

quite different from a rigid pendulum which “plows” into an inclined surface while maintaining its circular path.

When a revolute pin impacts against a normal surface which has a lateral velocity, the friction force causes the pin to rebound obliquely. A rigid pendulum, on the other hand, is constrained and must rebound normally. Assuming that its contact zone is symmetrical about a tangential plane, the pendulum would neither lose nor gain energy since the resultant friction force travels through no distance and hence does no work on the pendulum.

## Mechanical Couplings—A Geometrical Theory<sup>1</sup>

**E. J. F. Primrose.**<sup>2</sup> The authors have presented a most interesting paper, which uses purely geometrical arguments to throw light on the behaviour of mechanical couplings. However, in my opinion, they go too far in stating their Theorems 1 and 2, which I believe are untrue in their present form. A critical example is that of the elliptic trammel and its generalization, the spatial *PSSP*, which the authors discuss in Part 2. Taking the spatial *PSSP*, the degree of the ruled surface  $\Sigma$  generated by the *SS* line is 4, and the order of the curve  $\Gamma$  (an ellipse) described by a point  $Q$  on the *SS* line is 2, as the authors say. This would, of course, disprove Theorem 1, but the authors claim that the order of  $\Gamma$  is really 4, on the grounds that the plane of  $\Gamma$  meets  $\Sigma$  in a curve of order 4, consisting of  $\Gamma$  and the line at infinity counted twice. This is perfectly true, but I maintain that the line at infinity is not part of the locus of  $Q$ . It arises because there are two (imaginary) positions of the linkage in which the *SS* line is at infinity (this can be established by setting up a suitable coordinate system). In each of these two positions, the point  $Q$  is at infinity, of course, and these two positions of  $Q$  are the (imaginary) points at infinity on the ellipse  $\Gamma$ .

There is, therefore, an essential difference between the degree of the surface  $\Sigma$  and the order of the curve  $\Gamma$ , which disproves Theorem 1. Theorem 2 now falls to the ground because “feather” is no longer an invariant property of a connectivity-1 algebraic coupling.

### Author's Closure

Dr. Primrose is correct in his assertion that the locus of point  $Q$  does not include the line at infinity when the word “locus” is taken to mean only the mechanically produced path of the point  $Q$ . The alternative to the “mechanical” interpretation of a locus is to define it either by an algebraic equation or (for a point locus) by the intersection of two surfaces; the fourth-order curve to which Dr. Primrose refers is obtained by intersection. We have to admit that we do not adequately define the word “locus.” In order to maintain the truth of the theorems we must state that in special situations such as the elliptic trammel the point locus must be taken as the complete intersection of two surfaces and not merely as a “mechanical” locus.

The two surfaces in these special situations are, first, a line-series generated by a line of the moving body which passes through the tracing point, and, second, a surface which intersects the line-series in the mechanically-traced curve. Situations where the mechanically-traced curve is not the complete intersection arise most often when the second surface is a plane; then one or more of the generators of the line-series, whether finitely accessible or not, may themselves lie in the plane. This happens with the elliptic trammel, and also, for

instance, with the *CSE* linkage. When the *CE* link is fixed a line of the *CS* link generates a fourth-order ruled surface, but the mechanical locus of a point of the same link is an ellipse. However, the complete locus of the point is an ellipse together with the line coinciding with the minor axis of the ellipse taken twice; this is the intersection of the plane of the ellipse with the ruled surface generated by the line which passes through the tracing point and intersects the axis of the cylindrical pair at a right-angle.

A further example, given to us earlier by Dr. Primrose, illustrates that the second surface need not always be a plane. The locus is not here illustrated by a linkage, but rather by a moving line which rotates about a fixed line along  $Oz$  which it intersects at right-angles. As the moving line rotates about  $Oz$  (starting at an initial position  $\theta = 0$ ) the point of intersection with the fixed line is constrained to move up and down  $Oz$  so that the line generates a surface expressible as  $z = a \sin 2\theta$ . This conoidal surface is known as the cylindroid, and it has degree 3; however in a complete rotation through 360 deg the line generates the cylindroid twice, and thus the ruled surface generated by this device must be regarded as having degree 6. On the other hand the mechanical locus of a point attached to the moving line has order 4. This point locus, however, lies on a circular cylinder whose axis is  $Oz$ . The algebra shows that the cylinder not only intersects the complete mechanically-generated ruled surface in the same fourth-order curve twice but also has a real double circle at infinity in common with it. This accounts for the entire twelfth-order curve which the cylinder should have in common with the sixth-degree ruled surface.

These examples emphasize that the theorems presented in the papers are essentially geometrical and require considerable care when they are applied to certain mechanical movements. The limitations on the application of the theorems are mentioned from time to time in the papers and are discussed at some length in reference [19] (Part 1). We agree, however, with Dr. Primrose in that the basis of these limitations (i.e., when the mechanical locus is not always the complete intersection of the second surface with the line-series) is not always easy to appreciate or to understand fully. In spite of these problems of interpretation we believe that the theorems remain intact and retain considerable predictive usefulness. Discrepancies arise for point loci which lie on special surfaces, and for spatial loci of planes [19] (Part 1). They do not, however, arise for loci of lines, and these loci do, in general, give an unambiguous indication of feather. Finally, Theorem 2 (Part 1) provides a good reason for identifying feather with the order of the complete point locus (or with the degree of the line series). For example, the spin-surface of the elliptic trammel is of eighth order, a result which may be confirmed by straightforward algebraic means. If, however, the feather of the elliptic-trammel connection were taken as 2, then the spin-surface's order would be an exception.

We thank Dr. Primrose for his valuable criticisms and for this opportunity to clarify certain aspects of our papers.

## A Regenerative Compressor<sup>1</sup>

**J. W. Hollenburg.**<sup>2</sup> The authors are to be commended for a fine piece of experimental work in which they have demonstrated the attainability of greater than 50 percent efficiency in “advanced” regenerative turbo-machinery, a possibility which was reported in this writer's paper<sup>3</sup> almost two years ago.

The tests which were run, however, make it difficult to exactly attribute to what the improvement in efficiency over past experience (below 50 percent) is mainly due. The authors suggest that blade

<sup>1</sup> By E. F. Fichter and K. H. Hunt, *JOURNAL OF ENGINEERING FOR INDUSTRY*, TRANS. ASME, Series B, Vol. 99 pp. 77-87.

<sup>2</sup> University of Leicester, England.

<sup>1</sup> By H. Sixsmith and H. Altmann, *JOURNAL OF ENGINEERING FOR INDUSTRY*, TRANS. ASME, Series B, Vol. 99, No. 3, Aug. 1977.

<sup>2</sup> Consulting Engineer, Hoboken, N.J., Mem. ASME.

<sup>3</sup> Hollenburg, J. W., “A New Compressor for Hydrogen Pipeline Transmission,” University of Miami Hydrogen Energy Fundamentals Symposium, Miami, Florida, March, 1975.