

# Sampling/stochastic dynamic programming for optimal operation of multi-purpose reservoirs using artificial neural network-based ensemble streamflow predictions

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## ABSTRACT

Due to limited water resources and the increasing demand for agricultural products, it is significantly important to operate surface water reservoirs optimally, especially those located in arid and semi-arid regions. This paper investigates uncertainty-based optimal operation of a multi-purpose water reservoir system by using four optimization models. The models include dynamic programming (DP), stochastic DP (SDP) with inflow classification (SDP/Class), SDP with inflow scenarios (SDP/Scenario), and sampling SDP (SSDP) with historical scenarios (SSDP/Hist). The performance of the models was tested in Zayandeh-Rud Reservoir system in Iran by evaluating how their release policies perform in a simulation phase. While the SDP approaches were better than the DP approach, the SSDP/Hist model outperformed the other SDP models. We also assessed the effect of ensemble streamflow predictions (ESPs) that were generated by artificial neural networks on the performance of SSDP/Hist. Application of the models to the Zayandeh-Rud case study demonstrated that SSDP in combination with ESPs and the K-means technique, which was used to cluster a large number of ESPs, could be a promising approach for real-time reservoir operation.

**Key words** | artificial neural networks, dynamic programming, ensemble streamflow predictions, reservoir operation, stochastic optimization, Zayandeh-Rud Basin

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## INTRODUCTION

Optimal operation of surface water reservoirs is of great importance due to inadequate water resources and frequent droughts. Reservoir operation is about setting releases from a reservoir in different time steps to meet some predefined operational objectives. Reservoir operators must constantly decide on the amount to be released or stored to meet present or future demands. The problem becomes more complicated in presence of uncertainties about future inflows and demands (Kim *et al.* 2007).

Mathematical optimization approaches such as linear programming, nonlinear programming, and dynamic programming (DP) have been widely used in real-world applications (Labadie 2004; Simonovic 2009). The sequential nature of reservoir operation decisions and the ease of handling nonlinear relationships and uncertain streamflows

have all made DP attractive for use in reservoir operation optimization. By using the principle of optimality (Bellman 1957), DP provides a systematic procedure for decomposing a complex multi-stage optimization problem into simpler single-stage sub-problems (Yakowitz 1982).

The decision process becomes more difficult due to the variability and uncertainty of natural streamflow. This feature has made stochastic dynamic programming (SDP) promising with many successful applications in reservoir operation optimization (e.g. Mousavi *et al.* 2004a, b; Kim *et al.* 2007; Mujumdar & Nirmala 2007). Several SDP applications have dealt with the representation of streamflow persistence and forecast information in the decision-making process by incorporating a hydrologic state variable into the SDP formulation. The common choices of the

hydrologic state variable are the previous or current period's inflow and seasonal streamflow forecast (Stedinger *et al.* 1984; Karamouz & Vasiliadis 1992). Bras *et al.* (1983) showed that incorporation of current hydrologic forecast information in an online SDP model can lead to a more efficient operation of High Aswan Dam. In this application a real-time control problem was formulated and solved by SDP.

Reinforcement learning (RL) is another approach dealing with the uncertainty of streamflow. RL solves the recursive equation of SDP through simulation. In RL, the SDP's future-value function (FVF) is estimated and adjusted iteratively using a systematic algorithm such as Q-learning (Lee & Labadie 2007; Castelletti *et al.* 2010, 2013; Pianosi *et al.* 2012).

To improve the SDP performance with the consideration of the current hydrologic state of a basin, Kelman *et al.* (1990) proposed sampling stochastic dynamic programming (SSDP) and derived optimal operation policies for Feather Reservoir in California. The SSDP approach employs multiple streamflow hydrographs or scenarios, as an empirical distribution of streamflows, rather than using an explicit probabilistic description of the streamflow stochastic process. Such an advantage is also taken in RL for avoiding the curse of modeling. This key feature makes the SSDP approach more efficient when used both off-line, for deriving long-term optimal stationary policies, and on-line for real-time operation. In real-time operation, inflow forecasts are updated as new information becomes available. The SSDP formulation enables different types of streamflow forecasts, such as ensemble streamflow predictions (ESPs), to be easily incorporated into the model structure. As a result, SSDP can take full advantage of ESPs. For example, the National Weather Service (NWS) of the United States produces streamflow forecasts in the form of multiple hydrographs, each a possible realization of seasonal streamflow (Day 1985). Faber & Stedinger (2001) successfully combined SSDP with NWS's ESP forecasts to build a decision support system and derive optimal weekly reservoir operation policies. Kim *et al.* (2007) applied SSDP with monthly ESP forecasts to optimal operation of a multi-reservoir system in the Geum River Basin, Korea. Eum *et al.* (2011) combined SSDP/ESPs with hedging rules to derive optimal operation policies under drought conditions. In all of the mentioned

studies, use of SSDP combined with ESPs resulted in more efficient operational policies.

This paper focuses mainly on comparing the SDP and SSDP models and also on the way that SSDP models are integrated with ESPs generated by a data driven technique for real-time reservoir operation. In this regard, the performance of four optimization approaches including DP, standard SDP (SDP/Class), SDP with inflow scenarios (SDP/Scenario), and SSDP with historical inflow scenarios (SSDP/Hist) are compared by simulating their policies and assessing performance indices.

Artificial neural network (ANN), a well-known data-driven technique, is employed to generate a large number of ESPs that cover a wider range of possible future inflow scenarios. However, to make the SSDP approach computationally tractable, a limited, representative number of ESPs should be selected. This task is done by using the K-means clustering technique. Afterwards, the clustered scenarios and the best SSDP model are integrated for real-time operation of Zayandeh-Rud Reservoir in Iran. Figure 1 shows the schematic representation of the described procedure.

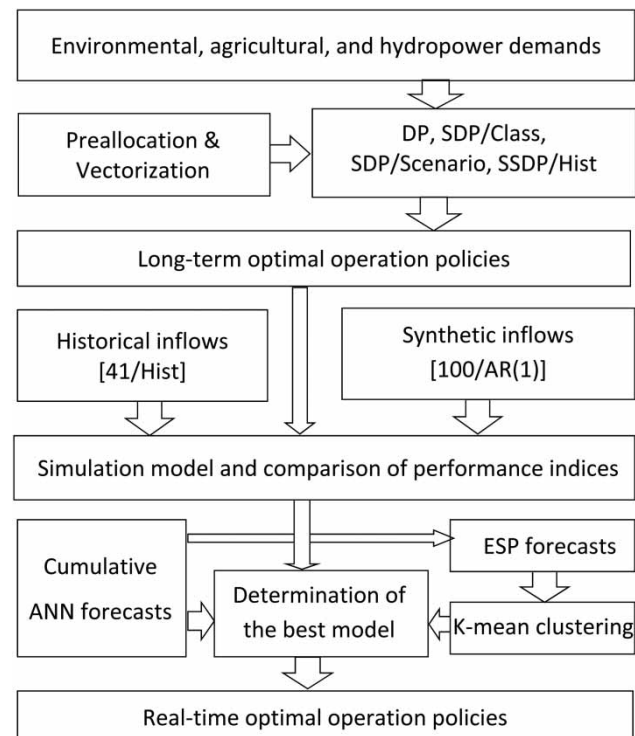


Figure 1 | Schematic representation of different stages of the study.

The rest of the paper is organized as follows. The employed methods as well as the formulation of different optimization models are presented in the next section. The results are then discussed. A summary and conclusions are presented in the final section.

## MATERIALS AND METHODS

### Study area

Zayandeh-Rud Basin is located in the central part of Iran between  $50^{\circ}20'$  to  $53^{\circ}17'$  eastern longitude and  $31^{\circ}08'$  to  $33^{\circ}44'$  northern latitude. It covers an area of  $41,500 \text{ km}^2$ . The Zayandeh-Rud River is the main resource for agricultural production, hydropower generation, as well as domestic and industrial water demand satisfaction in Isfahan metropolitan area. Having a storage volume of 1,470 million cubic meter (MCM), it is the largest surface reservoir constructed on the Zayandeh-Rud River. More than 70% of demand is for irrigating eight irrigation units, i.e. Nekooabad Left Bank (LB), Nekooabad Right Bank (RB), Mahyar, Borkhar, Abshar LB, Abshar RB, Rudasht, and a collection

of small-scale units with a total area of 205,000 ha. Wheat, barley, sugar beet, alfalfa, and rice are the primary crops being cultivated in this area (Murray-Rust et al. 2004). Figure 2 shows the location of the study area and the irrigation units.

Table 1 presents the monthly domestic, industrial, and environmental demands downstream of Zayadeh-Rud Dam. Table 2 also summarizes the characteristics of Zayadeh-Rud Dam and its power plant.

For all DP and SDP models, reservoir capacity was discretized into 10 storage indices using the Savaransky scheme with a good converging property (Doran 1975). The release from the reservoir was also discretized into 20 discrete points. Historical time series of inflow data for a period of 24 years from 1981–1982 to 2004–2005 were used to derive optimal operation policies. While the seasonal long-term average streamflows were used in deterministic DP, SDP/Class fitted the log-normal distribution to seasonal historical flows. The resulting conditional distributions were discretized then into five class intervals.

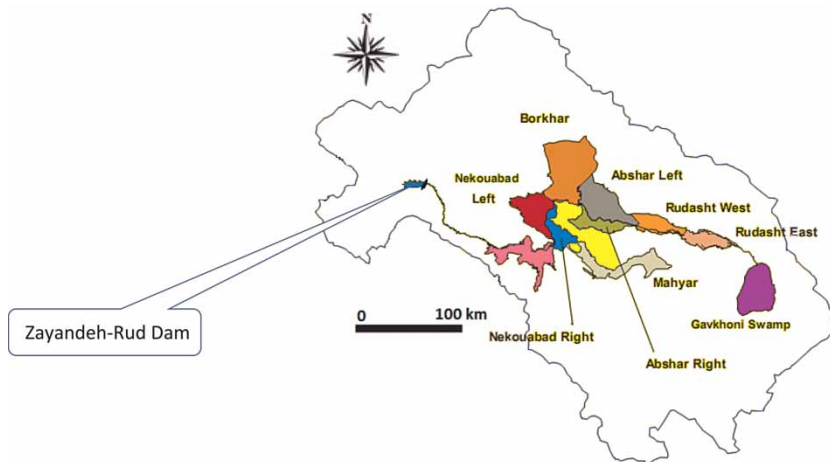


Figure 2 | Zayandeh-Rud River Basin and its irrigation units.

Table 1 | Monthly demands downstream of Zayandeh-Rud Dam (Morid et al. 2004)

Downstream demands	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.
Domestic, industrial and environmental (DIE) demands (MCM)	21	35	30	17	17	26	48	66	71	57	57	41
Agricultural demands (MCM)	24	41	20	0	3	61	216	297	172	94	78	11

**Table 2** | Characteristics of Zayandeh-Rud Dam and its power plant

Maximum reservoir storage (MCM)	1,500	Tail water level (TWL) (m)	1978
Minimum reservoir storage (MCM)	200	Plant factor (PF)	25%
Installed capacity (MW)	55	Efficiency (Eff)	95%

### Formulation of optimization models

This section presents the general formulation of the optimization model for reservoir operation which includes objective function and constraints as well as some explanations of various DP-based methods.

#### Objective function

The objective function of the optimization model is to minimize the shortages occurring in supplying water to domestic, environmental, industrial, agricultural, and hydropower demands. The shortage in hydropower generation is defined as the difference between a desired dependable energy and the produced energy when it is less than the desired one. Therefore, the model's objective function, which is defined as the sum of one-period immediate cost functions,  $cost_t$ , can be stated as follows:

$$\text{Min} \sum_t^{N \times T} cost_t = \sum_t \left( \frac{D_{\text{Agri}_t} + D_{\text{DIE}_t} - R_t}{\max_t(D_{\text{Agri}} + D_{\text{DIE}})} \right)^2 + \left( \frac{E_f - E_t}{E_f} \right)^2 \times I_t, \quad (1)$$

$$I_t = \begin{cases} 1, & \text{if } E_f > E_t \\ 0, & \text{otherwise,} \end{cases}$$

$$E_t = 2.725 \times \text{Eff} \times R_t \times \bar{H}_t \quad (2)$$

$$\bar{H}_t = \left( \frac{H_t + H_{t+1}}{2} \right) - \text{TWL} \quad (3)$$

where  $t$  = index for season or month ( $t = 1, \dots, T$ ),  $N$  = number of years,  $D_{\text{Agri}_t}$  = agricultural demand,  $R_t$  = water release from reservoir,  $D_{\text{DIE}_t}$  = sum of domestic, industrial, and environmental (DIE) demands all in period  $t$ ,  $\max_t(D_{\text{Agri}} + D_{\text{DIE}})$  = maximum of monthly ( $D_{\text{Agri}} + D_{\text{DIE}}$ ) values,  $E_f$  = desired dependable energy,  $E_t$  = hydropower produced in period  $t$ ,  $\text{Eff}$  = efficiency,  $H_t$  or  $H_{t+1}$  = reservoir water

head at the beginning or end of period  $t$ ,  $\bar{H}_t$  = average head on the turbine in period  $t$ ; and  $\text{TWL}$  = downstream tailwater level.

#### Constraints

The state of the reservoir system in the next stage resulting from a particular decision in the present stage is calculated based on the reservoir storage mass balance equation

$$S_{t+1} = S_t + Q_t - R_t - e(S_{t+1}, S_t) \quad (4)$$

The lower and upper bounds on reservoir storage and release as well as the upper bound on energy production ( $E_{\text{max}} = \text{Icap} \times n\text{hours}$ ) can be stated as follows:

$$S_{\text{min}} \leq S_{t+1} \leq S_{\text{max}} \quad (5)$$

$$\text{Min}\{Q_t, D_{\text{DIE}_t}\} \leq R_t \leq S_{\text{max}} - S_{\text{min}} \quad (6)$$

$$E_t \leq \text{Icap} \times n\text{hours} \quad (7)$$

where  $S_t/S_{t+1}$  = reservoir storage levels at the beginning/end of period  $t$ ,  $Q_t$  = inflow to reservoir in period  $t$ ,  $S_{\text{min}}$  = reservoir's minimum storage level,  $S_{\text{max}}$  = reservoir's maximum storage level,  $e(S_{t+1}, S_t)$  = evaporation loss in period  $t$ ,  $\text{Icap}$  = power plant's installed capacity, and  $n\text{hours}$  = number of hourly time steps in period  $t$ .

It is worth mentioning that the constraint placed to meet the DIE demands with the highest priority (Equation (6)) is satisfied by the careful selection of the smallest discrete release. In other words, it is enough to set the lower bound of the release variable to the smaller value between inflow and DIE.

Based on the above optimization model, we have formulated and tested various DP and SDP models depending on the way that the uncertainty of the streamflow process is dealt with. The models include DP, SDP/Class, SDP/Scenario, SSDP/Hist, and SSDP/ESP. The DP model is deterministic, whereas the SDP/Class model describes streamflow with a discrete lag-one Markov process. In SDP/Class, inflow variables are discretized into a number of class intervals and seasonal transition (conditional) probabilities are incorporated in the recursive equation of SDP.

Both SDP/Scenario and SSDP/Hist utilize transition probabilities between sequences of streamflows (scenarios), rather than streamflow points, in their formulations. Nevertheless, SDP/Scenario finds a solution by solving a single recursive equation backward in time similar to SDP/Class. However, SSDP/Hist solves two successive equations in each time period to maintain streamflow persistence better. The SSDP/ANN model investigates the effect of cumulative forecast scenarios, obtained by ANN models, on the performance of SSDP/Hist. The ANN cumulative forecasts are also used to generate ESP scenarios synthetically. The generated ESPs are combined with the SSDP model adaptively for real-time reservoir operation. More details about each of the optimization methods and their recursive equations are presented below.

*DP.* DP is an optimization technique which is consistent with the sequential decision-making nature of reservoir operations (Yeh 1985). In this approach, the operation time horizon is divided into a number of stages ( $t$ ). The state of the system at each stage  $t$  may be described by the reservoir storage ( $S_t$ ). For each stage and state, a release of water ( $R_t$ ) is determined so as to minimize the sum of the immediate cost function,  $\text{cost}_t(S_t, Q_t, R_t)$ , and the future-value or cost-to-go function,  $f_{t+1}^n(S_{t+1})$ . Note that the FVF depends on the state of the system at the beginning of the next stage,  $S_{t+1}$ . Taking advantage of the form of the objective function (Equation (1)), which is additively separable with respect to decision variables (releases), one can decompose the multi-stage optimization problem to a number of single-stage sub-problems. This can be accomplished according to Bellman's principle of optimality by solving the following recursive equation (Loucks et al. 1981):

$$f_t^n(S_t) = \min_{R_t} \{ \text{cost}_t(S_t, Q_t, R_t) + f_{t+1}^{n-1}(S_{t+1}) \} \quad (8)$$

$$\forall S_t \text{ and } t = 1, \dots, T, n = 1, \dots, N \times T$$

where  $f_t^n(S_t)$  is the optimal return (cost) from beginning of period  $t$  to the end of operation horizon with  $n$  periods remaining, and the reservoir storage is at state  $S_t$ . Note that while  $t = 1, \dots, T$  is cyclic and is repeated every year,  $n$  is associated with the total number of periods (stages) remaining.

Equation (8) is solved backward in time for discrete values of state ( $S_t$ ) and decision ( $R_t$ ) variables by discretizing these variables into a number of class intervals. Computations start at an arbitrary ending stage with a known terminating FVF,  $f_{T+1}^0(S_{T+1})$ , and continue over a number of cycles (years) until it reaches the steady-state conditions. The conditions are met when release policies and annual operation costs remain constant for all of the discrete storages ( $S_t$ ) and seasons ( $t$ ) in two successive cycles (iterations). We assumed that  $f_{T+1}^0(S_{T+1}) = 0$ , which is a typical assumption in long-term reservoir operation (Kelman et al. 1990; Faber 2000).

*SDP/Class.* To explicitly model the uncertainty of inflows, one can extend DP to SDP where streamflows are described by a cyclic first-order Markov process. The SDP recursive equation can be stated as follows (Loucks et al. 1981):

$$f_t^n(S_t, Q_t) = \min_{R_t} \{ \text{cost}_t(S_t, Q_t, R_t) + \sum_{Q_{t+1}} P[Q_{t+1}|Q_t] \cdot f_{t+1}^{n-1}(S_{t+1}, Q_{t+1}) \} \forall S_t, Q_t \text{ and } t = 1, \dots, T \quad (9)$$

where  $P_t[Q_{t+1}|Q_t]$  is the inflow transition probability of period/season  $t = 1, \dots, T$  and other variables and functions are defined previously. The state space is two-dimensional with  $S_t$  and  $Q_t$  as state variables. In fact, a favorite feature of the SDP model is its potential capability of incorporating hydrologic state variables into the model formulation by which the spatio-temporal correlations among streamflows are captured.

In the above formulation as  $t = 1, \dots, T$  is cyclic, the transition probabilities  $P_t[Q_{t+1}|Q_t]$  are cyclic too. Similar to Equation (8), the recursive Eq. (9) is solved for all of the discrete values of state ( $S_t$  and  $Q_t$ ) and decision ( $R_t$ ) variables over a number of cycles until it reaches the steady-state conditions. This means that besides  $S_t, Q_t$  is also discretized into a number of class intervals and the transition probabilities are estimated using a proper method of frequency analysis. That is why we have named this model SDP/Class.

*SDP/Scenario.* In SDP/Scenario, the discrete one-step transition probabilities of SDP/Class are replaced by a number of inflow sequences or scenarios, where each scenario is a

year of inflow data representing one T-month (12-month) realization of the corresponding stochastic process (Kelman et al. 1990). Unlike SDP/Class, SDP/Scenario uses multiple scenarios of streamflow process as an empirical distribution of streamflow variability, rather than employing an explicit probabilistic representation of streamflow stochastic process. Therefore, the recursive equation of SDP/Scenario may be written as follows:

$$f_t^m(S_t, i) = \min_{R_t} \{ \text{Cost}_t(S_t, Q_t(i), R_t) + E_{j|i} [f_{t+1}^{m-1}(S_{t+1}, j)] \} \quad (10)$$

$$\forall S_t, i, t \in \{1, \dots, T\}$$

where  $t$  = time period or stage starting from 1 to final period  $T$ ,  $i$  = an index for streamflow scenario ( $i = 1, \dots, m$ ) with a length equal to  $T$ , and  $Q_t(i)$  = inflow to reservoir in period  $t$  for scenario  $i$ . Other functions and variables including  $\text{Cost}_t(S_t, Q_t, R_t)$ ,  $S_t$  and  $R_t$  have already been defined. In this formulation, the hydrologic state of the system is represented by streamflow scenario  $i$ , which includes an entire history of the streamflow process rather than a single discrete level of streamflow. Note that  $j$  is the scenario following scenario  $i$ . The expected FVF is estimated based on the probability that the scenario  $i$ 's streamflows are followed by those of scenario  $j$  in the next period (Faber & Stedinger 2000).

To employ SDP/Scenario, one must specify the probability of the remainder of scenario  $j$  starting in period  $t + 1$  given scenario  $i$  in period  $t$ , i.e.  $P_t[\text{scenario } j | \text{scenario } i]$ . To do so, one can use Bayes' theorem as presented in Equation (11) (Kelman et al. 1990)

$$P[Q_{t+1}(j)|Q_t] = \frac{P[Q_t|Q_{t+1}(j)]P[j]}{\sum_{i=1}^m P[Q_t|Q_{t+1}(i)]P[i]} \quad (11)$$

where  $P[i]$  and  $P[j]$  are empirical streamflow distributions obtained by weighting equally all of the historical flow series, i.e.  $1/m$ . The likelihood function  $P[Q_t|Q_{t+1}(j)]$  can be estimated by using a regression relationship between historical flows in time period  $t + 1$ , scenario  $j$ , and the flow values in time period  $t$ , scenario  $i$ .

*SSDP/Hist.* The formulation of SSDP/Hist is similar to that of SDP/Scenario with one major distinction. In an effort to maintain streamflow persistence better, the optimal decision

$R_t$  for each scenario is chosen according to Equation (12). However, the FVF is updated for an intact scenario (Equation (13)). Therefore, the SSDP/Hist formulation is defined as follows (Faber & Stedinger 2000):

$$\min_{R_t} \{ \text{Cost}_t(S_t, Q_t(i), R_t) + E_{j|i} [f_{t+1}(S_{t+1}, j)] \} \quad (12)$$

$$\forall S_t, i \text{ and } t \in \{1, \dots, T\}$$

$$f_t(S_t, i) = \text{Cost}_t(S_t, Q_t(i), R_t) + f_{t+1}(S_{t+1}, i) \quad (13)$$

$$\forall S_t, i \text{ and } t \in \{1, \dots, T\}$$

where  $Q_t(i)$  is the streamflow of scenario  $i$  in period  $t$ ,  $i = 1, \dots, m$ , and  $j$  is the streamflow scenario following scenario  $i$ .  $P_t[j|i]$  can be estimated by using Bayes' theorem (Equation (11)). For a more comprehensive discussion on this issue, one can refer to Faber & Stedinger (2000).

According to above formulations, one can conclude that in SDP/Class, seasonal inflow variables are discretized into a number of class intervals with a representative value selected for each class interval. Then, the state transition probabilities among the inflow values (not inflow scenarios) or conditional density functions are estimated for each season (month). These probabilities are used subsequently in the recursive equation of SDP. However, both SDP/Scenario and SSDP/Hist make use of the probabilities of either historical or synthetic inflow scenarios directly. However, SDP/Scenario searches for an optimal solution by calculating the FVF using a single recursive equation (Equation (10)), whereas SSDP/Hist finds the solution by solving two separate equations (Equation (12), Equation (13)) to maintain the streamflow persistence better. Note that SSDP utilizes streamflow scenarios that include temporal structure of streamflow process. Therefore, SSDP/Scenario is in fact an SDP that benefits from such an advantage.

It is worth mentioning that all of the described models including DP, SDP/class, SDP/scenario, and SSDP/Hist provide an off-line solution in the form of feedback control/release policies. In the following section, we demonstrate how we can use ANNs to generate synthetic ESP forecasts and integrate SSDP and ESPs for real-time reservoir operation.

*Real-time operation: integration of ESP forecasts and SSDP.* The general idea of using ESP forecasts is similar to that

proposed by Faber & Stedinger (2001), but with a clear distinction. In the current study, ANNs are used to generate monthly cumulative forecasts as the fundamental elements in ESP generation, rather than using a linear regression approach. Therefore, in the following paragraphs we present how ANN and K-means clustering techniques are employed to generate and select synthetic ESPs. Afterwards, we discuss the way that the selected ESPs are combined with SSDP for real-time reservoir operation.

First, we carried out a correlation analysis to select the most appropriate input variables for the ANN models. Based on available data, the maximum, mean, and minimum temperature ( $T_{\max}$ ,  $T_{\text{mean}}$ ,  $T_{\min}$ ), precipitation ( $R$ ), and past inflows ( $Q$ ) were selected as the most relevant input variables (predictors). Then, the next 1-month to 12-month cumulative inflows were forecast by using different ANNs. The models were of three-layer feed-forward network type with a sigmoidal activation function trained by the Levenberg–Marquardt (LM) algorithm. Seventy percent and 30% of data were used for training and testing purposes, respectively. The number of neurons in the hidden layer was determined by trial and error. Moreover, input and output data were rescaled in [0.1 – 0.9] range (American Society of Civil Engineers (ASCE) 2000; Wang et al. 2006; Besaw et al. 2010). Finally, the performance of ANN models was compared based on the indices of determination coefficient ( $R^2$ ), root mean squared error (RMSE), and mean absolute error (MAE).

After generating cumulative inflow forecasts by ANNs, one can estimate the conditional probability of inflow forecast given the cumulative future inflow,  $P[QF_t|Y_{t+1}]$ .  $QF_t$  is the cumulative inflow forecast made at period  $t$  and  $Y_{t+1}(j)$  represents the historical cumulative inflow from period  $t + 1$  to  $T$  (Faber 2000). Then, the probability of  $Y_{t+1}(j)$  corresponding to scenario  $j$  given the inflow forecast  $QF_t$  can be estimated by using Bayes' theorem as follows:

$$P[Y_{t+1}(j)|QF_t] = \frac{P[QF_t|Y_{t+1}(j)] \times P[j]}{\sum_{i=1}^m ([QF_t|Y_{t+1}(i)] \times P[i])} \quad (14)$$

where  $P[j]$  and  $P[i]$  are the probabilities of occurring streamflow scenarios  $j$  and  $i$ , respectively. The likelihood function  $P[QF_t|Y_{t+1}]$  can be approximated by regressing  $QF_t$  on  $Y_{t+1}$  (Faber 2000).

Due to a large number of ESP scenarios and the resultant computational burden of SSDP/ESP, we employed the K-means clustering technique to select a limited, representative number of ESPs. The K-means algorithm was introduced by Cox (1957) and Fisher (1958) as a hard partitioning algorithm. It is an iterative process whereby the data are initially partitioned randomly, and iteratively re-assigned to a cluster based on the nearest distance to the cluster's center. The procedure terminates when it becomes impossible to re-assign any data from one cluster to another (Weatherill & Burton 2008). The K-means algorithm used in this study employs the Euclidean distance between the objects (Van der Heijden et al. 2004) as a dissimilarity measure. For more details on the K-means algorithm, one can refer to Goktepe et al. (2005).

Finally, to make real-time decisions for reservoir operation, we employed the SSDP model in an adaptive scheme being continuously re-applied with updated hydrologic information, i.e. ESP forecasts. In other words, the ESP forecasts are made in each month and the relevant optimal operation policies are continuously derived by the SSDP model.

### Performance indices

To evaluate how well optimal policies perform in real-world situations, one should simulate the policies and estimate some performance indices such as temporal or volumetric reliability of meeting demands, mean annual shortage, and mean value of simulated objective function (MVSOF). Reliability is defined as the percentage of time that the system operates without failure (Hashimoto et al. 1982) that may be calculated as follows:

$$\alpha_t = \frac{\sum_{t=1}^{T \times N} Z_t}{T \times N}, \quad Z_t = \begin{cases} 1, & \text{if } R_t \geq D_t, \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

where  $t$  is the index for month ( $t = 1, \dots, T$ );  $N$  is the number of years;  $D_t$  is downstream demands;  $R_t$  is release from the reservoir;  $Z_t$  is an indicator equal to one when the demands are satisfied and zero otherwise.  $\alpha_t$  is the temporal reliability index which determines the relative number of time periods when the demands are met.

Volumetric reliability ( $\alpha_Q$ ) is expressed as the ratio between the total volume released and the total demand volume

$$\alpha_Q = \frac{\sum_{t=1}^{T \times N} \min(R_t, D_t)}{N \times \sum_{t=1}^T D_t} \quad (16)$$

The index of mean annual shortage is calculated based on the difference between the water or energy demand and the amount of water or energy supplied in each period. The monthly differences are added up and then divided by the number of years.

Finally, MVSOF is an important index that measures the suitability of the policies in meeting the predefined operational objectives. It is determined based on the evaluation of cost<sub>*t*</sub> in the simulation phase.

## RESULTS AND DISCUSSION

### Comparison analysis

To evaluate the performance of optimal policies derived from an optimization model in real-world situations, one should simulate the policies and determine the performance indices for comparing different policies. These indices can be estimated reliably if we are in a data-rich situation. On the other hand, historical streamflow data may not include enough extreme values, whereas extreme flows could significantly affect the resultant system performance. Therefore,

we have synthetically generated a time series of streamflow whose size (length) is much larger than that of historical time series. This could help us better understand the differences between various DP-based models. Nevertheless, the data generated by a statistical model that fits best the historical data are not perfect in terms of modeling the correlation structure of historical streamflows. Consequently, we evaluate the system performance against both historical and synthetic data sets.

The first set is a 41-year historical streamflow time series from 1971–2011 named 41/Hist and the second one is a 100-year synthetically-generated streamflow using a first-order auto-regressive (AR) model named 100/AR(1). Before using AR models, the seasonality and trend non-stationary components were removed. In this regard, the Mann–Kendall test (Yue & Wang 2004) was used to determine if the trend component was significant. It was found that inflows of Oct., Dec., Jan., Aug. and Sep. had a significant trend at a 95% confidence level. Figure 3 shows the original and detrend flows for a sample month (Oct.).

Then, the lag numbers of detrend and deseasonalized monthly flows were determined based on the autocorrelation function (ACF) and partial ACF (PACF) and the appropriate lag number for each month was identified at a 95% confidence interval. For example, the ACF and PACF were significant for lag numbers up to 1 for October (Figure 4).

Afterwards, a series of 100-year streamflows was generated and the removed seasonal and trend components were added back to the generated data. Eventually, checks were made to see if the statistical characteristics of the generated data match with those of the historical data.

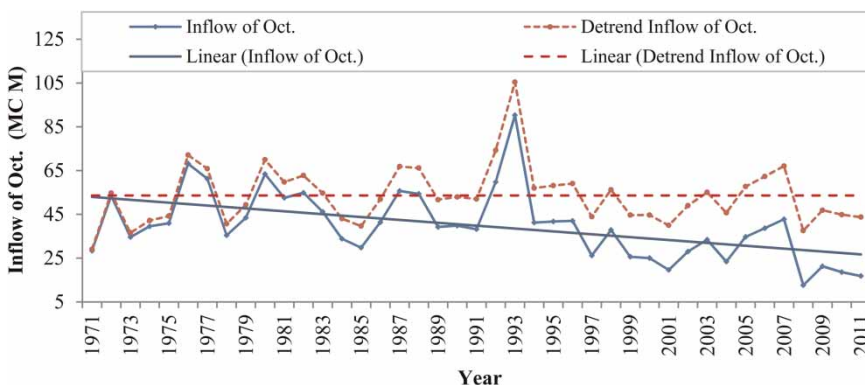


Figure 3 | Observed and detrend streamflows for October.



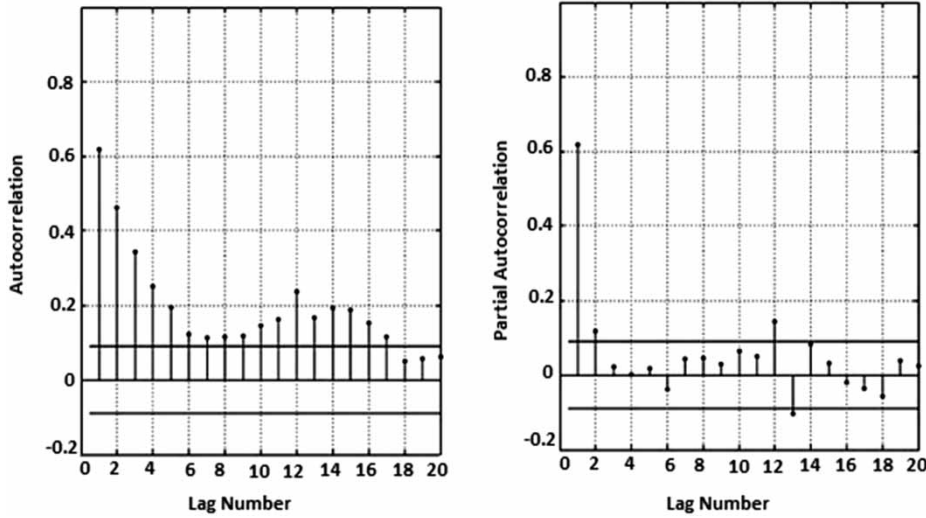


Figure 4 | ACF and PACF for October streamflows.

Having determined the historical and synthetically-generated inflows, the policies derived from each of the optimization models were simulated using interpolation and re-optimization techniques (Tejada-Guibert et al. 1993). This is because the state variables, either storage or inflow, can accept any values not necessarily equal to the discrete values. Doing interpolation within the optimal policy table is one of the techniques to decide what to release in each time step (stage) of the simulation model. Tejada-Guibert et al. (1993) introduced the idea of re-optimization at each stage with the actual system storage, rather than interpolating within the policy table. With re-optimization, a one-stage optimization model is solved at each simulation stage to determine the best release value of that stage depending on the system’s state. In each of the one-period optimization models, the optimal FVF is the

steady-state solution determined by the DP and SDP/SSDP models. Figures 5 and 6 show the results obtained for different models and their corresponding MVSOF and mean annual shortage indices.

Considering the MVSOF index, Figure 5 proves the SSDP/Hist approach as the best model. The difference between the worst and the best model, i.e. DP and SSDP/Hist, is also shown in Figure 5 (the fifth bar in the figure).

Figure 6 compares the suitability of the models based on the index of mean annual shortage (MCM). As expected, all of the stochastic optimization models are better than the DP model. However, the SSDP/Hist model is the best among the stochastic models.

Figures 7–9 compare the models based on the indices of temporal reliability of meeting DIE, agricultural, and hydro-power demands, respectively.

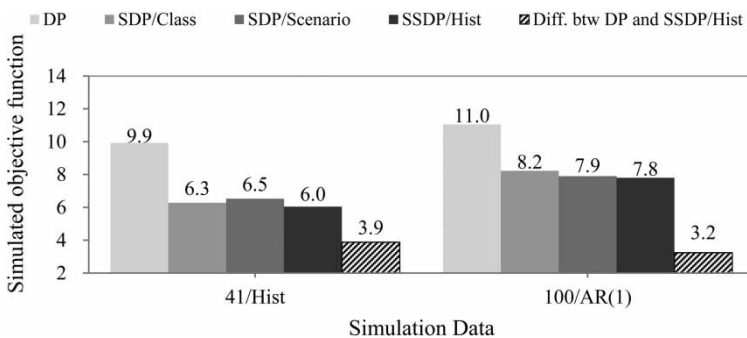


Figure 5 | Comparison of the optimization models based on the MVSOF index.

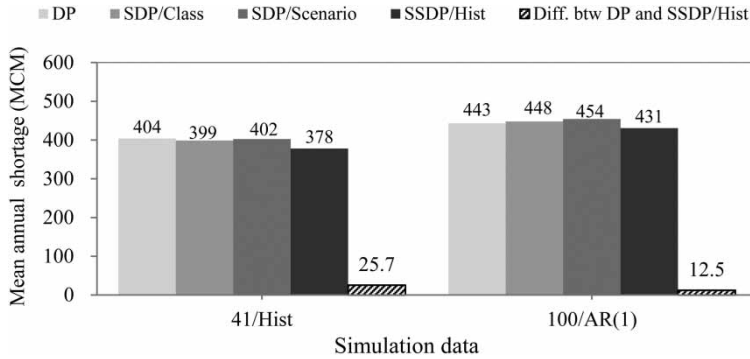


Figure 6 | Comparison of the optimization models based on the index of mean annual shortage.

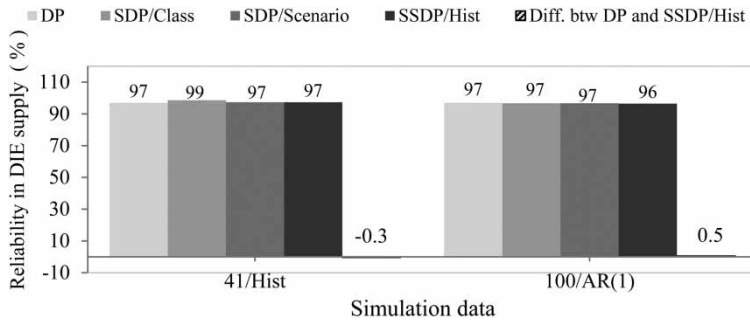


Figure 7 | Comparison of the optimization models based on the index of temporal reliability of meeting DIE demand.

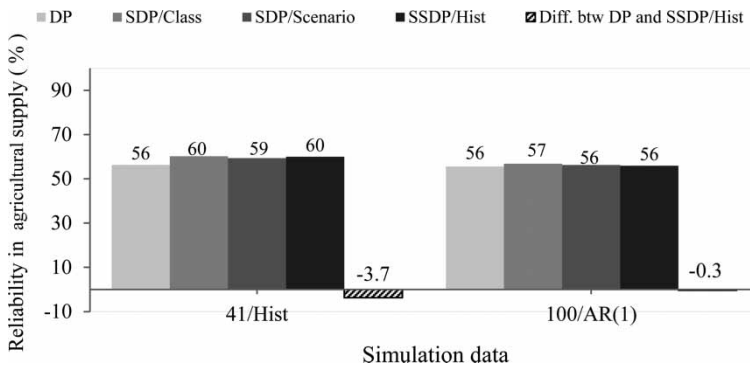


Figure 8 | Comparison of the optimization models based on the index of temporal reliability of meeting agricultural demand.

The results illustrate that all of the optimization models have performed almost the same when compared against the indices of temporal reliabilities of meeting DIE and agricultural demands. However, DP has performed a bit better based on the index of reliability of meeting hydro-power demand. This could be due to the allocation of less water to other demands that leads to more water storage and consequently a higher long-term average head

on the turbines. One can verify this statement by comparing the average long-term reservoir storage levels (elevations) of different models obtained as 430 (2027), 351 (2023), and 345 (2022) for the DP, SDP/Class, and SSDP/Hist models, respectively. The values within the parentheses are average elevations in meters. Figure 10 also shows the performance of the models based on the volumetric reliability index.

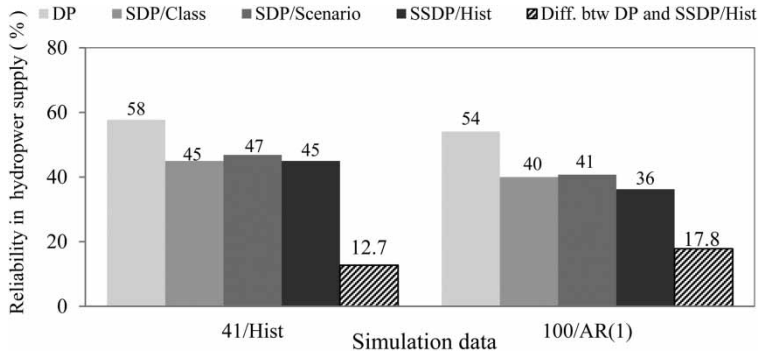


Figure 9 | Comparison of the optimization models based on the index of temporal reliability of meeting hydropower demand.

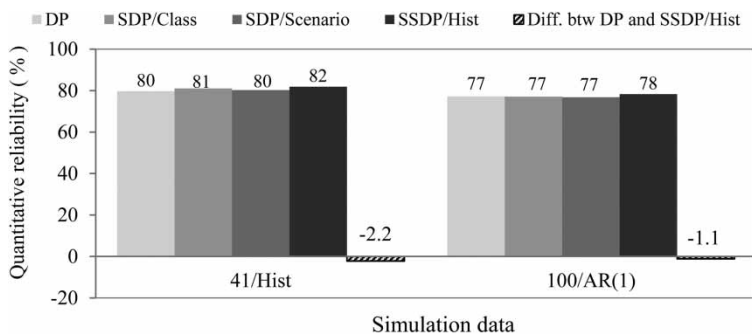


Figure 10 | Comparison of the optimization models based on the index of volumetric reliability.

The effect of using re-optimization, rather than interpolation, was also investigated on the simulation results as presented in Figures 11 and 12.

One can see from the figures that the use of re-optimization, compared to interpolation, has not led to a better performance. This is the main reason for only using the

interpolation technique in the rest of the paper. Note that while doing re-optimization, the FVF still needs to be interpolated. It is worth mentioning that the outcome and efficiency of the re-optimization and interpolation techniques depend on the complex interaction of some factors including number of discrete values of state variables, type

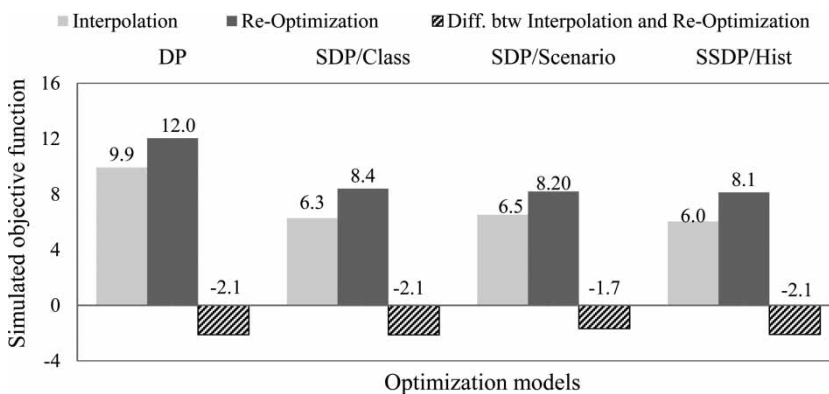


Figure 11 | Simulation results with and without using re-optimization based on the index of MVSOE.

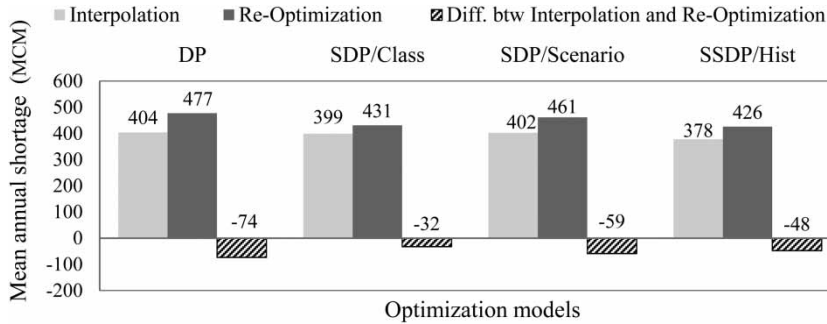


Figure 12 | Simulation results with and without using re-optimization based on the index of mean annual shortage.

of discretization scheme, type of interpolation scheme (linear or nonlinear), degree of nonlinearity of objective function, and the form of recursive equation (discounted or undiscounted).

### Impact of ANN forecasts on the SSDP/Hist model

In this section, we investigate the effect of cumulative forecasts on the performance of SSDP/Hist. In this regard, the historical inflows of scenario *j* were replaced by the cumulative inflow forecasts obtained by an ANN model. In other words, the probability function of  $P[QF_t|Y_{t+1}]$  was specified and used in the SSDP/Hist formulation where  $Y_{t+1}(j)$  represents the historical cumulative inflow from period  $t + 1$

to  $T$  in scenario *j*, and  $QF_t$  is a cumulative inflow forecast made at period  $t$ . The policies derived from this model, named SSDP/ANN hereafter, were simulated using the 41/Hist data set and interpolation technique. Table 3 summarizes the details about inflow forecasts calculated by ANN models including the input variables (predictors), the models architecture, and evaluation indices. The indices are  $R^2$ , RMSE and MAE, all in MCM. The architectures of the ANN models were finalized using trial and error.

Table 3 illustrates the performance of ANNs in forecasting cumulative flows for the next months. According to the results, the evaluation indices have improved from the first month of forecast, October, to the last month, September. This is due to the long lead-time (12 months) for the forecast

Table 3 | ANN model characteristics and performance in forecasting monthly cumulative flows for different months

Month	Input variables (predictors)	ANN architecture	Training phase			Test phase		
			RMSE	$R^2$	MAE	RMSE	$R^2$	MAE
Oct.	Tmean_Sep, Tmax_Jun, Tmax_Sep	3-2-1	260	0.6	207	310	0.59	238
Nov.	Tmean_Oct, Tmin_Oct	2-2-1	279	0.58	230	275	0.6	213
Dec.	Tmin_Oct, Tmin_Nov, Q_Nov	3-2-1	266	0.6	141	275	0.53	226
Jan.	Tmin_Oct, Tmin_Nov, Tmax_Dec	3-2-1	162	0.84	116	212	0.82	146
Feb.	Tmin_Oct, Tmin_Nov, Tmax_Jan	3-2-1	209	0.77	137	177	0.84	169
Mar.	Tmin_Oct, Tmin_Nov, Tmax_Jan	3-2-1	204	0.76	136	176	0.75	169
Apr.	Tmin_Oct, Tmax_Jan, Q_Mar	3-2-1	178	0.77	123	126	0.79	109
May.	Tmean_Apr, Tmax_Apr, Q_Apr	3-2-1	110	0.84	94	119	0.79	97
Jun.	Tmax_May, Q_Apr, Q_May	3-2-1	52	0.912	39	75	0.75	63
Jul.	Q_Apr, Q_May, Q_Jun	3-2-1	13	0.98	10	32	0.86	29
Aug.	Q_May, Q_Jun, Q_Jul	3-2-1	13	0.95	11	11	0.95	9
Sep.	Q_Jun, Q_Jul, Q_Aug	3-2-1	2	0.98	2	7	0.93	5

**Table 4** | Comparison of the SSDP/Hist and SSDP/ANN models based on the performance indices

Performance index	Optimization model		Degree of improvement (%)
	SSDP/Hist	SSDP/ANN	
Reliability of meeting DIE demand (%)	97	99	1.28
Reliability of meeting agricultural demand (%)	60	62	3.47
Reliability of meeting hydropower demand (%)	45	49	7.87
Volumetric reliability (%)	82	81	-0.99
Mean annual shortage (MCM)	378	368	-2.55
MVSOF	6.0	5.6	-7.46

model of the first month that results in less accurate forecasts. Table 4 represents the simulation results of the SSDP/ANN compare to SSDP/Hist. Based on the results, one can assess the improvement of the SSDP/Hist model as a result of using ANN cumulative forecasts.

The results demonstrate that the ANN forecasts have improved the temporal reliability indices associated with DIE, agricultural, and hydropower demands for about 1.28, 3.47, and 7.87%, respectively. However, the volumetric reliability index has decreased slightly (1%). Moreover, SSDP/ANN has performed better than SSDP/Hist with lower mean annual shortage (2.6%) and MVSOF (7.5%).

### ESP forecasts and the SSDP/ESP model

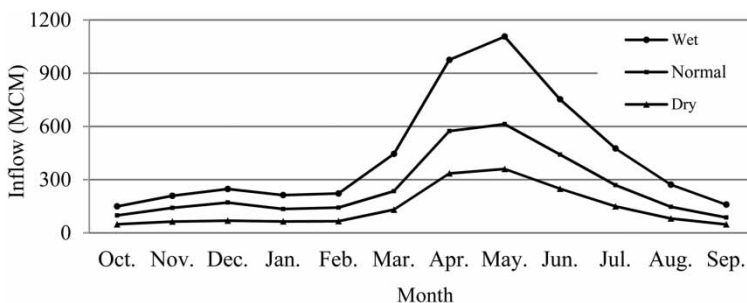
In this section, the monthly cumulative forecasts determined by ANN are used to produce the ESP scenarios

synthetically. Then, the generated ESPs are updated at the beginning of each month and the SSDP policies are applied only for a single time period. Subsequently, the ESP forecasts are updated and the procedure moves forward over time continuously. Due to the large number of ESPs (225), the SSDP/ESP model is computationally intensive. Therefore, we used the K-means clustering algorithm to reduce the number of ESP traces. The generated ESPs were grouped into three clusters representing dry, normal, and wet conditions. Figure 13 shows the average monthly inflows for the first class of ESPs.

Afterwards, eight patterns, each including inflows from October to September, were selected from each cluster. Finally, the selected ESPs (24 traces) were input to the SSDP/ESP model. Table 5 compares the performance indices resulting from the SSDP/ESP model with those obtained from the SSDP/Hist and SSDP/ANN models.

One can see in Table 5 that the real-time SSDP/ESP model outperforms the SSDP/Hist and SSDP/ANN models since it satisfies DIE, agricultural, and hydropower demands more reliably. Moreover, the indices of mean annual shortage and MVSOF have improved about 3.5 and 8%, respectively, compared to the SSDP/Hist model. These results also match the previous findings (Faber & Stedinger 2001; Kim et al. 2007). The better performance of the real-time SSDP/ESP model is due to the fact that ESP traces provide a better description of the conditional distribution of future inflows than that represented by historical streamflows.

Finally, it is notable that all computer codes developed for the optimization models benefitted from the two

**Figure 13** | Average monthly inflows for the first class of ESPs.

**Table 5** | Results of the SSDP/ESP model compared to those of the SSDP/Hist and SSDP/ANN models

Performance indices	Optimization models			Degree of improvement	
	SSDP/Hist	SSDP/ANN	SSDP/ESP	Diff. between SSDP/ESP and SSDP/Hist(%)	Diff. between SSDP/ESP and SSDP/ANN(%)
Reliability of meeting DIE demand (%)	97	99	99	1.76	0.47
Reliability of meeting agricultural demand (%)	60	62	63	5.00	1.48
Reliability of meeting hydropower demand (%)	45	49	52	15.56	7.12
Volumetric reliability (%)	82	81	83	1.37	2.39
Mean annual shortage (MCM)	378	368	365	- 3.46	- 0.94
MVSOF	6.0	5.6	5.6	- 7.96	- 0.54

techniques of vectorization and preallocation. This reduced the amount of computations substantially.

## SUMMARY AND CONCLUSIONS

In this paper, we investigated the performance of dynamic programming (DP), stochastic DP (SDP), and sampling SDP (SSDP) in combination with ESPs for optimal operation of a multi-purpose water reservoir system. Four optimization models including DP, SDP with inflow class intervals (SDP/Class), SDP with inflow scenarios (SDP/scenario), and SSDP with historical scenarios (SSDP/Hist) were developed for monthly operation of the Zayandeh-Rud multi-purpose reservoir system. The models were compared by simulating their optimal policies using interpolation and re-optimization techniques. While the SDP approaches performed better than the DP approach, the SSDP/Hist model outperformed the other SDP models.

The impact of inflow forecasts was also investigated on the performance of the best model (SSDP/Hist) by extending SSDP/Hist to SSDP/ANN, in which cumulative inflow forecasts were generated by ANNs. Moreover, we improved the performance of SSDP for real-time reservoir operation by combining the SSDP approach with ESPs, generated by ANNs, and the K-means clustering technique. We concluded that the proposed SSDP model in combination with ESP forecasts could provide a promising framework for real-time reservoir operation.

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