Realistic Nucleon-Nucleon Potentials Expressed in Terms of the Gaussian Basis

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(Received March 9, 1994)

The Yukawa functions appearing in meson-theoretical potentials are expressed by superposing the Gaussian basis functions with high accuracy. Two kinds of OBEP with and without retardation described by the Gaussian basis are constructed. The two potentials give excellent fits to the two-nucleon data.

Realistic nucleon-nucleon potentials based on the meson-exchange model have been proposed by many authors.1) The aim of this work is to confirm that the radial dependence of a class of one-boson-exchange potentials (OBEP) whose radial dependence is smooth and finite at the origin, is well simulated by superposing the Gaussian basis functions. We call this simulation the Gaussian expansion and show its validity by applying it to the Funabashi potential2) which is a nonstatic OBEP with the Gaussian soft core.

By using both the realistic Gaussian potential (GOBEP) and the Gaussian expansion of wave functions, numerical calculations and analytic estimation of the nuclear problems will be very simple.

First, we perform the Gaussian expansion of the individual Yukawa type functions from each meson (π, η, ρ, ω, δ, σ). Then, two kinds of OBEP with and without retardation, which we denote as GOBEP (R) (or (R)) and GOBEP (NR) (or (NR)) are constructed.

The r-space OBEP is usually derived from the p-space OBEP. The p-space OBEP $V(p', p)$ is expanded up to the order $O(p^2/M^2)$. The meson propagator $\tilde{A}_a(k^2)$ is also approximated as

$$\tilde{A}_a(k^2) = P \frac{1}{k^2 + m_a^2 - (E' - E)^2} \approx \frac{1}{k^2 + m_a^2 + (E' - E)^2},$$

where $k = p' - p$, $E = M + p^2/2M$, $M$ ($m_a$) is the nucleon (meson) mass and the suffix $a$ stands for the exchanged meson. By performing the Fourier transformation of $V(p', p)$, the r-space OBEP is given in the form:

$$V_{NN} = V_{cc} + S_{12} V_{c} + (L \cdot S) V_{LS} + W_{12} V_{w} + L^2 V_{LL} - \frac{1}{M} (F^2 V_{p} + V_{p} F^2),$$

where $V_i = v_i + w_i$; $i = C, T, LS, W, LL, p$,

$$v_i = \frac{1}{2} \left( v_i^a + v_i^b \right), \quad w_i = \left( v_i^a - v_i^b \right).$$

The two potentials give excellent fits to the two-nucleon data.
first (second) term of the r.h.s. of Eq. (1). To correct the overestimation of nonstatic effects in the approximation of the order $O(p'/M^2)$ in the $V(p', p)$, i.e., $V_w^a$, $V_{LL}^a$ and $V'_p^a$ are multiplied by a reduction parameter $\lambda = 0.5$. Details of the potential from each meson are given in Ref. 2) in which the radial dependence of the potential is described by the following Yukawa type functions:

$$E = e^{-x}, \quad Y = \frac{e^{-x}}{x}, \quad X = \left(\frac{1}{x} + \frac{1}{x^2}\right)Y, \quad Z = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)Y, \quad W = \frac{Z}{x^2}, \quad (4)$$

where $x = mr$ (hereafter $m = m_a$).

The Gaussian expansions of the functions in Eq. (4) are performed as follows. A series of the Gaussian range $\{r_n\}$ is chosen to be a geometric progression:

$$r_{n+1} = b \cdot r_n; \quad b = (r_n/r_1)^{1/[N-1]}, \quad (5)$$

where $N$ is the number of Gaussian basis functions. In our case $N = 10$, $r_1 = 0.315$ fm and $r_{10} = 7.806$ fm. However the four basis functions with $r_7 \sim r_{10}$ are not used in the cases of heavy mesons except $\pi$-meson, for the potential ranges by heavy mesons are much shorter than that by $\pi$-meson. Then, for example, the function $Y(mr)$ is expanded as

$$Y(mr) = \frac{\exp(-mr)}{mr} = \sum_{n=1}^{N} a_n \cdot \exp\left(-\frac{(r/r_n)^2}{2}\right). \quad (6)$$

The coefficients $a_n$ are determined by the $\chi^2$-search. To see accuracy of the Gaussian expansion, the exact Yukawa functions and the expanded ones for the $\pi$- and $\omega$-mesons are shown in Fig. 1. Excepting the short-range region ($r \leq 0.2$ fm), excellent results are obtained for the functions in Eq. (4) over all mesons.

Both potentials (R) and (NR) are constructed by $\sum_\alpha (v^\alpha_i + w^\alpha_i)$ and $\sum_\alpha v^\alpha_i$, respectively, where $\alpha = \pi, \eta, \rho, \omega, \delta, \sigma$. Using the Gaussian basis functions, the radial dependence of GOBEP has the following form:

$$U_i(r) = \sum_{n=1}^{N} A_n^i \cdot \exp\left(-\frac{(r/r_n)^2}{2}\right); \quad i = C, T, LS, W, LL, p, \quad (7)$$

where $A_n^i$ are the coefficients of the Gaussian expansion of GOBEP, which are related with the meson-nucleon coupling constants and the coefficients $a_n$ in Eq. (6). For example, $A_n^C$ is constituted by such term as $-m_\sigma g_\sigma^2 (1 - m_\sigma^2/4M^2) a_n$ which is the contribution from $\sigma$-meson. The reduction parameter $\lambda$ has been included in $A_n^i (i = W, LL, p)$.

Since there are some ambiguities of OBEP at the core region ($\leq 1$ fm), we treat them phenomenologically. First, the Gaussian function with the range $r_1$ is used as a cutoff function which makes $U_i = 0$ at $r = 0$. From Eq. (7), the radial part of GOBEP with the cutoff is expressed as follows:
Second, the soft core $U_c^0$ with the range $r_2 (=0.45 \text{ fm})$ is introduced into the central term:

$$U_i(r) = \sum_{n=1}^{N} A_n^i \cdot [\exp\{-(r/r_n)^3\} - \exp\{-(r/r_2)^3\}] .$$

(8)

and following modifications are made in the other terms:

$$U_i^0 = B_0^i \cdot \exp\{-(r/r_2)^3\} ; \quad i = C ,$$

(9)

Consequently, the GOBEP is written as follows:

$$V_{NN} = V_c + S_{12} V_T + (L \cdot S) V_{LS} + W_{12} V_w + L^2 V_{LL} - \frac{1}{M} \left( P^2 V_p + V_p P^2 \right) ,$$

(11)

$$V(x) = U_i(x) + U_i^0(x) = \sum_{n=1}^{N} C_n^i \cdot \exp\{-(r/r_n)^3\} ,$$

(12)

where $C_n^i$ consists of $A_n^i$ and $B_0^i$.

The meson-nucleon coupling constants and the core strength $B_0^i$ except $B_0^p$ which is fixed to zero, are determined by the $\chi^2$-search so as to reproduce the two-nucleon data. The coupling constants are common in both (R) and (NR). The numerical values, $M=938.9 \text{ MeV}$ and $\hbar c=197.327 \text{ MeV} \cdot \text{fm}$, are used in the calculations. The results are listed in Table I. The spin-$S$-isospin-$T$ states are represented as $^1O(S=0, T=0)$, $^1E(S=0, T=1)$, $^3E(S=1, T=0)$ and $^3O(S=1, T=1)$.

The expansion coefficients $C_n^i$ for (R) are shown in Table II and those for (NR) will be given in our next paper. The derivative of function $V_p(r)$ appearing in the Green's method is given with good accuracy as follows:

$$\frac{dV_p}{dr} = -2r \sum_{n=1}^{N} C_n^p \cdot \frac{1}{r_n^2} \exp\{-(r/r_n)^3\} .$$

(13)

Some radial dependence of both the potentials, (R) and (NR), is illustrated in
Table II. Coefficients \( C_n \) for GOBEP (R).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( C_n^c )</th>
<th>( C_n^{L^c} )</th>
<th>( C_n^p )</th>
<th>( C_n^c )</th>
<th>( C_n^{L^c} )</th>
<th>( C_n^p )</th>
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<tr>
<td>1</td>
<td>-0.60213E+4</td>
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<td>-0.671835E+1</td>
<td>-0.859211E+0</td>
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<td>0.56092E+4</td>
<td>0.244972E+3</td>
<td>0.128622E+1</td>
<td>0.139472E+4</td>
<td>0.145892E+2</td>
<td>0.391443E+0</td>
</tr>
<tr>
<td>3</td>
<td>0.99362E+3</td>
<td>0.163198E+2</td>
<td>0.128622E+1</td>
<td>0.139472E+4</td>
<td>0.145892E+2</td>
<td>0.391443E+0</td>
</tr>
<tr>
<td>4</td>
<td>0.14082E+2</td>
<td>0.827635E+0</td>
<td>0.571465E-2</td>
<td>0.382323E+3</td>
<td>0.405068E+1</td>
<td>0.159512E+0</td>
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<tr>
<td>5</td>
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<td>0.359194E+2</td>
<td>0.535470E+2</td>
<td>0.161642E+1</td>
<td>0.275487E-1</td>
<td>0.106675E+0</td>
</tr>
<tr>
<td>6</td>
<td>0.26360E+2</td>
<td>0.11998E+2</td>
<td>0.751465E+2</td>
<td>0.106675E+0</td>
<td>0.766688E+3</td>
<td>0.39522E-4</td>
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<tr>
<td>7</td>
<td>0.13617E-2</td>
<td>0.478875E-6</td>
<td>0.11998E+2</td>
<td>0.751465E+2</td>
<td>0.106675E+0</td>
<td>0.766688E+3</td>
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<tr>
<td>8</td>
<td>-0.26661E+5</td>
<td>0.838961E+3</td>
<td>0.278714E+4</td>
<td>0.143459E+3</td>
<td>-0.176805E+4</td>
<td>-0.843274E+0</td>
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<tr>
<td>9</td>
<td>0.35821E+4</td>
<td>0.853164E+3</td>
<td>0.246770E+4</td>
<td>0.132378E+3</td>
<td>-0.177131E+4</td>
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<td>10</td>
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<td>0.100066E+2</td>
<td>0.541183E+1</td>
<td>0.141554E+0</td>
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<td>11</td>
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<td>-0.155891E+1</td>
<td>0.283747E-2</td>
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<tr>
<td>12</td>
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<td>0.100080E+0</td>
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<td>0.250641E+0</td>
<td>0.240764E-1</td>
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<td>13</td>
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<td>0.100080E+0</td>
<td>0.373327E-3</td>
<td>0.250641E+0</td>
<td>0.240764E-1</td>
</tr>
<tr>
<td>14</td>
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<td>0.100080E+0</td>
<td>0.373327E-3</td>
<td>0.250641E+0</td>
<td>0.240764E-1</td>
</tr>
</tbody>
</table>

Figs. 2 and 3. Potentials for each state are expressed as \( 2^{s+1} V_P \), where \( P=(-)^L \).

The calculated phase shifts with (R) and (NR) are shown in Figs. 4~6. They are in good agreement with the results of phase shift analysis. The low energy parameters, deuteron binding energy \( E_d \), electric quadrupole moment \( Q_s \) and \( D \)-state probability \( P_o \) calculated with (R) and (NR) are listed in Table III.

In conclusion, we summarize as follows. 1) The Yukawa type functions from each meson are well reproduced by superposing the Gaussian basis functions. Then the OBEP is accurately described in terms of the Gaussian basis. 2) An advantage of
Fig. 2. $V_+^C$ and $V_{+T}^S$ of (R) (solid lines) and (NR) (dotted lines).

Fig. 3. $V_+^C$ and $V_{+T}^S$; notations are the same as in Fig. 2.

Fig. 4. Calculated phase shifts with (R) (solid lines) and (NR) (dotted lines) for $^1E$ and $^1O$.

Fig. 5. The same as in Fig. 4 but for the $^3E$.

Fig. 6. The same as in Fig. 4 but for the $^3O$.

Table III. Low energy parameters, $E_d$, $Q_d$ and $P_o$.

<table>
<thead>
<tr>
<th></th>
<th>GOBEP (R)</th>
<th>GOBEP (NR)</th>
<th>Exp. val.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\alpha_p$ (fm)</td>
<td>-23.755</td>
<td>-23.741</td>
<td>-23.748(10)</td>
</tr>
<tr>
<td>$^1\gamma_p$ (fm)</td>
<td>2.628</td>
<td>2.617</td>
<td>2.75(5)</td>
</tr>
<tr>
<td>$^3\alpha$ (fm)</td>
<td>5.433</td>
<td>5.419</td>
<td>5.419(7)</td>
</tr>
<tr>
<td>$^3\gamma$ (fm)</td>
<td>1.783</td>
<td>1.764</td>
<td>1.754(8)</td>
</tr>
<tr>
<td>$E_d$ (MeV)</td>
<td>-2.22445</td>
<td>-2.22472</td>
<td>-2.224575(9)</td>
</tr>
<tr>
<td>$Q_d$ (fm$^2$)</td>
<td>0.270</td>
<td>0.264</td>
<td>0.2860(15)</td>
</tr>
<tr>
<td>$P_o$ (%)</td>
<td>5.550</td>
<td>5.325</td>
<td>4~7</td>
</tr>
</tbody>
</table>
the Gaussian expansion is that the strength of potential at an arbitrary range is locally adjustable. In fact the two-nucleon data have been well reproduced by adjustment of the Gaussian basis with the range \( r_2 (=0.45 \text{ fm}) \), where the short-range part of our OBEP is modified. 3) We emphasize that \( ^3V^+_{w} \) and \( ^3V^+_{LL} \) influence both the agreement of \( \delta(\pi D_2) \) and \( \epsilon_1 \) with experimental values.