Relativistic Calculation of Two-Body Correlations in Finite Nuclei

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Two-body correlations in a nucleus $^{16}\text{O}$ are investigated in the framework of relativistic formulation. Equations are solved by using the wave packet expansion method. For nucleon-nucleon interactions, six kinds of mesons are considered. The calculated values are compared with the experimental values. The contribution of ladder processes cancels almost the exchange energies in the Hartree-Fock. The simultaneous contributions of the exchange and ladder processes are not even to the single-particle energies and the total energy: owing to their contributions the single-particle energies get smaller while the total energy larger than the predictions of the Hartree, improving the fits to the data.

§ 1. Introduction

One of the most important tasks of nuclear physics is to understand systematically not only the static but also the dynamical properties of nuclei. To attain this purpose one needs to pursue two fields of study, that is, to investigate the nucleon-nucleon interactions and to solve many-body problems. The former field has been got clearer recently: nucleon-nucleon scattering up to 1 GeV and deuteron structure are well described by the meson exchange model (OBEP). This model adopts six kinds of mesons with masses lighter than 1 GeV. Though many other models of nuclear forces which reproduce the experimental data on a similar level are proposed, the meson exchange model has some advantages, i.e., it is based on the origin of the nuclear attractive interactions. Since the meson exchange description reproduces two-nucleon data, it is reasonable to assume that the relativistic quantum field theory is the most suitable tool to treat the nuclear many-body problems. In fact, mesons satisfy the relativistic field equation, that is, the Klein-Gordon equation while the motions of nucleons are described by the Dirac equation.

From the following two viewpoints nuclear matter is usually chosen as a first test ground of nuclear many-body models before one investigates finite nuclei. First, it is easier to handle especially because of the translational invariance. Secondly, the results of the nuclear matter study can be applied to estimate the quantities for finite nuclei by using a procedure like the local density approximation. Since Walecka showed that the relativistic mean-field approximation succeeds in reproducing the saturation properties of nuclear matter in the $\sigma-\omega$ model, much attention has been paid to his model. Recently Brockmann et al. formulated the Dirac-Brueckner approach to nuclear matter and reproduced the nuclear matter saturation properties using two-body interactions which reproduce the nucleon-nucleon scattering data and deuteron properties.

The application of the relativistic quantum field theory to finite nuclei has been performed by many authors either in the Hartree or in the Hartree-Fock approximation (see, for instance, the review article 4)). Horowitz and Serot showed that the
relativistic Hartree model can reproduce some data of the ground states of spherical nuclei. But, in their model the parameters are so adjusted as to reproduce the bulk properties of the nuclei while fits to the nucleon-nucleon scattering data are lost. Furthermore one question arises: why does their model work well without taking into account Brueckner correlations? In fact, two-body correlations play an important role in the non-relativistic nuclear physics. The inclusion of the Brueckner correlations may damage the success of the relativistic Hartree model. To answer this question we must take into account the two-body correlations in the relativistic framework for finite nuclei.

Although the Brueckner correlations are important in nuclear physics, there has been no literature that solved exactly the relativistic Brueckner problems for finite nuclei. It is because the calculation to solve the differential Brueckner equations is exceedingly difficult. Therefore some approximations were usually introduced. For example Mütther et al. recently solved the relativistic Brueckner Hartree-Fock equations by using the effective density approximation. In order to solve the equations correctly we expand the solutions in terms of the wave packet basis functions.

Recently the non-relativistic effective interaction theory succeeded to reproduce the experimental data of $^{16}$O starting with the nonrelativistic Bonn potential. They use sophisticated many-body techniques including the Brueckner correlations. We intend not to study the magnitude of the relativistic effects, but to study how the relativistic Hartree result is modified by including higher order effects. We take into account only positive energy nucleons to describe a nuclear system.

§ 2. Formulation

2.1. One-particle Green's functions

In this section, we consider a nucleus to be a composite system of nucleons and mesons: our model contains not only nucleons but also for the moment two kinds of isoscalar mesons, $\sigma$ and $\omega$, the $\sigma$-$\omega$ model. Other kinds of mesons are taken in later sections.

The lagrangian density is expressed as follows:

\[
L = L_N + L_\sigma + L_\omega + L_{N\sigma} + L_{N\omega},
\]

\[
L_N = -\bar{\phi}(\gamma_\mu \partial_\mu + M)\phi,
\]

\[
L_\sigma = -\frac{1}{2}(\partial_\mu \phi \partial_\mu \phi + m_\sigma^2 \phi^2),
\]

\[
L_\omega = -\frac{1}{4} F_{\mu \nu} F_{\mu \nu} - \frac{1}{2} m_\omega^2 V_\mu V_\mu,
\]

\[
L_{N\sigma} = g_\sigma \bar{\phi}\phi \phi,
\]

\[
L_{N\omega} = ig_\omega \bar{\phi}\gamma_\mu \phi V_\mu
\]

with
Relativistic Calculation of Two-Body Correlations in Finite Nuclei

\[ F_{\mu\nu} = \frac{\partial V_{\nu}}{\partial x_{\mu}} - \frac{\partial V_{\mu}}{\partial x_{\nu}}, \] \hspace{1cm} (2.7)

where \( M, m_\sigma \) and \( m_\omega \) denote the masses of nucleon, \( \sigma \)- and \( \omega \)-mesons. The fields \( \phi, \phi \) and \( V_\mu \) are of nucleon, \( \sigma \)- and \( \omega \)-mesons. Our convention is the same as that of Chin.\(^{11}\)

From this lagrangian the total energy of the nuclear system is given as

\[ E = E_N + E_\sigma + E_\omega, \] \hspace{1cm} (2.8)

\[ E_N = \int d^3 x \lim_{y \to \pm \infty} \text{Tr} \left[ \gamma_4 \frac{\partial}{\partial x_0} G(x, y) \right], \] \hspace{1cm} (2.9)

\[ E_\sigma = -\frac{i}{2} \int d^3 x \lim_{y \to \pm \infty} \left[ \nabla_x^2 + \left( \frac{\partial}{\partial x_0} \right)^2 - m_\sigma^2 \right] \Delta(x, y), \] \hspace{1cm} (2.10)

\[ E_\omega = -\frac{i}{2} \int d^3 x \lim_{y \to \pm \infty} \left[ \nabla_x^2 + \left( \frac{\partial}{\partial x_0} \right)^2 - m_\omega^2 \right] D_{\omega}(x, y), \] \hspace{1cm} (2.11)

where \( G, \Delta \) and \( D_{\omega} \) are the full one-particle Green's functions of nucleon, \( \sigma \)- and \( \omega \)-mesons. When one takes into account negative energy nucleons (Dirac sea), the contributions to the total energy of these one-particle Green's functions become infinite. To make them finite, it is necessary to renormalize them. For this purpose, some counter terms must be introduced. But the calculation of the vacuum polarizations of a finite nucleus is not yet feasible.\(^{15}\) We, avoiding this problem in this paper, take into account only positive energy nucleons.

The one-particle Green's function for nucleons satisfies the Dyson equation,

\[ G(x, y) = G^0(x, y) + \int d^4 y_1 d^4 y_2 G^0(x, y_1) \Sigma(y_1, y_2) G(y_2, y), \] \hspace{1cm} (2.12)

with the proper self-energy \( \Sigma \) and the free one-particle Green's functions \( G^0 \). A diagramatic demonstration of this equation is given in Fig. 1. The solution \( G \) of this integral equation may be expressed by the following bilinear form of single particle wave functions \( U_n(x) \),\(^6\)

\[ iG(x, y) = \sum_n U_n(x) \bar{U}_n(y) \exp[-i\epsilon_n(x_0 - y_0)] \times \left[ \theta(x_0 - y_0) \theta(\epsilon_n - \epsilon_F) - \theta(y_0 - x_0) \theta(\epsilon_F - \epsilon_n) \right], \] \hspace{1cm} (2.13)

where \( \epsilon_n \) and \( \epsilon_F \) are the single particle energies and the Fermi energy. Equation (2.12) is rewritten as a Dirac equation by using the expression (2.13);

\[ \left[ -i \gamma \cdot \nabla + \beta M \right] U_n(x) + \int d^3 x_1 [\beta \Sigma(x, x_1, \epsilon_n)] U_n(x_1) = \epsilon_n U_n(x), \] \hspace{1cm} (2.14)

with

Fig. 1. The diagramatic expression of the Dyson equation for the full Green's function of nucleons.
The full meson propagators $\Delta$ and $D_{s\lambda}$ in (2·10) and (2·11) are defined so as to cancel the halves of the meson exchange interaction energies in the sum of the nucleon single particle energies. 13,14

2.2. Two-body ladder correlations

In the previous subsection we briefly reviewed the Green's function approach. We now wish to obtain the expression of the self energy $\Sigma$ extending the Hartree and Hartree-Fock formulations to include in it the contribution of ladder processes (see Figs. 2 and 3). We introduce the scattering matrix $\Gamma$ which satisfies the Bethe-Salpeter (BS) equation. In free space, $\Gamma$ reduces to the $N-N$ free scattering amplitude. In the nuclear medium this is modified by the effects of the nuclear average potentials and the Pauli exclusion.

The self energy is given by the scattering matrix $\Gamma$ as

$$\Sigma(x, y, k_0) = \int d(x_0 - t_1) \exp[ik_0(x_0 - t_1)]\Sigma(x, x_1). \quad (2·15)$$

where the states $n$ denote hole states. The scattering matrix $\Gamma$ is determined by the BS equation for a pair of nucleons in the finite nucleus,

$$\Gamma(xz; yw|k)_{\sigma\gamma; \rho\tau} = V(xz; yw)_{\sigma\gamma; \rho\tau} - \int d^3x_1\cdots d^3x_4 V(xz; x_3x_4)_{\sigma\gamma; \rho\tau} \times \frac{\sum_{a_1\geq a_2} U_{a_1}(x_3)\sigma U_{a_2}(x_4)\rho \bar{U}_{a_1}(x_3)\rho \bar{U}_{a_2}(x_4)\sigma}{k + \epsilon_{a_1} + \epsilon_{a_2} - i\eta} \times \Gamma(x_1x_2; yw|k)_{\rho\tau; \gamma}, \quad (2·17)$$

where Greek subscripts $a$ denote single particle quantum numbers including Dirac spinor and isospin component indices. For intermediate states we neglect the hole-hole propagations as is often done in this sort of calculations. 19 We use the lowest order of approximation to the kernel $V$ of the BS equation, i.e., the sum of one-meson propagators. The sum of one-meson propagators in the $\sigma$-$\omega$ model is

$$V(xz; yw)_{\sigma\gamma; \rho\tau} = \delta(x - y)\delta(z - w)[g_\sigma^2(I)_{ab}(I)_{\sigma\gamma}d^0(x - z; m_\sigma)$$

$$- g_\omega^2(\gamma_\mu)_{ab}(\gamma_\nu)_{\sigma\gamma}d^0(x - z; m_\omega)] \quad (2·18)$$

with
\( \Delta^0(x-z; m) = \int \frac{d^3k}{(2\pi)^3} \frac{-\exp[ik(x-z)]}{k^2 + m^2} \),

(2.19)

where we neglect the retardation effect for simplicity. This does not affect the Hartree energies at all but very slightly the Fock energies.16

Equations (2.14) and (2.16) are solved self-consistently as follows. When determine \( G \) (or \( \epsilon_n \) and \( U_n \)), we substitute the self energy \( \Sigma \) into Eq. (2.14) and when determine \( \Sigma \), we substitute single nucleon wave functions \( U_n \) into Eq. (2.16). These coupled equations have been solved recently for nuclear matter. There, because of the translational invariance, the nuclear wave functions are plane waves modified only by the effective nucleon mass.13 This simplicity makes it feasible to solve the Dirac equations. For finite nuclei, however, this simplicity is not met. So a new method to solve these equations must be required. For this purpose, we use the wave packet expansion method which will be explained in the following subsection.

2.3. Self-consistent Dirac equation and the wave packet expansion method

In order to solve the relativistic equations numerically, we use the wave packet basis function method.8,9 The nucleon single particle wave functions are expanded as

\[
U_{n\ell m}(x) = \sum_{n_{\ell m}=1}^{\text{max}} \left[ c_{n_{\ell m}+}^{(\dagger)} u_{n_{\ell m}+}(x) + c_{n_{\ell m}^+}^{(\dagger)} u_{n_{\ell m}^+}(x) + c_{n_{\ell m}^-}^{(\dagger)} u_{n_{\ell m}^-}(x) \right]
\]

(2.20)
in terms of the wave packet basis functions

\[
u^{(\dagger)}_{n\ell m}(x) = \sum_{m_{\ell n}(\ell)} (1/2) \delta_{l m} \int d^3p F_{n\ell m}(p) \psi_{\ell n+}(p) \exp[ip \cdot x],
\]

(2.21)
which are made of the free Dirac spinor functions \( \psi_{\ell n}(p) \). The harmonic oscillator functions \( F_{n\ell m} \) in the momentum representation are used for the form factors of the wave packets. The set of wave packet basis functions consists of positive energy wave packets \( \nu^{(+)}_n \) and negative energy ones \( \nu^{(-)}_n \). It is verified easily that these wave packets are eigen functions of total angular momentum \( \vec{l} \), its third component \( l_z \) and the parity.

On the basis of the wave packets the Dirac differential equation (2.14) is reduced to the matrix equation,

\[
\sum_{n_{\ell m}} (T_{n_{\ell m}} + \Gamma_{n_{\ell m}}(c)) c_{n_{\ell m}} = \epsilon_n c_{n_{\ell m}}
\]

(2.22)
with the matrices

\[
T_{n_{\ell m}} \equiv \int d^3x \nu_{n_{\ell m}^+}(x)[-i\sigma \cdot \nabla + \beta M] \nu_{n_{\ell m}^+}(x),
\]

(2.23)
\[
\Gamma_{n_{\ell m}}(c) \equiv \int d^3x d^3y \nu_{n_{\ell m}^+}(x) [\nu_{n_{\ell m}^+}(y) - \epsilon_n - \epsilon_n] \sigma_{\ell m} \nu_{n_{\ell m}^+}(x)\nu_{n_{\ell m}^+}(y) + \Gamma(xz; xw) \nu_{n_{\ell m}^+}(x) \nu_{n_{\ell m}^+}(y) U_\ell \nu_{n_{\ell m}^+}(y)\nu_{n_{\ell m}^+}(z),
\]

(2.24)
where the one-body potential \( \Gamma_{n_{\ell m}} \) is a functional of the expansion coefficients \( c_n \) of the single particle wave functions not only through the Hartree-Fock energies \( \epsilon_n \), but
also through the renormalized two body interactions $\Gamma$.

We express the scattering matrix $\Gamma$ in terms of the wave packet basis functions

$$\Gamma(\alpha_1; \alpha_2 | k) = \int d^3x d^3y d^3w \bar{u}_{\alpha_1}(x) \bar{u}_{\beta_1}(y) \Gamma(\alpha_1; \alpha_2 | k) \Gamma(\beta_1; \beta_2 | k)$$

where the wave functions $u$ are wave packet basis functions and $\alpha_1, \beta_1, \alpha_2, \beta_2$ denote a set of the quantum numbers $n_l j m$. Equation (2·17) is transformed into the following matrix equation,

$$\Gamma(\alpha_1; \alpha_2 | k) = V(\alpha_1; \alpha_2) - \sum_{\beta_1, \beta_2} V(\alpha_1; \beta_1) \Gamma(\beta_1; \beta_2 | k) \Gamma(\alpha_2; \beta_2 | k)$$

with

$$V(\alpha_1; \alpha_2) = \int d^3x d^3y d^3w \bar{u}_{\alpha_1}(x) \bar{u}_{\beta_1}(y) \Gamma(\alpha_1; \alpha_2 | k) \Gamma(\beta_1; \beta_2 | k)$$

We decompose this equation into those expressed in the channels with a definite angular momentum $J$ for the pair of nucleons. The result is

$$\Gamma(\alpha_1; \alpha_2; J | k) = V(\alpha_1; \alpha_2; J | k)$$

with

$$V(\alpha_1; \alpha_2; J | k) = \int d^3x d^3y d^3w \bar{u}_{\alpha_1}(x) \bar{u}_{\beta_1}(y) \Gamma(\alpha_1; \alpha_2; J | k) \Gamma(\beta_1; \beta_2 | k)$$

where the labels $\alpha_1, \alpha_2, \beta_1, \beta_2$ in $V$ denote a set of quantum numbers except for the third component of the angular momentum $J$. We can solve the matrix equation by the matrix inversion.

Equations (2·17) and (2·22) are to be solved self-consistently as follows.

1) Use the wave packet basis functions $u_n$ and the Hartree energies for the starting single nucleon states $U_n$ and energies $\epsilon_n$ to solve Eq. (2·17) for $\Gamma$.

2) Substituting the obtained $\Gamma$ into the secular equation (2·22) to solve it by diagonalizing $T + \Gamma$, determine the eigen energies $\epsilon_n$ and eigen functions $U_n$.

3) Solve again Eq. (2·17) with the above obtained eigen energies $\epsilon_n$ and eigen functions $U_n$.

4) Solve again Eq. (2·22) substituting the above obtained $\Gamma$.

5) Repeat this process until input and output quantities get sufficiently close together.

§ 3. Numerical results

3.1. The $\sigma$- and $\omega$-meson model

In this section we report the numerical results obtained in the model which contains, for simplicity, only $\sigma$- and $\omega$-mesons. We adopt the parameters in Ref. 6), namely
with which the empirical saturation point of nuclear matter has been reproduced in the mean field calculation. The solutions of the coupled equations (2.17) and (2.22) depend on the maximum radial quantum number \( n_{\text{max}} \) for positive and negative energy free nucleon space at which the wave packet basis is truncated. Therefore we have checked the convergence in the Hartree approximation. For the range parameter of the harmonic oscillator functions \( F_{\text{nlm}} \) we adopt the value of 1.81 fm. It is verified that the final results are insensitive to the variation of this value, when we take the basis space with \( n_{\text{max}} \geq 5 \). In this calculation we do not take into account the Coulomb force, the correction of center of mass motion nor the finite size effect of nucleons (see Table I).

The convergence of the results within the Hartree approximation is attained at \( n_{\text{max}} = 6 \) indicating that our basis function space is sufficient for the self-consistent calculation. In Table II the values of the expansion coefficients are shown for positive energy components \( c_{\uparrow} \) and negative ones \( c_{\downarrow} \). The mixing of the negative energy sector is only a few percent but the effect on the physical observables is so large that the negative energy components are necessary. It is known that the similar situation is also seen in the relativistic treatment of hydrogen atoms where the wave function of the bound electron expanded in terms of plane waves contains some negative energy components.

The inclusion of the Fock terms complicates the calculation due to the non-locality of the one body potential, for which the wave packet method is very suitable. The results of the wave packet method in the Hartree-Fock and Brueckner calculations are seen in Table III, where the nucleon exchange contribution makes the \( ^{16}_0 \) nucleus less bound and where the reduction of the one body potential by the repulsive Fock term also reduces the \( l-s \) splitting roughly by 40 percent. The contributions of \( \sigma \)- and \( \omega \)-mesons to the \( l-s \) potential is expressed as

\[
V_{l-s} = -\frac{d}{dr}(U_\sigma + U_\omega) S \cdot L, \tag{3.1}
\]

where \( U_\sigma(r) \) and \( U_\omega(r) \) represent the one body central potentials due to the corresponding mesons. The Fock terms reduce both \( U_\sigma \) and \( U_\omega \).

### Table I. The Hartree results \( (n_{\text{max}}=7) \) of single particle energies, total energy in MeV and the root mean-square radius in fm of \( ^{16}_0 \).

| \( S_{1/2} \) | \(-41.8\) |
| \( P_{3/2} \) | \(-20.8\) |
| \( P_{1/2} \) | \(-12.6\) |
| \( E \) | \(-95.4\) |
| \( \sqrt{\langle r^2 \rangle} \) | \(2.59\) |

### Table II. The expansion coefficients \( c_{\uparrow} \) and \( c_{\downarrow} \) of the single particle wave function \( 1S_{1/2} \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{\uparrow} )</td>
<td>(-0.992)</td>
<td>0.117</td>
<td>0.00121</td>
<td>(-0.0264)</td>
</tr>
<tr>
<td>( c_{\downarrow} )</td>
<td>(-0.0298)</td>
<td>0.0195</td>
<td>(-0.00992)</td>
<td>0.00407</td>
</tr>
</tbody>
</table>
Table III. The results of the Hartree, Hartree-Fock, 2nd order ladder and Brueckner calculations ($n_{max}=3$) using $\sigma$ and $\omega$ mesons. $S_{1/2}$, $P_{3/2}$, $P_{1/2}$: single particle energies, $\langle r^2 \rangle$: root mean-square radius, $E$: total energy, $E_{dir}$: direct energy, $E_{exc}$: exchange energy and L-S: $l$-$s$ splitting are shown in MeV and fm.

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>HF</th>
<th>4th Order</th>
<th>Full Ladder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1/2}$</td>
<td>-42.2</td>
<td>-21.2</td>
<td>-25.3</td>
<td>-24.9</td>
</tr>
<tr>
<td>$P_{3/2}$</td>
<td>-20.7</td>
<td>-8.74</td>
<td>-12.1</td>
<td>-12.2</td>
</tr>
<tr>
<td>$P_{1/2}$</td>
<td>-11.5</td>
<td>-3.53</td>
<td>-6.49</td>
<td>-6.79</td>
</tr>
<tr>
<td>$\langle r^2 \rangle$</td>
<td>2.58</td>
<td>2.99</td>
<td>2.88</td>
<td>2.90</td>
</tr>
<tr>
<td>$E$</td>
<td>-95.7</td>
<td>-3.37</td>
<td>-25.1</td>
<td>-26.7</td>
</tr>
<tr>
<td>$E_{dir}$</td>
<td>-285</td>
<td>-238</td>
<td>-284</td>
<td>-284</td>
</tr>
<tr>
<td>$E_{exc}$</td>
<td>0</td>
<td>72.0</td>
<td>85.3</td>
<td>85.8</td>
</tr>
<tr>
<td>L-S</td>
<td>9.2</td>
<td>5.2</td>
<td>5.6</td>
<td>5.4</td>
</tr>
</tbody>
</table>

We should give a brief account how we treat the particle states in the calculation of the two-body ladder correlations. We include the single particle angular momenta up to $l=3$ with radial quantum numbers up to $n_{max}=3$. The obtained numerical results of the $\sigma$-$\omega$ model show the general property of the contributions from Fock and ladder processes, although this model without taking into account iso-vector mesons $\pi$ and $\rho$ lacks an important constituent of the nuclear force, that is, the tensor force. We improve the model in the following section. The calculation in the $\sigma$-$\omega$ model with Walecka parameters does not reproduce details of the experimental values.

### 3.2. Inclusion of other mesons

Mesons other than $\sigma$ and $\omega$ contribute to the Fock exchange and ladder correlation processes so that these mesons are important ingredients beyond the Hartree approximation. In fact, the exchange interactions of these mesons are expected to affect various aspects of nuclear structure.

From the investigation of the two body problems in free space such as nucleon-nucleon scattering up to 1 GeV and deuteron properties, it has become clear that meson exchange models of nuclear interactions work well. One of the models takes into account six mesons, that is, $\sigma$, $\omega$, $\pi$, $\rho$, $\eta$ and $\delta$ with masses below 1 GeV. We can assume that the contributions of heavier mesons than these six mesons are expected to be negligible in low energy properties of nuclei.

We take the six-meson model. The interaction lagrangian density for the additional mesons is as follows:

\[
L_{N\pi} = -i \frac{g_\pi}{2M} \bar{\psi} (\gamma_5 \gamma_\rho) \tau \sigma_{\mu} \pi, \tag{3.2}
\]

\[
L_{N\rho} = ig_\rho \bar{\psi} \gamma_\rho \tau \psi \rho_\mu + \frac{f_\rho}{4M} \bar{\psi} \sigma_{\mu \nu} \tau \psi \left( \frac{\partial \rho_\nu}{\partial x_\mu} - \frac{\partial \rho_\mu}{\partial x_\nu} \right), \tag{3.3}
\]

\[
L_{N\eta} = -i \frac{g_\eta}{2M} \bar{\psi} (\gamma_5 \gamma_\rho) \phi \partial_\mu \eta, \tag{3.4}
\]
where the fields $\pi$, $\rho$, $\eta$ and $\varphi$ are of $\pi$, $\rho$, $\eta$ and $\delta$ mesons and $\tau$ are the isospin Pauli matrices. We adopt the parameters in Ref. 5), that is, Bonn A,
\[ \begin{align*}
  m_\pi &= 550 \text{ MeV}, & m_\omega &= 782.6 \text{ MeV}, & m_\rho &= 548.8 \text{ MeV}, \\
  m_\sigma &= 138.03 \text{ MeV}, & m_\rho &= 769 \text{ MeV}, & m_\pi &= 983 \text{ MeV}.
\end{align*} \]

With these parameters two-nucleon systems are well reproduced. We use the form factors of the form \((A^2 - m^2)/(A^2 + q^2)\), where \(A\) is the cutoff mass and \(q^2\) is the squared momentum transfer. The values of the cutoff parameters are
\[ \begin{align*}
  A_\sigma &= 2.0 \text{ GeV}, & A_\omega &= 1.5 \text{ GeV}, & A_\rho &= 1.5 \text{ GeV}, \\
  A_\pi &= 1.05 \text{ GeV}, & A_\rho &= 1.3 \text{ GeV}, & A_\pi &= 2.0 \text{ GeV}.
\end{align*} \]

With this form factor the values of the “renormalized” coupling constants
\[ g^R = g \frac{A^2 - m^2}{A^2} \] (3.6)
are
\[ \begin{align*}
  g^R_\pi &= 9.448, & g^R_\omega &= 11.54, & g^R_\rho &= 8.124, \\
  g^R_\sigma &= 13.45, & g^R_\rho &= 2.293, & g^R_\pi &= 2.361
\end{align*} \]
and the vector-tensor ratio for $\rho$-meson is given by \(f_\rho/g_\rho = 6.1\).

We first apply this model to the mean field (Hartree) calculation, the result of which is surprisingly satisfactory in spite of the lack of the Fock terms and the ladder correlations (see Table IV). This overall good trend of the results suggests the success of Walecka model but it is on an insecure basis because it neglects the Fock exchange energies which are as large as half the direct energies.

Next we carry out the Hartree-Fock calculation but find that the result is not

| Table IV. The results of the Hartree, Hartree-Fock, 2nd order ladder and Brueckner calculations using six kinds of mesons are compared with the experimental values. Single particle energies, root mean-square radius, total energy, kinetic energy and $l$-$s$ splitting are shown in MeV and fm. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | H               | HF              | 4th Order       | Full Ladder     | Exp             |
| $S_{1/2}$      | -52.8           | -34.2           | -44.6           | -44.6           | -47             |
| $P_{1/2}$      | -24.3           | -15.0           | -22.0           | -22.6           | -21.8           |
| $P_{3/2}$      | -12.3           | -9.71           | -16.7           | -17.3           | -15.7           |
| $\langle r^2 \rangle$ | 2.27           | 2.75            | 2.60            | 2.61            | 2.73            |
| $E$            | -101            | -51.6           | -108            | -112            | -128            |
| $E_{dir}$      | -353            | -261            | -344            | -347            |                 |
| $E_{exc}$      | 0               | 16.8            | 30.7            | 31.2            |                 |
| $T$            | 252             | 193             | 205             | 204             |                 |
| L-S            | 12.0            | 5.29            | 5.26            | 5.20            | 6.3             |
convergent: The value of the binding energy fluctuates when we run the self-consistent calculation. These numerical fluctuations are due to large contributions of two isovector mesons ($\pi$ and $\rho$). In fact the $d(r)$ terms of the $\pi$ and $\rho$ meson exchanges affect the Fock terms so repulsively that the nuclear system gets unbound.

We now discuss the results of the calculation including the ladder correlations. A numerical instability is also seen in the calculation which uses the same parameters. Our calculation of the correlations is up to $l=3$. It is expected that when we cover the full space of nucleon configurations, the complete treatment of short range correlations may suppress partly the effect of the zero range interactions. Therefore in our truncated space calculation we subtract partly the contributions of the central $\delta$ part of $\pi$ and $\rho$ exchanges by introducing one parameter to suppress the $\delta$ parts.

In our Hartree-Fock and Brueckner results the extension of the configuration space is replaced effectively by the 40 percent subtraction of the $\delta$ parts. We show in Table IV forth order calculation which implies the calculation up to second order of Bethe-Salpeter kernel and the full ladder calculation which contains the infinite series (see Fig. 3). These two calculations give nearly the same result, indicating that higher order corrections are small. The full ladder calculation reproduces the experimental data well. The large $l-s$ splitting and the too small charge radius in the Hartree result are improved by the full ladder.

The relativistic total kinetic energy is defined as

$$T = \sum_n \int d^3x \ U_n^*(x)[-i\alpha \cdot \nabla + (\beta - 1)M]U_n(x), \quad (3.7)$$

when the single particle states $U_n$ are obtained. The sum of the direct energy $E_{dir}$ and exchange energy $E_{exc}$ including the ladder corrections is

$$E_{dir} + E_{exc} = \frac{1}{2} \sum_n \int d^3x d^3y d^3z d^3w U_n(x)\bar{U}_k(z) \times [\Gamma(xz; yw|\epsilon_n - \epsilon_k) - \Gamma(xz; wy|\epsilon_n - \epsilon_k)]U_n(y)U_k(w). \quad (3.8)$$

By using Eqs. (2.22), (3.7) and (3.8) the total energy $E = T + E_{dir} + E_{exc}$ is related to the sum of single particle energies and the kinetic energies $T$ as follows:

$$2E - \sum_n \epsilon_n = T. \quad (3.9)$$

This equation yields an estimate of the value of $T$ about 150 MeV by substituting the data for the sum of single particle energies $\epsilon_n$ and the total energy $E$. The theoretical value of $T$ in the Hartree is much larger than the value 150 MeV. The improvement of the value $T$ when we go from the Hartree to full ladder calculation is seen in Table IV. This reflects the improvement of the calculated quantities in the ladder calculation.

§ 4. Concluding remarks

We have generalized the relativistic self-consistent theory to take into account two-body scattering correlations. The self-consistent Dirac equations including
excited single particle states are so difficult to solve that the standard treatment as differential equations does not work well. Our alternative treatment which uses the wave packet basis functions makes it easy to solve the problem. The good convergence of our results shows that the method is suitable for this problem.

This method is used to solve the Hartree-Fock problem. The value of the total energy obtained with the Bonn A parameters does not converge. The Fock terms yield repulsive effects and damages the validity of the Hartree calculation. It suggests that the ladder correlations are necessary ingredients in the study of the relation between the two body interactions and the nuclear bulk properties. We are compelled to introduce the δ part subtraction. This subtraction may be unnecessary in the full configuration calculation.

Our calculation shows that the mean field theory (Hartree theory) is apparently successful due to the large cancellation between the contributions of the Fock terms and the two-body correlations. The joint corrections due to the Fock terms and the ladder correlations decrease the single particle energies and increase the total energy, modifying the predictions of the Hartree to improve the fits to the data. The simultaneous fits to the empirical values of the total energy and the root-mean-square radius are not attained by the effective density approximation of Müther et al. The tensor force which is effective in the correlations is expected to affect the various aspects of the nuclear physics. The correlations may play a critical role in dynamical nuclear processes, that is, collective motions, nuclear fission processes and so on. These remain to be done in our future study.

References

14) S. Ohnaka, Ph. D. Thesis (1991), University of Tsukuba.