Wat. Sci. Tech. Vol. 28, No. 1, pp. 91–101, 1993.

0273–1223/93 $6.00 + 0.00

SPECIFIC CAKE RESISTANCE: MYTH OR REALITY ?

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ABSTRACT

The specific cake resistance is considered to be the key factor in the characterization of sludge dewaterability. The use of this parameter without knowing how it is derived and the conditions under which it can be used has been creating a great deal of confusion in the literature. Moreover, the lack of a universally agreed test method to determine its value causes additional problems. Hence, attempts to go from the laboratory data to the full scale industrial equipment using this concept may end up in failure.

This paper starts with the origin of the specific cake resistance in the historical perspective of filtration theory. Currently used experimental techniques in the determination of specific cake resistance are reviewed, advantages and disadvantages are succinctly summarized. An alternative way of characterization of sludges using a resistance function rather than a specific cake resistance is proposed.

KEYWORDS

Filtration; sludge dewatering; specific cake resistance; resistance function.

INTRODUCTION

Sludge dewatering is defined as the removal of enough of the liquid portion so that the sludge behaves as a solid (Vesilind, 1979). Over the last decade, the use of pressure filters in dewatering of sludges has substantially increased. Since most sludges are difficult to filter, chemical conditioning is usually used prior to filtration. Both the design of these filters and the determination of the optimum amount of conditioning agents require characterization of sludges. In literature, the three most commonly used parameters for sludge characterization are as follows:

1. Buchner-funnel filtration time (Tenney *et al.*, 1970).
2. Capillary suction time (CST) (Baskerville and Gale, 1968; Vesilind, 1988).
3. Specific cake resistance (Coackley and Jones, 1956; Swanwick and Davidson, 1961; Gale, 1967; Kavanagh, 1980).

Since its introduction to sludge dewatering literature in 1956 by Coackley and Jones, specific cake resistance has been extensively used for characterization of sludges. It is generally preferred since it is considered to have a sound theoretical basis. In order to better understand the specific resistance concept, it is necessary to examine the development of the filtration equation.

THE DEVELOPMENT OF THE CAKE FILTRATION EQUATION

The cake filtration equation based on the concept of specific cake resistance is

$$u_o = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\mu}\frac{A(\Delta P_T)}{(\langle r_w\rangle cV + AR_m)} \tag{1}$$

where u_o is the superficial liquid velocity at the exit of filter cake, i.e., superficial filtrate velocity; A is the cross-sectional area of filter; V is the filtrate volume; t is time; μ is the filtrate viscosity; ΔP_T is the total pressure drop across the filter; c is the mass of solids per unit volume of filtrate; and R_m is the filter medium, or septum, resistance. The term $\langle r_w\rangle$ is the average specific cake resistance based on the mass of solids in the cake. The literature on water and wastewater filtration usually attributes the development of Eq. (1) to either Carman (1938) or to Coackley and Jones (1956). It appears, however, that Sperry (1916) was the first to develop an equation similar to Eq. (1). Later, Ruth (1935) developed Eq. (1) in its current form. Ruth's analysis of filtration is based on heuristic analogy with Ohm's law for two resistances in series, namely the cake resistance, R_c, and the filter medium resistance, R_m.

Ohm's Law and Series Resistance

According to Ohm's law the voltage, V, is proportional to the current, I, the resistance being the constant of proportionality. This relationship is expressed as

$$I = \frac{V}{R} \tag{2}$$

The resistance, R, is a characteristic of the conductor and given by

$$R = \rho\frac{L}{A} \tag{3}$$

where L is the length of the conductor, A is the uniform cross-sectional area, and ρ is the resistivity or specific resistance of the conductor.

Electrical resistance analogy of filtration is shown in Fig. 1. Therefore, the filtrate rate is expressed as

$$\mu u_o = \frac{\Delta P_T}{R_c + R_m} \tag{4}$$

in which u_o is the superficial liquid velocity defined by

$$u_o = \frac{1}{A}\frac{dV}{dt} \tag{5}$$

The resistance of the cake, R_c, is generally expressed as

$$R_c = \langle r_w\rangle\frac{W_s}{A} \tag{6}$$

where W_s is the total mass of solids within the cake; and $\langle r_w\rangle$ is the average specific cake resistance based on the mass of solids. Note that this definition of $\langle r_w\rangle$ is analogous to the definition of electrical resistivity given in Eq. (3). Substitution of Eqs. (5) and (6) into Eq. (4) gives

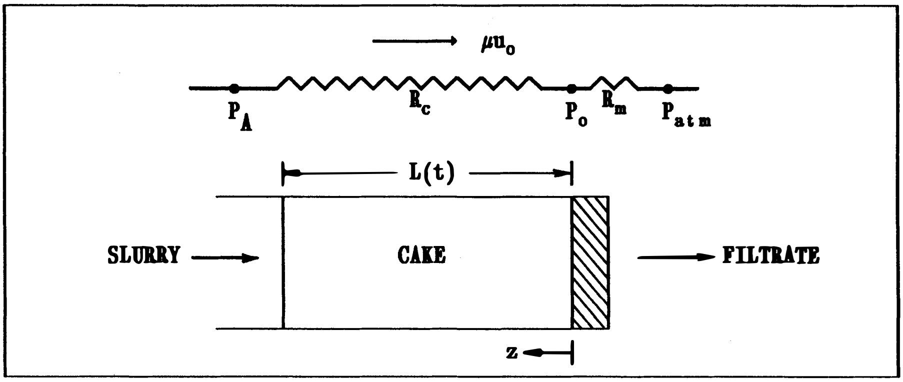

Fig. 1. Electrical resistance analogy of filtration.

$$\frac{1}{A}\frac{dV}{dt} = \frac{1}{\mu}\frac{A(\Delta P_T)}{(\langle r_w\rangle W_s + AR_m)} \tag{7}$$

Since the volume of filtrate, V, is easier to measure than the mass of solids in the filter cake, W_s, then the macroscopic mass balance

$$\begin{bmatrix}\text{Mass of Slurry}\\ \text{Filtered}\end{bmatrix} = \begin{bmatrix}\text{Mass of Wet}\\ \text{Cake}\end{bmatrix} + \begin{bmatrix}\text{Mass of}\\ \text{Filtrate}\end{bmatrix} \tag{8}$$

is used to make the conversion. Symbolically, Eq. (8) is

$$W_s = cV \tag{9}$$

where c represents the mass of solids per unit volume of filtrate and is given by

$$\frac{1}{c} = \left[\frac{1}{s} - 1\right]\frac{1}{\rho} - \left[\frac{\langle\epsilon\rangle}{1-\langle\epsilon\rangle}\right]\frac{1}{\rho_s} \tag{10}$$

and where ρ and ρ_s are the densities of liquid and solid, respectively; s is the mass fraction of solids in slurry; and $\langle\epsilon\rangle$ is the average cake porosity. Elimination of W_s between Eqs. (7) and (9) results in the filtration equation, Eq. (1).

Since 1935 Eq. (1) has been used for cake filtration analysis. For a constant pressure filtration, integration of Eq. (1) by considering c, $\langle r_w\rangle$ and R_m constant gives

$$\frac{t}{V} = \left[\frac{\mu\langle r_w\rangle c}{2A^2(\Delta P_T)}\right]V + \frac{\mu R_m}{A(\Delta P_T)} \tag{11}$$

Equation (11) indicates that the average specific cake resistance, $\langle r_w\rangle$, and the medium resistance, R_m, can be obtained, respectively, from the slope and the intercept of the straight line when t/V is plotted versus V.

Relationship Between Local and Average Specific Cake Resistance

It is apparent from Fig. 1 that the cake and filter medium resistances are given by

$$R_c = \frac{\Delta P_c}{\mu u_o} \tag{12}$$

$$R_m = \frac{\Delta P_m}{\mu u_o} \tag{13}$$

where ΔP_c and ΔP_m are the pressure drops across the cake and the medium, respectively. Using Eqs. (6) and (12) the expression for the average specific cake resistance becomes

$$\langle r_w \rangle = \frac{A \Delta P_c}{\mu u_o W_s} \tag{14}$$

In the literature, the local value of the specific cake resistance, r_w, is obtained from Eq. (14) by assuming that this equation holds locally, i.e.,

$$r_w = \frac{A}{\mu u_o} \frac{\partial P}{\partial W} \tag{15}$$

At any instant, integration of Eq. (15) over the total cake mass gives

$$W_s = \int_0^{W_s} dW = \frac{A}{\mu u_o} \int_{P_o}^{P_A} \frac{dP}{r_w} \tag{16}$$

Comparison of Eqs. (14) and (16) indicates that

$$\langle r_w \rangle = \frac{\Delta P_c}{\int_{P_o}^{P_A} \frac{dP}{r_w}} \tag{17}$$

Equation (17), according to Svarovsky (1985), is rather unusual but a widely used definition of the average specific cake resistance. It is indeed unusual because mathematically the right side of Eq. (17) is equal to $1/\langle 1/r_w \rangle$ and, in general, $\langle 1/r_w \rangle \neq 1/\langle r_w \rangle$. This point has been mentioned in literature (Tiller and Horng, 1983; Tiller and Yeh, 1987). They concluded that no changes are necessary because: (i) usual equations become clumsy if $\langle 1/r_w \rangle$ is used instead of $1/\langle r_w \rangle$; and (ii) it is necessary to maintain customary practice. These, of course, are expedient but not conceptually honest conclusions.

As can be seen from the development presented above, Eq. (14) is the logical consequence of the analogy with Ohm's law. If Eq. (14) is assumed to hold locally, and if the superficial liquid velocity, u, is considered constant throughout the filter cake at any instant, i.e., $u = u_o(t)$, then Eq. (17) is obtained automatically. Therefore, no changes in Eq. (17) can be made without revising the original formulation of Ruth.

Basic Filtration Equation and Darcy's Law

It should be noted that the development of the basic cake filtration equation is purely based on the analogy with Ohm's law. Nowhere in the derivation has Darcy's law been used. Darcy's law is a macroscopic equation expressed in the form

$$u_o = \frac{\langle K \rangle}{\mu} \frac{\Delta P_c}{L} \tag{18}$$

where $\langle K \rangle$ is the average permeability of the cake. Comparing Eq. (18) with Eq. (14) and making use of the relationship

$$W_s = AL(1-\langle \epsilon \rangle)\rho_s \tag{19}$$

relates the average permeability to the average cake resistance in the form

$$\langle K\rangle^{-1} = \langle r_w\rangle(1-\langle\epsilon\rangle)\rho_s \tag{20}$$

Thus, Eq. (20) is the only link between Darcy's law and the basic equation of filtration. If Darcy's law is assumed to hold locally, then Eq. (18) can be written as

$$u_o = \frac{K}{\mu}\frac{\partial P}{\partial z} \tag{21}$$

In this case, comparison of Eq. (21) with Eq. (15) and the use of the relationship

$$dW = A(1-\epsilon)\rho_s dz \tag{22}$$

gives the following expression between the local values of permeability and specific cake resistance:

$$K^{-1} = r_w(1-\epsilon)\rho_s \tag{23}$$

Examination of Eqs. (20) and (23) implies another mathematical inconsistency because if $f = ab$, then $\langle f\rangle \neq \langle a\rangle\langle b\rangle$.

Modified Forms of Average Specific Cake Resistance

Deviations from parabolic behavior are usually attributed to the variation of the average specific cake resistance during the course of filtration and the modifications proposed for its definition are summarized in Table 1.

As shown in Table 1, the starting point in the analyses is Darcy's law. Three different forms of Darcy's law have been used in the filtration literature. Equation (A) in Table 1 assumes that superficial liquid velocity is constant throughout the filter cake at any instant. Equation (B) considers the variation of superficial liquid velocity within the cake. Equation (C), on the other hand, takes into account the variations in superficial liquid and solid velocities. Expressing permeability in terms of specific cake resistance and cake length in terms of cake mass yields Eqs. (E), (F), and (G). At any instant, these equations can be integrated from the cake-septum interface, i.e., $z = 0$, to the cake surface, i.e., $z = L$, to get Eqs. (H), (I), and (J). Equations (L), (M), and (O) are obtained when the term $(A/\mu u_o)$ in Eqs. (H), (I), and (J) is substituted into the general definition of the average specific cake resistance defined by Eq. (K). Equations (M) and (O) are derived by Tiller and Shirato (1964) and Shirato *et al.* (1969), respectively. The terms J_1 and J_2 are the correction factors which are less than unity and are dependent on the slurry concentration. Therefore, among the three definitions of the average specific cake resistance, only Eq. (L) is independent of slurry concentration.

It should be noted, however, that apparent flaws exist in the the development of Eqs. (M) and (O). The development of these equations requires the use of Eqs. (D) and (K) which correspond to Eqs. (23) and (14), respectively. Examination of the derivations of these equations indicates that they are valid if and only if the superficial liquid velocity is constant throughout the filter cake at any given instant, i.e., $u = u_o(t)$. The use of these equations for cases in which superficial liquid velocity is not constant may be in error.

EVALUATION OF AVERAGE SPECIFIC CAKE RESISTANCE

Buchner funnel test is the most commonly used method to determine the average specific cake resistance. In this method, after placing a filter paper into a funnel, 100-300 mL of sludge is poured into it. A vacuum is then applied and the filtrate is collected into a calibrated reservoir where the volume is recorded as a function of time. These data are plotted as t/V versus V and the average specific cake resistance is obtained from the slope of the resulting straight line according to Eq. (11).

TABLE 1 Development of the Various Forms of Average Specific Cake Resistance

$$u_o = \frac{K}{\mu}\frac{\partial P}{\partial z} \quad (A)$$

$$u = \frac{K}{\mu}\frac{\partial P}{\partial z} \quad (B)$$

$$u - \frac{\epsilon}{1-\epsilon}u_s = \frac{K}{\mu}\frac{\partial P}{\partial z} \quad (C)$$

$$K^{-1} = r_w(1-\epsilon)\rho_s \quad ; \quad dW = A(1-\epsilon)\rho_s dz \quad (D)$$

$$u_o = \frac{A}{\mu r_w}\frac{\partial P}{\partial W} \quad (E)$$

$$u = \frac{A}{\mu r_w}\frac{\partial P}{\partial W} \quad (F)$$

$$u - \frac{\epsilon}{1-\epsilon}u_s = \frac{A}{\mu r_w}\frac{\partial P}{\partial W} \quad (G)$$

INTEGRATION OVER THE CAKE MASS AT ANY INSTANT

$$\frac{A}{\mu u_o} = \frac{W_s}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (H)$$

$$\frac{A}{\mu u_o} = \frac{\int_0^{W_s}\frac{u}{u_o}\,dW}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (I)$$

$$\frac{A}{\mu u_o} = \frac{\int_0^{W_s}\left[\frac{u}{u_o} - \frac{\epsilon}{1-\epsilon}\frac{u_s}{u_o}\right]dW}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (J)$$

GENERAL DEFINITION OF THE AVERAGE SPECIFIC CAKE RESISTANCE

$$\langle r_w \rangle = \left[\frac{A}{\mu u_o}\right]\frac{\Delta P_c}{W_s} \quad (K)$$

$$\langle r_w \rangle = \frac{\Delta P_c}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (L)$$

$$\langle r_w \rangle = J_1\frac{\Delta P_c}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (M)$$

where

$$J_1 = \int_0^1\left[\frac{u}{u_o}\right]d\left[\frac{W}{W_s}\right] \quad (N)$$

$$\langle r_w \rangle = J_2\frac{\Delta P_c}{\int_{P_o}^{P_A}\frac{dP}{r_w}} \quad (O)$$

where

$$J_2 = \int_0^1\left[\frac{u}{u_o} - \frac{\epsilon}{1-\epsilon}\frac{u_s}{u_o}\right]d\left[\frac{W}{W_s}\right] \quad (P)$$

Various factors affecting the value of the average specific cake resistance are reported by Christensen and Dick (1985) as: (i) concentration of solids in slurry, s; (ii) initial time and filtrate volume readings; (iii) the method used to determine the dry mass of cake per unit volume of filtrate, c. It is obvious that all of these factors have a significant effect on the reported values of the average specific cake resistance. However, the most important issue is whether or not Buchner funnel experiments actually simulate filtration.

The analysis of filtration process by the use of Eq. (11) requires a continuous supply of slurry with constant solids concentration, s. On the other hand, once the slurry is poured into the funnel most of the solids settle down and form a filter cake as a result of both applied vacuum and sedimentation. After a certain period of time, the supernatant liquid at the top of the filter cake is almost free of solids and from that point on the process resembles flow through a packed bed rather than filtration. If this period constitutes a significant fraction of the total time, the overall process cannot be regarded as filtration. It is also important to note that once the level of the supernatant liquid reaches the surface of the cake the process is no longer filtration but cake dewatering, and the start of this period is very difficult to determine.

MULTIPHASE FILTRATION THEORY

The multiphase filtration theory which is developed by Willis and Tosun (1980) has several advantages over the the classical filtration analysis. These advantages, as stated by Tosun and Willis (1989), are as follows: (i) it permits a thorough investigation of the assumptions normally made regarding the magnitude of terms in the governing differential equations; (ii) the resulting accurate mathematical description permits a more rational experimental program; (iii) it provides practicing engineers with a mechanism, clogging medium, and a set of parameters (design, material, operation) that can be used to alter filtration operations; (iv) it provides a rigorous framework for further study.

The classical filtration theory considers the medium resistance negligible and assumes that the controlling factor is the resistance of the cake. This is the reason why the average specific cake resistance has been given so much attention in the literature. The multiphase filtration theory, on the other hand, indicates that the region of high drag which occurs at the cake-septum interface controls the filtrate rate. This points out the fact that it is important to specify the type of septum, in addition to the liquid and solid phases.

The multiphase filtration theory combines the volume-averaged equations of change with the experimental observations to deduce the filtration mechanism. The details of this development are given elsewhere (Willis and Tosun, 1980; Chase *et al.*, 1990; Willis *et al.*, 1991). The governing equation for the multiphase filtration theory is given by

$$\frac{dt}{dV} = \left[\frac{G}{A^2 \Delta P_c} \frac{(\lambda/\epsilon^2)_{z=0}}{J_o}\right] V \tag{24}$$

The term J_o is the dimensionless pressure gradient at the cake-septum interface defined by

$$J_o = (\partial P^* / \partial \xi)_{z=0} \tag{25}$$

in which the dimensionless variables P^* and ξ are given as

$$P^* = \frac{P - P|_{z=0}}{\Delta P_c} \tag{26}$$

$$\xi = \frac{z}{L} \tag{27}$$

Equation (24) relates the cake pressure drop to the filtrate rate and filtrate volume. The dimensionless pressure gradient J_0 accounts for the effect of the pressure gradient, or drag, on the deformation of the solids. The variations in the solids concentration of slurry are accounted for by the term G, which is defined by

$$G = \frac{c}{(1-\langle\epsilon\rangle)\rho_s} \tag{28}$$

The term λ is the resistance function that relates the drag force to the velocity difference. It depends on the viscosity of liquid and the surface area of the solids. It is a more appropriate parameter than the average specific cake resistance to characterize the filter cakes.

When dt/dV is plotted versus V for a constant cake pressure drop filtration, Eq. (24) reveals that the variations in J_0 and $(\lambda/\epsilon^2)_{z=0}$ are responsible for the deviations from linearity. The interpretation of rate data by using Eq. (24) is given in detail by Willis *et al.* (1991).

EXPERIMENTAL

Feed Solution

Constituents of the reproducible, soluble wastewater used in this study are listed in Table 2. Bacto-peptone and glucose serve as the energy and carbon source. A phosphate buffer is used to provide a pH of 7.0 and to serve as a source of phosphorus; and additional nitrogen is added as ammonium chloride.

TABLE 2 Composition of Wastewater

Constituent	Concentration (mg/L)
Glucose	935
Bacto-peptone	400
K_2HPO_4	800
NH_4Cl	300
$MgSO_4.7H_2O$	150
$CaCl_2$	20
$NaHCO_3$	240
$MnSO_4.3H_2O$	5
$FeSO_4.7H_2O$	5
$ZnSO_4.7H_2O$	5

Reactor Operation

The activated sludge in this study is originated from the raw sewage sample obtained from the influent of the Middle East Technical University wastewater treatment plant. The seed is allowed to increase to a suspended solids concentration of about 1500 mg/L on a batch feeding basis. Then, the reactor with an effective volume of 4 L is fed in a fill-and-draw system with 1 L synthetic wastewater every day.

Reactor temperature is kept constant at 25°C. Air is supplied to the reactor through a number of porous stone diffusers. The amount of air delivered is adjusted so as to maintain the mixed liquor suspended solids (MLSS) in suspension.

Filtration Tests

The sludge samples for filtration tests are prepared by removing the supernatant after settling under quiescent conditions. As shown in Fig. 2, the filtration cell consists of a cylindrical plexiglas cylinder, 3 cm in diameter, and a porous support plate on which the filter paper is placed. This cell is used for both down-flow and up-flow filtration modes which correspond to Buchner funnel and filter leaf tests, respectively.

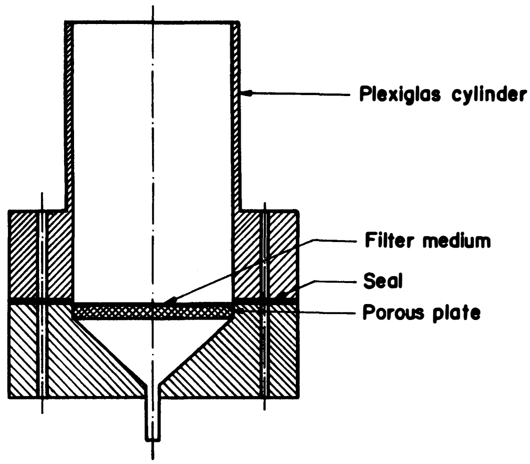

Fig. 2. Filter cell.

In down-flow filtration tests, a small sample from a well-mixed sludge is poured into the filter cell and timing is initiated as soon as the desired vacuum is applied (20 in. Hg). Monitoring of the filtrate volume is continued until no more filtrate comes out of the unit.

In up-flow filtration tests, the filter cell is turned upside down and submerged in a large sludge container in which continuous mixing is provided to maintain a uniform concentration. These tests are continued until the filtration rate is almost zero.

Both down-flow and up-flow experiments are carried out for two different types of filter paper, namely, Whatman # 40 (pore size = 8 μm) and Whatman # 41 (pore size = 20-25 μm). The resulting data are plotted as t/V versus V in Fig. 3 and the calculated values are given in Table 3. The results indicate that the slope of the t/V versus V line is strongly affected by the filter medium as predicted by the multiphase filtration theory. On the other hand, for the same type of septum, the change in the slope of t/V versus V plot for up-flow and down-flow experiments indicates the significance of the variations in the slurry solids concentration.

CONCLUSION

The average specific cake resistance is introduced into the classical filtration theory as a result of its analogy with Ohm's law. This theory does not explain the mechanism of filtration because of the inconsistencies and contradictions in its development. Moreover, the controlling factor in filtration is not the resistance of the cake but the resistance of the cake-septum interface. This point has been clearly demonstrated in this work by showing the effect of the filter medium on the filtrate volume-time data.

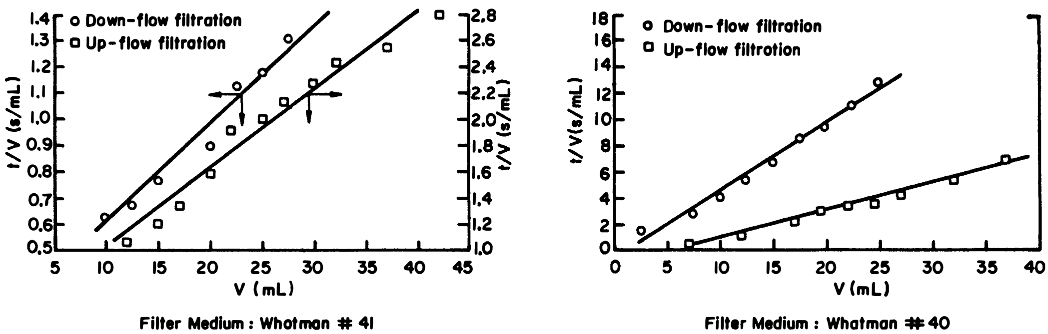

Fig. 3. Time-Filtrate volume data.

TABLE 3 Numerical Values of the Slopes of t/V versus V Plots

Type of Filtration	Slope of t/V vs. V line	
	Whatman # 40	Whatman # 41
Up-flow	0.210	0.059
Down-flow	0.518	0.038

In most of the cases it is doubtful whether Buchner funnel tests actually simulate filtration process. The experimental study presented in this work supports this point. Buchner funnel tests can be used to get a qualitative information on the filterability of sludges. However, quantitative conclusions are subject to question.

The appropriate parameter in the characterization of sludges is the resistance function. This value, however, is not only dependent on the characteristics of the solid phase. The type of filter medium as well as the liquid and solid phases should be specified.

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