\phi}}(\xi_t)} \Psi_{I_{\phi M_{\phi}}(\xi_R)} u_{n_{1l_1 t_{1M_1}}(r_1)} Y_{1i_{1l_1 t_{1M_1}}(r_1)}$$ \hspace{1cm} (11)

and

$$\Psi_{I_{\phi M_{\phi}}(\xi_p, \xi_c, r_2)} = \sum_{[\delta_a]} \theta_{A(a,c)}(I_{1l_2 M_{2m_2} j_{2l_2 t_{2M_2}} j_{2l_2 t_{2M_2}}}) (j_{2l_2 t_{2M_2}} M_{2p} M_{A}) \times \Psi_{I_{\phi M_{\phi}}(\xi_p)} \Psi_{I_{\phi M_{\phi}}(\xi_c)} u_{n_{2l_2 t_{2M_2}}(r_2)} Y_{1i_{2l_2 t_{2M_2}}(r_2)}$$ \hspace{1cm} (12)

with $[\delta_t] = [I_{2M_{2l_2} M_{2m_2} j_{2l_2 t_{2M_2}}}]$ and $[\delta_a] = [I_{2M_{2l_2} M_{2m_2} j_{2l_2 t_{2M_2}}}]$. The radial wave functions for the relative motion between $R$ and $t$ in the target $T$ and between $p$ and $t$ in the emitted particle $a$ are $u_{n_{1l_1 t_{1M_1}}}(r_1)$ and $u_{n_{2l_2 t_{2M_2}}}(r_2)$, respectively. The expansion coefficient $\theta_{A(a,c)}$ is the fractional parentage coefficient (fcp) for $A = B + C$. Similarly, the wave functions $\Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_1)$ and $\Psi_{I_{\phi M_{\phi}}(\xi_a, \xi_c, r_2)$ for the target and residual
nuclei are expanded in terms of the corresponding fractional parentage coefficients $\theta_{T(a,c)}$ and $\theta_{R(b,c)}$, respectively. The direct and exchange spectroscopic amplitudes $S^D$ and $S^E$ are expressed by $N_D \cdot \theta_{T(r,R)} \cdot \theta_{d(p,t)}$ and $N_E \cdot \theta_{T(a,c)} \cdot \theta_{R(b,c)}$, respectively. Inserting Eqs. (5) and (6) into Eq. (4), we get the total transition amplitude $M$ and the differential cross section. 

In charged particle induced reactions, the reaction cross section $\sigma(E)$ at and far from the Coulomb barrier depends predominantly on the barrier penetration. For nonresonant reactions, the reaction cross section reduces to approximately

$$\sigma(E) = S(E) \cdot \exp(-2\pi\eta/E)$$

with $\eta = Z_p Z_r e^2 / (\hbar v)$ where $2\pi\eta$ and $S(E)$ are the Sommerfeld parameter and the astrophysical $S(E)$ factor, respectively. The quantity $S(E)$ defined by Eq. (13) represents the effects of the nuclear reaction only apart from the effect of the Coulomb barrier in the reaction cross section $\sigma(E)$.

§ 3. Numerical calculation

In this section, the differential cross sections of the reaction $^{19}$F($p, a$)$^{16}$O(g.s.) with $Q=8.12$ MeV at $E_p=250, 350$ and $450$ keV are calculated by assuming both of the direct process and the exchange process in the FR-DWBA.

Radial wave functions $u_{nl}(r_i)$ of the bound states are solved in a real Woods-Saxon potential with the Coulomb potential of the uniformly charged sphere of radius $r_c$. The parameters of the radius $r_0$, the diffuseness $a$ of the Woods-Saxon potential and the radius $r_c$ of the Coulomb potential are taken from Refs. 2), 12), 14) and 15) for the bound states of $^{19}$F$\rightarrow^{16}$O+$t$ and $a$+$p$+$t$ in the direct process and $^{19}$F$\rightarrow^{15}$N+$a$ and $^{16}$O$\rightarrow^{15}$N+$p$ in the exchange process, respectively. The strengths $V$ of the potential are determined so as to fit the experimental separation energy $E_B$ of the cluster in the nucleus in the ground state. The potential parameters are listed in Table I. Here, we assume simply the bound states $^{16}$O+$t$ and $^{15}$N+$a$ to be the $4s$ and $4p$ states, respectively.

The interaction potentials $V_{pt}$ and $V_{pr}$ for the direct process and $V_{pa}$ and $V_{pc}$ for the exchange process are taken as the real Woods-Saxon form without the Coulomb interaction. The parameters for the potential are listed in Table II. 12), 16), 17)

First, in order to study in a consistent way the reaction mechanism of the $(p, a)$ reaction at keV and MeV energy ranges, the value of the spectroscopic amplitudes $S^D$ and $S^E$ mentioned above are obtained phenomenologically from the analysis of the experimental angular distribution for the reaction $^{19}$F($p, a$)$^{16}$O(g.s.) at $E_p=18$ MeV.

<table>
<thead>
<tr>
<th>System</th>
<th>State</th>
<th>$E_B$(MeV)</th>
<th>$V$(MeV)</th>
<th>$r_0$(fm)</th>
<th>$a$(fm)</th>
<th>$r_c$(fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{19}$F$\rightarrow^{16}$O+$t$</td>
<td>4s</td>
<td>11.70</td>
<td>108.96</td>
<td>1.31</td>
<td>0.65</td>
<td>1.31</td>
<td>2)</td>
</tr>
<tr>
<td>$a$+$p$+$t$</td>
<td>1s</td>
<td>19.82</td>
<td>60.99</td>
<td>1.43</td>
<td>0.30</td>
<td>1.43</td>
<td>12)</td>
</tr>
<tr>
<td>$^{19}$F$\rightarrow^{15}$N+$a$</td>
<td>4p</td>
<td>4.01</td>
<td>92.58</td>
<td>1.43</td>
<td>0.56</td>
<td>1.43</td>
<td>14)</td>
</tr>
<tr>
<td>$^{16}$O$\rightarrow^{15}$N+$p$</td>
<td>1p</td>
<td>12.13</td>
<td>60.98</td>
<td>1.13</td>
<td>0.66</td>
<td>1.15</td>
<td>15)</td>
</tr>
</tbody>
</table>
The parameters for the optical potential at $E_p=18$ MeV are listed in Table III. Here, we adopted the modified Woods-Saxon potential of Michel's type as the $a^{16}\text{O}$ optical potential of the final state. Perey’s effects which take account of the damping effects of the distorted wave functions in nuclear interior region are involved in calculating the distorted waves. The parameters $\beta$ for the entrance and exit channels are 0.85 fm and 0.54 fm, respectively. In Fig. 2, we show the calculated angular distributions together with the experimental data at $E_p=18$ MeV. The solid line is the calculated result by making use of the FR-DWBA including both direct and exchange processes. Compared with the results of Neu et al., our calculated results are in good agreement with the experimental data. The values of $S^D$ and $S^E$ are obtained to be 1.57 and 0.74, respectively.

Next, we calculate the differential cross sections of the reaction $^{19}\text{F}(p, a)^{16}\text{O}$ (g.s.) at $E_p=250$, 350 and 450 keV. In the entrance channel $p^{19}\text{F}$, we use the optical potential of the energy dependent Woods-Saxon type of Herndl et al. We can reproduce the experimental

Table II. Parameters for interaction potentials for the direct and exchange processes.

<table>
<thead>
<tr>
<th>Interaction</th>
<th>$V$(MeV)</th>
<th>$r_0$(fm)</th>
<th>$a$(fm)</th>
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<tr>
<td>$V_{pd}$</td>
<td>61.4</td>
<td>1.39</td>
<td>0.40</td>
</tr>
<tr>
<td>$V_{pp}$</td>
<td>47.3</td>
<td>1.25</td>
<td>0.64</td>
</tr>
<tr>
<td>$V_{pa}$</td>
<td>45.0</td>
<td>1.40</td>
<td>0.44</td>
</tr>
<tr>
<td>$V_{pc}$</td>
<td>43.9</td>
<td>1.13</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Table III. Optical potential parameters.

<table>
<thead>
<tr>
<th></th>
<th>$V$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_1$ (fm)</th>
<th>$a_1$ (fm)</th>
<th>$r_2$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p^{19}\text{F}$</td>
<td>39.63</td>
<td>1.61</td>
<td>0.66</td>
<td>5.95</td>
<td>1.96</td>
<td>0.84</td>
<td>1.25</td>
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<tr>
<td>volume type</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a^{16}\text{O}$</td>
<td>38.00</td>
<td>1.70</td>
<td>0.60</td>
<td>25.00</td>
<td>1.24</td>
<td>0.65</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table IV. Optical potential parameters.

<table>
<thead>
<tr>
<th>System</th>
<th>Real part</th>
<th>Imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^{19}\text{F}$</td>
<td>$V=V_0+V_1\cdot E_{lab}$</td>
<td>$W=W_0+W_1\cdot E_{lab}$</td>
</tr>
<tr>
<td></td>
<td>$V_0=56.5, V_1=-2.0$</td>
<td>$W_0=0.625, W_1=1.5$</td>
</tr>
<tr>
<td></td>
<td>$r_0=1.12, a_0=0.48$</td>
<td>$r_0=1.25, a=0.55$</td>
</tr>
<tr>
<td></td>
<td>$r_c=1.20$</td>
<td></td>
</tr>
<tr>
<td>$a^{16}\text{O}$</td>
<td>$V=150.0$</td>
<td>$W_{a^{16}O}=3.0$</td>
</tr>
<tr>
<td></td>
<td>$r_0=0.723, a_0=0.48$</td>
<td>$r_0=0.723, a=0.5$</td>
</tr>
<tr>
<td></td>
<td>$r_c=1.40$</td>
<td></td>
</tr>
</tbody>
</table>

a) surface type. (unit: $V$(MeV), $W$(MeV), $r$(fm), $a$(fm))
differential cross section\textsuperscript{21} for $p^{-19}$F elastic scattering as a function of proton bombarding energy of 600–1500 keV by making use of these optical potential parameters as is shown in Fig. 3. In the exit channel $\alpha + ^{16}$O, we use the potential with the surface absorption term by Mehta et al.\textsuperscript{22} These optical potential parameters are shown in Table IV.

In Figs. 4~6, the calculated results of the cross section at $E_p = 250$, 350 and 450 keV are shown with the contributions to the cross section from each process of the direct and the exchange, respectively. The solid line is the calculated result with the total amplitude $M$ in Eq. (4) with the $S^0$ and $S^E$ obtained at 18 MeV. Also, the dotted line and dashed line are the calculated results of the direct process and exchange process, respectively. We find that the calculated solid curve is in good agreement with the experimental data all over the whole angles. The contribution of the exchange process to the angular distributions at backward angles is remarkable. We
cannot reproduce the experimental data by the only direct process at all. In Fig. 7, we show the calculated results of the cross sections at $E_p=250$, $350$, $450$ keV and $18$ MeV, together with the experimental data. It is shown that the general feature of the calculated curves shows a good agreement with the data. The experimentally observed energy dependence is nicely reproduced by our calculations.

§ 4. Discussion and summary

The direct reaction process has been commonly accepted to be dominant in the transfer reaction at energies higher than about $20$ MeV. Compound mechanisms have frequently been suggested for astrophysically relevant reactions which proceed at sub-Coulomb energies. Our calculations show that the direct reaction mechanism is also important for astrophysically relevant transfer reactions in agreement with the results of Herndl et al. and Raimann et al. Furthermore, our FR-DWBA results show that the contributions to the cross sections from both effects of the direct and exchange processes are important in the reaction $^{19}\text{F}(p,\alpha)^{16}\text{O}(\text{g.s.})$ in thermonuclear energy. In particular, the shape of the angular distributions at the backward angles cannot be represented without the consideration of the exchange process. The FR-DWBA treatment is more appropriate than the ZR-DWBA for this reaction at sub-Coulomb energy.

The astrophysical $S$-factor in Eq. (13) can be parametrized in the analytical form

$$S(E)=S(0)+\dot{S}(0)E+\frac{1}{2}\ddot{S}(0)E^2.$$  
(14)

For nonresonant reactions, $S(E)$ factor is known to be generally constant in low energies. Our theoretical calculation yields $S(0)=7.685$ MeV·b, $\dot{S}(0)=-1.12$ b and $\ddot{S}(0)=2.00$ MeV⁻¹b in comparison with the values $S(0)=8.755$ MeV·b, $\dot{S}(0)=-3.48$ b and $\ddot{S}(0)=20.1$ MeV⁻¹b of the ZR-DWBA in Fig. 8.

In summary, we studied the experimental angular distributions for the $^{19}\text{F}(p,\alpha)^{16}\text{O}(\text{g.s.})$ reaction at incident...
energies of 250, 350 and 450 keV below the Coulomb barrier are analyzed with the finite-range distorted wave Born approximation formalism including both the direct and exchange processes. We treated in a consistent way the reaction mechanism of the reaction of the $^{19}$F$(p,\alpha)^{16}$O(g.s.) at keV and MeV energy ranges by adopting the spectroscopic amplitude obtained phenomenologically by the analysis of the same reaction at $E_p=18$ MeV. Good agreements between theoretical calculations and experiments were obtained for the shapes and the absolute magnitudes of the cross sections. Especially, the energy dependence of the differential cross section is nicely fitted to the data. The direct reaction mechanism is dominant for this reaction, in particular the exchange process plays an important role for reproducing the experimental angular distributions in keV energy ranges.

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References