by J. H. Perry, McGraw-Hill Book Company, Inc., New York, N. Y., third edition, 1950, p. 559.

2 "On the Evaporation of Falling Drops," by N. Frössling, *Gerlands Beitr. Geophys.*, vol. 52, 1938, p. 170.

3 "Studies of the Combustion of Droplets in a Fuel Spray—The Burning of Single Drops of Fuel," by G. A. E. Godsave, collected papers of the Fourth International Symposium on Combustion, Cambridge, Mass., September 1–5, 1952, The Williams and Wilkins Company, Baltimore, Md., 1953, p. 818.

4 "Vaporization Rates and Heat-Transfer Coefficients for Pure Liquid Drops," by R. D. Ingebo, *Chemical Engineering Progress*, vol. 48, 1952, p. 403.

5 "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, Inc., New York, N. Y., second edition, 1950; Chapter III, by H. C. Hottell and Appendix Table XIII, p. 396.

6 "Mass Transfer" (containing a critical review of references on "drop evaporation"), by R. L. Pigford, *Industrial and Engineering Chemistry*, vol. 45, 1953, p. 957.

7 "Evaporation From Drops," by W. E. Ranz and W. R. Marshall, Jr., *Chemical Engineering Progress*, vol. 48, 1952, pp. 141–173.

Discussion

O. A. Uyehara[3] and P. S. Myers.[4] The author is to be congratulated for his reminder that mass transfer affects heat transfer and for his thoughts on this matter after he has had time for reflection on his data as presented in reference (7) of the text.

In considering the evaporation of a drop of volatile liquid in high-temperature surroundings it must be remembered that the temperature of this liquid does not instantly rise from its initial temperature to its vaporizing or wet-bulb temperature. This situation is illustrated in Fig. 1 of this discussion where an experimental temperature-time history of a drop of fuel is presented. The data were obtained as described in the references[5,6] and are intended to be representative of the history of a drop suddenly introduced into hot air. It can be clearly seen in Fig. 1 that the time required for the drop to reach its wet-bulb temperature is an appreciable portion of the lifetime of the drop. It should be clear that the equations and concepts expressed by the author are applicable only to the steady-state portion of the curve.

As a matter of interest the accompanying references[5,6] also consider the effect of mass transfer on heat transfer. Although they were derived in a slightly different fashion the resulting equations are the same as those presented by the author. In this connection the N-factor of the author or the Z-factor of these references[5,6] are also plotted in Fig. 1. It can be seen that under these circumstances only a small fraction of the Q entering the film surrounding the drop reaches the liquid. Thus there is ample justification for the author's insistence of recognition of the effects of mass transfer on heat transfer. As a matter of interest, Equations [17] and [18] of the text were used in the computations and the computed results were in reasonable agreement with the experimental data.

Inasmuch as some of the peculiarities of heat transfer in the presence of mass transfer are being discussed, it might be well to mention the discrepancies in the definitions of the Nusselt number under these circumstances.

The Nusselt number is customarily defined as $h2r/k$; the symbols are as defined in the text. However, as is clearly pointed out in the text (and Fig. 1) the heat that is transferred to the edge of the film surrounding the drop and the heat that actually reaches the liquid surface may differ by a factor of 4 or more. This effect does not show up in the Nusselt number when it is defined as in the foregoing. If however, the Nusselt number is defined as

$$(dT^*/dr^*)_{r=r_1}$$

as in Equation [10] of the text where

$$dT^* = \frac{dt}{(T_2 - T_1)} \quad \text{and} \quad dr^* = \frac{dr}{2r_1}$$

then clearly the temperature gradient

$$\left(\frac{dT^*}{dr^*}\right)_{r=r_1}$$

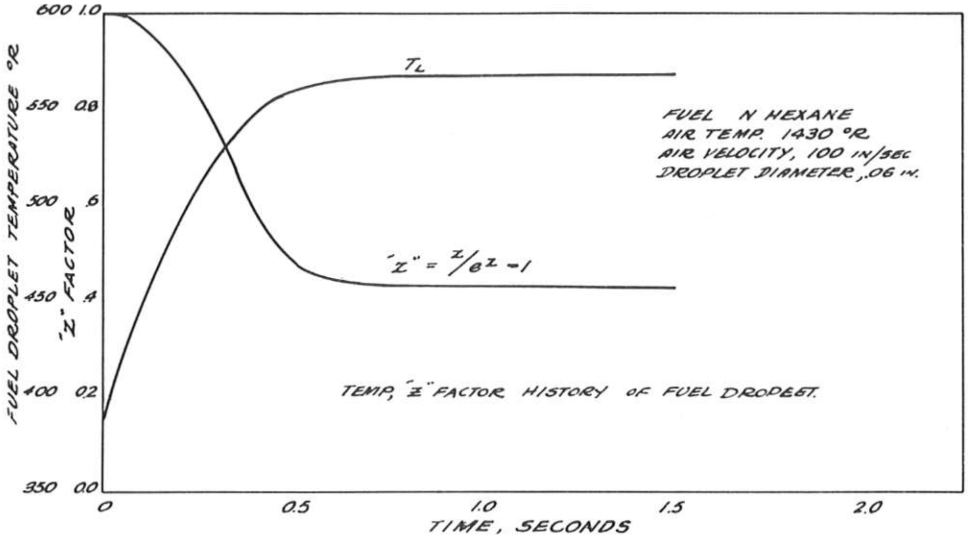

Fig. 1 Experimental Temperature-Time History of a Drop of Fuel

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[5] "A Theoretical Investigation of the Heating-Up Period of Injected Fuel Droplets Vaporizing in Air," by M. M. El Wakil, O. A. Uyehara, and P. S. Myers, NACA TN 3179, May, 1954.

[6] "Experimental and Calculated Temperature and Mass Histories of Vaporizing Fuel Droplets," by M. M. El Wakil, R. J. Priem, H. J. Brikowski, P. S. Myers, and O. A. Uyehara, NACA TN 3490.

will be different with and without mass transfer. Using this definition of the Nusselt number does take into account the effect of mass transfer on heat transfer. This definition also makes it clear (Equation [10]) that the thermal conductivity is to be evaluated at the surface and not at an average temperature somewhere between the air and liquid temperatures.

Equation [15] of the text presents another definition of the Nusselt number in terms of the "effective film thickness." If, as is suggested by the author, we assume that the effective film thickness remains unchanged for the same convective conditions then this definition also says that the Nusselt number is independent of mass transfer. Perhaps a discussion as to exactly what is meant by the Nusselt number would be helpful.

In conclusion it might be mentioned that Williams[7] has shown that internal reflection inside the drop can cause an appreciable increase in the absorptivity of the drop.

AUTHOR'S CLOSURE

The only rules that one should follow in defining the Nusselt number are that it be composed entirely of quantities which are known or wanted and that it be clearly defined. To restrict the definition may spoil the fun of authors and discussers. Indeed, the definition should not be restricted since the Nusselt number can be a generalized dimensionless heat-transfer rate which is related to the boundary conditions and finds many uses and definitions in many different types of problems.

Equation [10] of the text is not a definition of the Nusselt number but a consequence of the definition which is given in the sentence which follows the equation. In this case the Nusselt number is the average rate of heat transfer by conduction per unit area of interface (symbol q_{ai}) multiplied by a characteristic linear dimension (chosen as the drop diameter) and divided by the temperature difference across the transfer path and the thermal conductivity (chosen as the gas-phase thermal conductivity at the interface). Thus

$$N_{\text{Nu}} \equiv q_{ai}(2r_1)/(T_2 - T_1)k_1$$

Because of radial symmetry q_{ai} does not have to be found by integrating and averaging local values of the surface temperature gradients; and the defined dependent quantity can be represented in terms of the known independent quantities of Equation [10].

Another interesting way to look at the problem of heat transfer with mass transfer is from the standpoint of dimensional analysis. No longer can $h \equiv q_{ai}/(T_2 - T_1)$ be treated as a single quantity. q_{ai} and $(T_2 - T_1)$ are now independent of each other; and one new dimensionless group, N, must appear in the final correlation. A wide temperature variation in k also introduces a complication. Such a variation can be treated by another dimensionless group, e.g., the power on T in a $k - T$ correlation; but the experimental and theoretical problem becomes much more complicated, in most cases impossible of solution. The present paper tries to avoid the issue by employing an unspecified average conductivity. What this average should be, what its limits of use are, and whether its use is the most practical way of handling such problems are matters which must eventually be considered by those who would understand heat transfer.

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[7] "The Combustion of Droplets of Heavy Liquid Fuels," by H. C. Hottel, G. C. Williams, and H. C. Simpson, presented at the Fifth Symposium on Combustion, Pittsburgh, Pa., September, 1954.