Strömgren Sphere in the Expanding Universe and a Possible Seed of Structure Formation

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Formation of the Strömgren sphere by a massive star in the early universe after the recombination is studied. The quasi-stationary Strömgren sphere is formed in the stages of $z>10^{-2}$ for typical parameters of the universe model. The electron temperature $T_e$ in the HII region is kept equal to the cosmic background radiation temperature $T_r$ by the Compton process. After this epoch, the ionisation front expands continuously and the quasi-stationary Strömgren sphere is not formed. $T_e$ can increase above $T_r$ in this later stage. Transition feature is investigated in an analytic manner. In the case of the quasi-stationary Strömgren sphere, the HII region may generate a dense shell by the expansion due to pressure unbalance and a fragmentation of this shell could trigger further structure formation. This process occurs only for the small HII region.

§ 1. Introduction

We have little knowledge about the dark age of the early universe between the recombination at the redshift factor $z_{rec}\sim 10^3$ and the high redshift QSOs at $z_{QSO}\sim 5$. Highly ionized state of the intervening medium to these QSOs has suggested the early activity of the subgalactic objects including QSO themselves. Thus the formation of the energetic sources in this dark age is highly probable. The effects of the subgalactic energetic sources could be large even if their mass fraction is small.

The early formation of such objects is possible from exceptionally high peaks of the density fluctuation. For example, if we extrapolate the COBE-DMR amplitude down to $10^9 M_\odot$ according to the CDM spectrum and employ the Press-Schechter formalism, the mass fraction of collapsed objects amounts to $10^{-3}$ at $z=10^5$ with $h=0.5$. Anyhow, whatever we assume the CDM scenario or any other like the isocurvature perturbation scenario, the formation of the pre-galactic gas clouds greater than the Jeans mass should start just after the recombination. What is unclear is the mass fraction of the collapsed objects.

Due to formation of $H_2$ molecules and the resulting cooling by them in the collapsing clouds, they may fragment into ordinary mass stars. These stars will affect the enrichment of heavy elements, the heating of the medium, the generation of gas motion and density fluctuation via the Strömgren sphere and thus trigger the galaxy formation as the second generation objects.

One of the interesting effects is a biasing mechanism, breeding and/or suppressing of further star formation. The breeding mechanism might accelerate the star formation how small the mass-fraction of the primordially collapsed objects is. Here, we are interested in the formation of the subgalactic objects and not in the formation of the large-scale structure of the galaxy distribution, which may be due to the hierarchical clustering by gravity.

With this motivation, we investigate the effects of the ionisation photon flux on
the surrounding medium in the dark age of the early universe. The effect of the O, B stars in the interstellar medium is the formation of the Strömgren sphere and the breeding of star formation.\textsuperscript{8} The HII region heated by the photo-ionisation expands, forms the shocked shell around it and triggers the star formation by fragmentation of the shell. In this mechanism, the ionisation photon flux is utilized to form a thermally inhomogeneous state, which induces the density inhomogeneity via gas motion.

We seek for the condition under which such process would occur by assuming a radiation flux arbitrarily. At their last stage, massive stars will explode and affect also their surroundings by the kinetic energy.\textsuperscript{9} The so-called explosion scenario of the large-scale structure,\textsuperscript{10} takes into account this aspect only.

The locality of the photoionisation has been discussed by considering the overlapping of the Strömgren spheres in the expanding universe.\textsuperscript{11} The treatment of the uniform heating\textsuperscript{12,13} is justified in such a later stage as \( z < 10 \), where the ionisation photons are not blocked, the uniform ionisation of the medium will simply proceed and the kinetic temperature reaches \( \sim 10^4 \text{ K} \).

It is found that the quasi-stationary Strömgren sphere is formed only in such an early stage as \( z > 10^{1.2} \). In the stage before \( z \sim 10^2 \), the kinetic temperature in the HII region has been cooled to the temperature of the cosmological background radiation by the Compton process. Therefore the kinetic temperature does not stay constant and is far below \( 10^4 \text{ K} \) in contrary to the assumption in Ref. 11). This transition feature will be analysed.

Due to this Compton cooling in the HII region, its expansion will be moderate and the breeding of stars becomes rather difficult, although still marginally possible taking into account ambiguity of the various parameters.

In § 2, the basic equation for the motion of initial ionisation front is investigated and the evolution of the ionisation front is discussed in § 3. We will make lucid the key parameters involved in this process. In § 4, the cosmological application of the above results is discussed. In § 5, the evolution of the electron temperature under the photoionisation heating is discussed: the evolution of \( T_e \) exhibits a bouncing at \( z \sim 10^2 \) and the minimum temperature is \( \sim 10^2 \text{ K} \). In § 6, we discuss briefly the effect on the surrounding medium: the expansion of HII region by pressure unbalance and the mass fraction of the affected region by the Strömgren sphere.

§ 2. Initial ionisation front

If we define \( S(r, t) \) as the total number of photons flowing through the spherical shell of a radius \( r \) from the star per second with a frequency greater then the Lyman limit, it satisfies the

\[
\frac{\partial S(r, t)}{\partial r} = -4\pi r^2 x^2 n(t)^2 a^{(2)}, \tag{2.1}
\]

where \( a^{(2)} \) is the hydrogen recombination coefficient excluding direct captures to the ground level, \( x \) is the degree of ionisation, and \( n(t) \) is the hydrogen density.

Now we consider a radius of “initial” ionisation front \( r_{\text{i}}(t, t_{\text{i}}) \) which has started from the star at \( t_{\text{i}} \).\textsuperscript{8} The speed of the ionisation front is estimated as a rate of
inclusion of the neutral atoms within \( r_i \),

\[
\frac{dr_i(t, r_i)}{dt} = \frac{S(r_i, t)}{4 \pi r_i(t, r_i)^2 n(t)}. \tag{2.2}
\]

Then combining (2·1) and (2·2), we have

\[
\frac{dr_i}{dt} \frac{\partial S(r_i)}{\partial r_i} = \frac{dS(r_i)}{dt} = -x^2 n(t) a^{(2)} S(r_i). \tag{2·3}
\]

If \( x=1 \), we obtain

\[
S(r_i(t, t_i)) = S(0)e^{-r(t)} \tag{2·4}
\]

with

\[
\Gamma(t) = \int_t^\infty a^{(2)} n(t') dt'. \tag{2·5}
\]

Substituting (2·4) into (2·2), we obtain

\[
\frac{4\pi}{3} r_i(t, t_i)^3 = \int_t^\infty \frac{S(0)}{n(t')} e^{-r(t')} dt'. \tag{2·6}
\]

The initial ionisation front defined by (2·3) does not coincide with the ionisation front formed around the continuously shining star. This \( r_i(t, t_i) \) will give a clue to the behaviour of the ionisation front, on which we will touch in the last part of § 3.

In order to rewrite (2·6), we introduce the velocity defined by

\[
V_i = \left( \frac{3S(0)}{4\pi n(t)t^2} \right)^{1/3} = \left( \frac{9S(0)}{2\Omega_0 S_*} \right)^{1/3} c \tag{2·7}
\]

with

\[
S* = c^3/Gm_p = 10^{21.4} \text{ sec}^{-1}, \tag{2·8}
\]

\( m_p \) being proton mass. In the last expression of (2·7), we have used the cosmological relation \( n(t)t^2 = \Omega_0/6\pi Gm_p \) assuming the Einstein-de Sitter model with baryon density parameter of \( \Omega_0 \). In the following, we will assume this universe model and a pure hydrogen matter for simplicity.

If we take the flux corresponding to the Eddington luminosity with the stellar mass of \( M \) as

\[
S(M) = \frac{4\pi GMm_p c}{\sigma\chi_i},
\]

its ratio to the critical flux \( S_* \) is expressed by the fundamental physical constants as

\[
\frac{S(M)}{S_*} \sim a_r^{-7} \left( \frac{l_p}{\hbar/m_e c} \right) \frac{M}{M_{\text{Ch}}},
\]

where \( a_r = e^2/\hbar c \) is the fine structure constant, \( \chi_i = 13.6 \text{ eV} \) is the ionisation energy of hydrogen, \( l_p \) is the Planck length and \( M_{\text{Ch}} \) is the Chandrasekhar mass. Then, \( V_i \sim c \) for
\[ M \sim \alpha_f \left( \frac{\hbar}{m_e c} \right) M_{\text{ch}} \sim 10^8 M_\odot. \]  

(2.9)

If we normalize \( S \) by a typical flux of O, B stars, \( V_i \) is written as

\[ V_i = 10^{13} \text{ km sec}^{-1} \left( \frac{S(0)}{10^{48} \text{ erg sec}^{-1}} \right)^{1/3} \left( \frac{0.1}{Q_6} \right)^{1/3}. \]

Then \( r_1 \) is now written from (2.6) as

\[ r_1(t, t_i) = V_i t_i \Phi(t/t_i)^{1/3} \]  

(2.10)

with

\[ \Phi(\xi) = \int_1^\xi y^2 e^{-r(y)} dy. \]  

(2.11)

Furthermore, if we introduce the Strömgren radius for a static background density of \( n(t) \) as

\[ r_{\text{St}}(t) = \left( \frac{3S(0)}{4\pi a_{\text{St}}^3 n(t)^2} \right)^{1/3}, \]  

(2.12)

\( r_1 \) is rewritten also as

\[ r_1(t, t_i) = r_{\text{St}}(t_i) [\gamma(t_i) \Phi(t/t_i)]^{1/3} \]  

(2.13)

with

\[ \gamma(t) \equiv n(t) a_{\text{St}} t. \]  

(2.14)

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**§ 3. Evolution of the ionisation front**

We investigate evolution of \( r_1(t, t_i) \) described by (2.12). As we will discuss later in § 5, the electron temperature \( T_e \) in the HII region is kept equal to the temperature \( T_r \) of the cosmic background radiation due to the Compton cooling up to the redshift factor \( z \sim 10^2 \) and then \( T_e \) begins to increase in the later stage.

The recombination rate \( a^{(3)} \) has a form

\[ a^{(3)}(T_e) = a_0 \phi(T_e) \frac{T_e}{T_e^{1/2}} \propto t^{\beta}, \]

where \( \phi \) is a slowly changing function of \( T_e \), \( a_0 \approx 2.06 \times 10^{-11} \text{ cm}^3 \text{ sec}^{-1} \) and \( \phi \sim 3 \) in \( T_e \sim 10^2 \text{ K} \), \( \beta \approx 1/3 \) for \( T_e = T_r(t) \) and \( \beta < 0 \) in the later stage. Then, writing \( y = t/t_i \) and \( \gamma_i = \gamma(t_i) \),

\[ \Gamma(y) = \gamma_i + \frac{1}{1 - \beta} \left( 1 - \frac{1}{y^{1-\beta}} \right). \]  

(3.1)

Therefore \( r_1(t, t_i) \) is a function of \( t/t_i \) and \( \gamma_i \).

For \( \gamma_i = 0.1 \sim 10 \), the evolution of \( r_1(t, t_i) \) is shown in Fig. 1. The general feature is not so sensitive to \( \beta \). The curves show how the expansion velocity increases with the decrease of \( \gamma_i \). The behavior of \( \Phi(\xi) \) is mainly governed by \( \gamma_i \) and the dimen-
sional quantities are determined by $V_1$, $\gamma_1$ and $t_1$, those are the key parameters involved in this process.

For the limiting cases of $\gamma_1$, its behaviour is as follows: For $\gamma_1 \gg 1$, $r_1(t, t_i)$ approaches quickly $r_{s1}(t_i)$ as

$$r_1(t, t_i) \approx r_{s1}(t_i) \left( 1 - e^{-n(t - t_i)/t} \right)^{1/3}$$

(3·2)

and it is expanding still with velocity $V_1 \sim V_1 e^{-n(t_{1/3}/t)^{1/3}}$, which is very small for $\gamma_1 \gg 1$. For $\gamma_1 \ll 1$, on the other hand, it is shown that

$$r_1(t, t_i) \approx v_1 t \sim V_1 e^{-n(t_{1/3}/t)^{1/3}} \cdot 3^{1/3}$$

(3·3)

In the interstellar medium, the initial ionisation front is known to follow the relation (3·2), where the built-up time of the Strömgren sphere is $t_i/\gamma_1 = 1/n(t_i)a^{(3)}$.

The ionization front $R(t)$ formed around a continuously shining star during a finite period will be obtained by solving a radiation transfer process. We guess, however, that $R_1(t)$ is approximately given as

$$R_1(t) \approx \text{Max}\{r_1(t, t_i); t_i < t\},$$

(3·4)

that is, $R_1(t)$ is given by the locus of the curves $r_1(t, t_i)$ with $t_i < t$. According to this,

$$R_1(t) \approx r_{s1}(t) \quad \text{for} \quad \gamma(t) \gg 1,$$

$$R_1(t) \approx V_1 t/3^{1/3} \quad \text{for} \quad \gamma(t) \ll 1.$$  

Then we may say that the Strömgren sphere is formed in a quasi-stationary way in the stages of $\gamma(t) > 1$. In the stages of $\gamma(t) < 1$, however, the ionization photons are not blocked and the ionisation of the medium just spreads, by which the motion of gas and the density contrast are never generated. The treatment of the uniform ionisation and heating in the early universe is justified only in this later stage.

§4. Application to cosmological situation

As seen in §3, the key parameter is $\gamma(t)$ of (2·14). Taking $\phi=3$,

$$\gamma(t) = n(t)a^{(2)}t = 10^{1.8}(z+1)\left(\frac{T_r}{T_\phi}\right)^{0.8}(\hbar\Omega_\phi),$$

(4·1)
h being the Hubble constant $H_0$ in units of $100 \text{ km/sec/Mpc}$. It is crucial to know the evolution of $T_e$, which we will discuss in the next section. The condition of $\gamma(t) > 1$ is well satisfied for $z > 10^{1.5}$ even if the universe model parameter $hQ_b$ is as small as 0.05 and even if the departure from $T_e = T_r$ is taken into account.

We also note that the time scale around $z \sim 10^2$ is

$$t = 10^{6.8} h^{-1} \left( \frac{10^2}{1 + z} \right)^{1.5} \text{years}.$$  
\[(4.2)\]

Since we are considering a massive star as the ionisation photon source, its lifetime $t_{\text{star}}$ is the same order of the age of the universe at $z \sim 10^2$. For the Eddington model, which would be correct for $M > 10^2 \ M_\odot$, the lifetime is

$$t_{\text{Edd}} \simeq \epsilon \frac{\sigma T}{4 \pi GM P} = \epsilon \frac{10^{-3}}{10^{4.8}} \text{years},$$  
\[(4.3)\]

$\epsilon$ being the fraction of burned mass. For O, B stars, the main sequence life is $\sim 10^{-4}(10 \ M_\odot/M)$ years for $M = 10^{-2} \ M_\odot$. Then, all the early-born stars with $M < 10^2 \ M_\odot$ will survive down to $z \sim 10^2$ but, in $z \ll 10^2$, the life is smaller than the expansion time of the universe.

For the Eddington luminosity of mass $M$, the ionisation photon flux is the order of

$$S_{\text{Ed}} \simeq 10^{81} \frac{M}{10^2 \ M_\odot} \frac{X_t}{h \nu} \text{sec}^{-1},$$  
\[(4.4)\]

where $\nu = \langle \nu S(\nu) \rangle / S$ and $\langle \rangle$ denotes the integration over the photon energy $\nu$ and $S$ being the total ionization photon flux. The Strömgren radius for the Eddington luminosity is estimated as

$$r_{\text{str}} \simeq \frac{10^{1.5}}{h^4 Q_b^2} \left( \frac{M}{10^2 \ M_\odot} \right)^{1/3} \left( \frac{10^2}{1 + z} \right)^{1/3} \left( \frac{T_e}{T_r} \right)^{1/3} \left( \frac{X_t}{h \nu} \right)^{1/3} \text{pc}.$$  
\[(4.5)\]

When we write down the relation (2.1), the quasi-equilibrium between ionization and recombination has been assumed as follows:

$$(1 - x) n(t)/\sigma_t(\nu) S(\nu) / 4 \pi r^2 = x^2 n(t)^2 a^{(2)},$$  
\[(4.6)\]

where $\sigma_t(\nu)$ is the photo-ionisation cross section. The condition for the quasi-equilibrium is just given by $\gamma(t) > 1$ under the additional condition of sufficient ionisation up to the Strömgren radius, which leads to the minimum flux condition as

$$S \frac{36 \pi x}{\sigma_t n} \frac{a^{(2)}}{1 - x} \frac{T_b}{(1 + z)^3} (h^2 Q_b)^{-1} \text{sec}^{-1},$$  
\[(4.7)\]

taking $a^{(2)} \sim 10^{-11} \text{ cm}^3\text{sec}^{-1}$ and $\sigma_t \sim 10^{-18} \text{ cm}^2$. This condition is well satisfied for massive stars for $z > 10$ even if $h^2 Q_b$ is as small as 0.025.
§ 5. Energy balance in the HII region

By the photo-ionisation, the electrons get the average energy given by

\[ \varepsilon = \frac{\langle \sigma \nu S(\nu)(h\nu - \chi) \rangle}{\langle \sigma \nu S(\nu) \rangle}, \]  \hspace{1cm} (5·1)  

and the photo-ionisation acts as the heating source \( I_{\text{PI}} \).

The ionized matter is cooled by the Compton process \( \Lambda_{\text{Com}} \) and the free-free process \( \Lambda_{\text{ff}} \). The energy balance condition, \( I_{\text{PI}} = \Lambda_{\text{Com}} + \Lambda_{\text{ff}} \), is written as

\[ I_{\text{PI}} \left( 1 - \frac{kT_e g_{\text{ff}}}{\varepsilon \phi(T_e)} \right) = \Lambda_{\text{Com}} \]  \hspace{1cm} (5·2)  

with \( g_{\text{ff}} \) being the gaunt factor of the free-free process. Since \( kT_e / \varepsilon < 1 \) in most cases, we neglect the free-free cooling for a while. The condition \( I_{\text{PI}} = \Lambda_{\text{Com}} \) is rewritten as

\[ T_e^{1/2} [T_e - T_{r0}(1+z)] = B \phi(T_e) \frac{T_e}{1+z}, \]  \hspace{1cm} (5·3)  

where

\[ B = \frac{A}{\alpha r^2} \frac{\varepsilon \chi l^{1.5}}{k^{2.5} T_{r0}^{0.5}} \left( \frac{n_e}{n_\gamma} \right). \]  \hspace{1cm} (5·4)  

\[ A = \frac{5(2/\pi)^4 (6/\pi)^{0.5} \chi(3)}{0.96}, \]  

\( n_e/n_\gamma \) is the cosmic baryon number and \( T_{r0} = 2.7 \text{ K} \).

Equation (5·3) is a quadratic equation of \( (1+z) \) and it is solved as

\[ 1+z = T_e \pm \sqrt{T_e^2 - 4BT_{r0} \phi(T_e) / T_e^{0.5}} \frac{T_e}{2 T_{r0}}. \]  \hspace{1cm} (5·5)  

Therefore, to be a real solution, \( T_e \) must be

\[ \frac{T_e}{\phi(T_e)^{0.4}} \geq T_e \]

\[ = \left( \frac{4A}{\alpha r^2} \left( \frac{n_e}{n_\gamma} \right)^{0.4} (\varepsilon \chi l^{1.5})^{0.4} \right) \frac{k}{\varepsilon} \]

\[ \approx 10^{2.8} \left( \frac{\varepsilon}{\chi l} \right)^{0.4} \left( \frac{h^2 \Omega_B}{0.025} \right)^{0.4} \text{ K}. \]  \hspace{1cm} (5·6)  

Since \( \phi(T_e) \) is slowly decreasing with the increase of \( T_e \) and its value is the order of unity, \( T_e \) is regarded as the minimum temperature, which is realized at

\[ 1+z_e = \frac{T_e}{T_{r0}} \]  

Fig. 2. Evolution of the electron temperature in the HII region. The temperature and the scale factor of the universe expansion are normalized by the minimum temperature \( T_e \) defined by (5·6) and the temperature bounce epoch \( z_e \) defined by (5·7), respectively.
The plus sign solution of (5·5) applies for \( z \geq z_c \) and the minus sign solution does for \( z \leq z_c \). In the early period of \( z > z_c \), the electron temperature has been kept equal to \( T_e(t) \) due to the strong Compton cooling. But after \( z_c \), the electron temperature begins to increase because the heating dominates over the Compton cooling. This feature is shown in Fig. 2. From (5·5), it is found that \( \beta \) in (3·1) is \( \beta \sim 1/3 \) for \( z > z_c \) and \( \beta \sim -2/9 \) for \( z < z_c \).

The above argument is correct if \( kT_e \ll \epsilon \) and the above result about the bounce of temperature is well justified. For the later period of \( z \ll z_c \), the estimation of temperature must be modified by including the free-free process. By this process, \( T_e \) will be limited to \( T_e < 10^4 \) K.\(^{12,13}\)

§ 6. Effects on the surrounding medium

(a) Expansion of the HII region

In the interstellar medium, the Strömgren sphere expands due to the pressure unbalance and the dense shell is formed at the boundary. This process is especially interesting because it can generate the density contrast in the gas via non-gravitational way. This has been totally neglected in the uniform heating treatment.\(^{12,13}\) Here we investigate the condition of this process to occur for the Strömgren sphere in the early universe. To see a direct comparison, we restrict our discussion to the quasi-stationary Strömgren sphere of \( \gamma(t) > 1 \) (see (4·1)).

As is well known in interstellar space where the outer density \( n_1 \) stays constant, the radius increases as\(^{8}\)

\[
R(\tau) \approx R(0) \left[ 1 + \frac{\tau}{\tau_e} \right]^{4/7},
\]  

(6·1)

where \( \tau \) is the time since the stationary HII region has been formed and

\[
\tau_e = \frac{4}{7} \frac{R(0)}{v_{th}},
\]  

(6·2)

\( v_{th} \) being the thermal velocity in the HII region. This is derived from the relation \( R^3 n_H^2 \propto S \) (see (2·12)) and \( dR/d\tau = v_{th}(n_H/n) \)\(^{6,5}\) assuming that the inner pressure sufficiently dominates over the outer one.

The above expansion law holds when \( v_{th} > v_1 \) (see (3·3)). Considering the cases \( \gamma(t) > 1 \), we will take as \( R(0) = r_{th}(t) \) for the Strömgren sphere formed at \( t \).

The temperature in the HII region in the quasi-stationary Strömgren sphere in \( z > z_c \) is only slightly greater than the outer one \( T_l \) which has decreased as \( T_l \propto (1 + z)^2 \) since recombination. In such a case, it is not well justified to neglect the outer pressure. Then the estimation by (6·2) should be taken as the maximal expansion. In \( z < z_c \), however, the inner pressure becomes large enough but the other conditions become difficult to be satisfied. Therefore more detailed analysis is necessary for \( z < z_c \).
Since the expansion law (6·1) holds in \( t/t < 1 \), the maximum expansion by the pressure unbalance will be appreciable only if \( t/t > 1 \). The condition of \( \tau_e = r_{st}(t)/v_{th} < t \) is rewritten as

\[
S < S_{**} \gamma(t),
\]

where

\[
S_{**} = \frac{v_{th}^3}{G M_p} \sim 10^{46} \text{ erg sec}^{-1} \left( \frac{T_e}{10^4 \text{ K}} \right)^{1.5}.
\]

It is interesting to compare this critical flux with the another critical flux \( S_* \) defined in (2·8). The existence of the maximum \( S \) will be understood as follows: to get a sufficient expansion within the cosmic time (\( \tau_e < t \)), the radius \( r_{st}(t) \) must be small enough because \( v_{th} \) is fixed.

The condition (6·3) is rather discouraging to have a sufficient expansion for the \( \text{Strömgren sphere} \) of O, B stars in \( z < 10^2 \). The above consideration has suggested that the interesting stage in this respect is around \( z \sim 10^2 \) and a dynamic analysis including various processes together is necessary. This expansion is one of the necessary conditions for the breeding of stars.

(b) **Mass fraction affected by primordial stars**

Another interesting factor for the breeding of stars is the multiplicity of the breeding. One luminous source with mass \( M \) can form the HII region with mass \( M_{st} = 4\pi \rho(t) r_{st}(t)^3/3 \) and this mass gives also the order of magnitude of the compressed mass by the expansion. If we take the flux as \( fS_{60} \), the ratio of these masses is

\[
\frac{M_{st}}{M} = \frac{4\pi f G M_p^2 c}{n(t) a(t)^2 \sigma_{th} \epsilon} \sim 10^7 \left( \frac{10^2}{1 + z} \right)^{2.5} \left( \frac{T_e}{T_r(t)} \right)^{0.5} \frac{\chi_I}{\epsilon\Omega_b h^2}.
\]

To get the breeding trigger for the further generation of stars, this ratio should be \( M_{st}/M > 1 \) at least. Although it depends on various uncertain parameters, this has suggested that the latter stages like \( z \sim z_c \) seem more favorable for the breeding. However, in the later stage of \( z < z_c \), the conditions such as the HI medium, the quasi-stationary \( \text{Strömgren sphere} \) \( (\gamma(t) > 1 \text{ and } v_{th} < v_{t}) \) and the stationary source \( (t_{star} > t) \) will break down immediately. Therefore more detailed analysis combining all these effects is necessary for stages like \( z \sim 10^{1-2} \).

If we denote the mass fraction of the collapsed objects by \( z \) with mass larger than \( M \) as \( C(\geq M) \), the mass fraction covered by the HII region is \( P_H = C(\geq M) M_{st}/M \). If this number is larger than unity, the shells may collide with each other, it will accelerate the star formation but the further breeding would be suppressed. We can estimate \( C(\geq M) \) by the Press-Schechter formalism.\(^{14}\) If we take the Harrison-Zeldovich spectrum normalized by the COBE-DMR data\(^{15,16}\) and extrapolate it down to low mass using the CDM transfer function,\(^{17}\) \( C(\geq 10^2 M_\odot) \sim 10^{-3} \) at \( z \approx 10^2 \) for \( \Omega = 1, h = 0.5.\(^{18}\) (We notice that this value is rather sensitive to \( h \).) Here \( M \) is the total CDM mass and the baryon mass is \( \Omega_b M \). Therefore, within the ambiguity of the parameters, it seems marginal whether the mass fraction of stars including the breeding effect is the order of unity or tiny one.
References

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