Chiral Lagrangian with Higher Resonances and Flavour $SU(3)$ Breaking

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A chiral Lagrangian with $SU(3)$ breaking and higher resonances is proposed. In this model, the $SU(3)$ breaking structure in vector-pseudoscalar sector is realized with the decay constants of pseudoscalar mesons and the masses of vector mesons used as inputs. We examine whether the resulting $SU(3)$ breaking effect in the charge radii of pseudoscalar mesons is consistent with the experimental facts.

§ 1. Introduction

Quantitative prediction of rare $K$ decay rates based on the standard model is a necessary ingredient in the program for understanding the origin of $CP$ violation. However, since the typical energy scale in $K$ decays $\approx M_K (\approx 500$ MeV) is too low for perturbative QCD to be applicable, we need some method to estimate the long distance contributions from QCD. The chiral symmetry imposes strong constraints on low energy dynamics associated with $K$ decays.$^{1-7}$

In the usual formalism of chiral dynamics, the energy dependence of the amplitudes involving Goldstone bosons ($\pi, K$) is given as a power series expansion with respect to the external momenta and quark masses. The coefficients of $O(p^4)$ terms in the effective chiral Lagrangian are explicitly determined from the experimental data.$^8$ Later it has been recognized that these coefficients of $O(p^4)$ terms are saturated by the contributions from higher resonances (vector, axial vector and scalar mesons).$^4$ The importance of incorporating higher resonances can also be seen in the result that the $\Delta I = 1/2$ rule is explained if we take the propagation of scalar resonances into account.$^7$

These results motivate us to the present work: in this article we propose a model for the chiral Lagrangian with scalar and vector mesons.

In QCD the $SU(3)$ breaking effect is introduced by the quark mass term $\mathcal{L}_m = -\overline{q}m(0)q + \text{h.c.}$ In chiral dynamics, the scalar field plays the key role; it develops the vacuum expectation value (VEV) and induces the decay constants for pseudoscalar mesons. We shall see that the explicit $SU(3)$ breaking originating from current quark masses result in the differences among decay constants. We introduce vector meson following the hidden local Lagrangian approach.$^9$ The $SU(3)$ breaking effects in vector-pseudoscalar couplings are generated by those of decay constants of pseudoscalar mesons and the masses of vector mesons. The explicit form of the potential terms which gives the connection between the VEV's of scalars and current quark masses will be shown in § 2.

Let us consider the expansion of the full Lagrangian in powers of derivatives. At the order $\partial^2$ level, we have the kinetic term of pseudoscalars and the mass term of
vector mesons. There are an infinitely large number of terms which are consistent with chiral and hidden local symmetries. This is because arbitrary higher dimensional operators of order $\partial^2$ are allowed in our approach. In § 3 we argue first that we can restrict ourselves to four-dimensional operators since we are constructing an effective Lagrangian which is valid below the cutoff scale $\Lambda \approx 4\pi F_\pi \approx 1$ GeV.$^{10}$

For a phenomenological test of this model we calculate the electromagnetic form factors for pseudoscalar mesons and the $K_{e3}$ form factor in § 4. There, the charge radii of $\pi^+$, $K^+$ and $K^0$ will be compared to their experimentally measured values. We will also find that the normalization of $K_{e3}$ form factor $F_+(0) \approx 1$ under fairly natural assumptions. In § 5, we examine the possible effects to charge radii from the extra terms which have not been included tentatively in the reduced chiral Lagrangian. It is shown that these additional terms do not modify the results obtained above. Section 6 is devoted to discussion and a summary.

§ 2. Chiral Lagrangian

2.1. Preliminaries

Let us begin our discussion with recalling the way one derives the chiral perturbation theory.

First we consider $U(N)_L \times U(N)_R$ linear $\sigma$ model$^{11}$ which is defined by the Lagrangian$^{12,13}$

$$\mathcal{L} = \text{Tr}(\partial_\mu M^\dagger \partial^\mu M) + \mu^2 \text{Tr}(M^\dagger M) - \frac{\lambda_1}{4} \left(\text{Tr}(M^\dagger M)\right)^2 - \frac{\lambda_2}{4} \text{Tr}((M^\dagger M)^2)$$

$$+ \frac{1}{4 G_1} \text{Tr}(m^{(0)}(M + M^\dagger)),$$  \hspace{1cm} (2.1)

where $m^{(0)} = \text{diag}(m_u^{(0)}, m_d^{(0)}, \ldots)$ denotes current quark mass matrix, and the last term breaks chiral symmetry explicitly. Using a unitary matrix $\xi$ and a hermitian matrix $\Sigma$, we can decompose $N \times N$ complex matrix $M$ in the form

$$M = \xi \Sigma \xi^\dagger.$$  \hspace{1cm} (2.2)

The positivity of $\mu^2$ signals the occurrence of spontaneous breakdown of chiral symmetry. This can be seen by examining the vacuum expectation value (VEV) $\langle \Sigma \rangle$ of $\Sigma$ which is determined by minimizing the potential

$$V(\langle \Sigma \rangle) = -\mu^2 \text{Tr}(\langle \Sigma \rangle^2) + \frac{\lambda_1}{4} \left(\text{Tr}(\langle \Sigma \rangle^2)\right)^2 + \frac{\lambda_2}{4} \text{Tr}(\langle \Sigma \rangle^4) - \frac{1}{2 G_1} \text{Tr}(m^{(0)} \langle \Sigma \rangle).$$  \hspace{1cm} (2.3)

$^*)$ For simplicity, anomaly terms which explicitly break the conservation of $U(1)_A$ current are neglected. Anomalous decays including vector mesons have been treated, for example, by Ö. Kaymakcalan and J. Schechter, Phys. Rev. D31 (1985), 1109.
Then the associated Goldstone bosons $\pi^a (a=0, 1, \cdots, N^2-1)$ are non-linearly realized in $\xi$ as

$$\xi = \exp \left( i \sum_{a=0}^{N^2-1} \frac{\pi^a T^a}{F^a} \right)$$

(2.3)

with dimension-one constants $F^a (a=1, \cdots, N^2-1)$.

The lowest order chiral Lagrangian used in chiral perturbation theory can be obtained in the infinitely heavy $\Sigma'$ limit, where $\Sigma' = \Sigma - \langle \Sigma \rangle$ is the dynamical degree of freedom associated with the $\Sigma$ field. Remember that the systematic chiral expansion is a power series expansion in the external momenta and quark masses. From the potential (2.2) we can see that the splitting in the components of diagonal matrix $\langle \Sigma \rangle$ is higher-order. Hence $F^a = F_\pi (a=1, \cdots, N^2-1)$, and by setting $\langle \Sigma \rangle = (F_\pi / 2) \mathbb{I}$ we obtain

$$\mathcal{L} = \frac{F_\pi^2}{4} \{ \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + 2B_0 \text{Tr}(m_q^0(U + U^\dagger)) \} + \cdots,$$

where $U = \xi^a$, $B_0 = 1/(4G_F F_\pi)$ and the ellipsis contains the higher-order terms in chiral expansion, the irrelevant constant terms and the terms involving physical scalar fields.

Now let the coupling $\lambda_i$ and $\lambda_s$ go to infinity with $F_\pi$ kept fixed. Then scalar particles become so heavy that they decouple effectively. Hence, in this limit, only Goldstone bosons survive and their low energy behaviour is described by the well-known leading order Lagrangian

$$\mathcal{L} = \frac{F_\pi^2}{4} \{ \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + 2B_0 \text{Tr}(m_q^0(U + U^\dagger)) \} .$$

(2.4)

Consider the case of two quark flavours ($u$ and $d$) and apply the corresponding chiral perturbation approach. Then the chiral expansion is in powers of external momenta and the masses of these quarks which are about 10 MeV. Since the expansion parameter must be dimensionless, we have to divide the typical momentum scale ($p$) and the masses of quarks by some constant $\Lambda$ with mass dimension. This $\Lambda$ is considered to be the cutoff scale under which our effective chiral description is valid, and is about $4\pi F_\pi \approx 1.2$ GeV\(^10\) where $F_\pi \approx 93$ MeV is the decay constant of $\pi$ meson. Hence we expect, in this case, that the convergence of this expansion will be good as long as all external momenta are sufficiently smaller than $\Lambda(p^2/\Lambda^2 \lesssim 2m_u/\Lambda \approx \times 10^{-2})$.

For the purpose of describing $K$ mesons within the framework of chiral dynamics, we have to extend the above framework such as to include strange quark. In this case one more expansion parameter $(2m_s/\Lambda) = 0.18 \sim 0.36$ for $m_s = 100 \sim 200$ MeV has to be taken into account. Also, in $K$ decays, the magnitudes of the typical momenta are the order of $K$ mass $\approx 500$ MeV. Thus we have $(p/\Lambda)^2 \approx (M_K/\Lambda)^2 \approx 0.21$. Hence the expansion may not converge so rapidly as in the case of $SU(2)_L \times SU(2)_R$.

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\*1) $T^a (a=1, \cdots, N^2-1)$ are the generators of the fundamental representation of $SU(N)$, and $T^0$ denotes $U(1)$ generator; $T^0 = (1/\sqrt{2N})(1, \cdots, 1)$. All these matrices should be normalized such that $\text{Tr}(T^a T^b) = (1/2) \times \delta^{ab} (a, b = 0, 1, \cdots, N^2-1)$. 

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Let us give one such example. One of the successful challenge is the explanation of the large enhancement of $\Delta I=1/2$ component in the amplitude for $K^0\to2\pi$ by incorporating scalar resonances.\footnote{The argument holds when we introduce the width for the $\sigma$ particle.} There, the enhancement factor arises from the scalar ($\sigma$) propagator\footnote{The argument holds when we introduce the width for the $\sigma$ particle.}

\[ \frac{M_\sigma^2 - M_\pi^2}{M_\sigma^2 - M_K^2} = \frac{F_K}{3F_\pi - 2F_K} \approx 2.2. \]  \hfill (2·5)

In the systematic expansion of the chiral perturbation theory, this factor will be obtained as the series

\[ \frac{F_K}{3F_\pi - 2F_K} = 1 + \frac{3(F_K - F_\pi)}{F_K} + \cdots \]

\[ = 1 + 0.54 + (0.54)^2 + \cdots. \]

The convergence of this series is very slow. Note that the $SU(3)$ breaking gets amplified when it comes into the numerator. Hence it is crucial to understand how to include the $SU(3)$ breaking.

To do this we must go back to what we know about the low energy dynamics. One of the important facts is the vector meson dominance structure which can be seen in the pion-electromagnetic form factor $F_\pi(s)$ (s is the invariant mass squared of virtual photon). Under this hypothesis $F_\pi(s)$ is given by

\[ F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} = 1 + \frac{1}{6} \langle r^2 \rangle_{\pi^+} s + \cdots, \]  \hfill (2·6)

where $M_\rho$ is the mass of $\rho$ meson. Thus the charge radius $\langle r^2 \rangle_{\pi^+}$ is obtained as

\[ \langle r^2 \rangle_{\pi^+} = 6 \left( \frac{1}{M_\rho^2} \right) \approx 0.41 \text{ fm}^2, \]

which is in good agreement with its experimental value 0.44 fm$^2$. It is also known that the behaviour caused by the infinite series in Eq. (2·6) recapitulates the experimentally measured momentum dependence of $F_\pi(s)$.

In the systematic chiral expansion, QCD dynamics appears in the next-to-leading order terms, i.e., $\mathcal{O}(\rho^4)$ terms which accompany unknown constants $L_1, \cdots, L_{10}$.\footnote{Each of these constants requires an experimental input. For example, $L_9$ has been determined by using the charge radius $\langle r^2 \rangle_{\pi^+}$ of $\pi^+$ as its input. However, as was shown before, $\langle r^2 \rangle_{\pi^+}$ could be obtained from the presence of $\rho$ meson with the vector meson dominance hypothesis. More generally, $L_1, \cdots, L_{10}$ are all determined from the higher resonance contributions, consistent with their proper values.} Each of these constants requires an experimental input. For example, $L_9$ has been determined by using the charge radius $\langle r^2 \rangle_{\pi^+}$ of $\pi^+$ as its input. However, as was shown before, $\langle r^2 \rangle_{\pi^+}$ could be obtained from the presence of $\rho$ meson with the vector meson dominance hypothesis. More generally, $L_1, \cdots, L_{10}$ are all determined from the higher resonance contributions, consistent with their proper values.\footnote{We shall, therefore, take an approach in which all higher-order effects in the chiral perturbation theory can be reproduced by the introduction of vector and scalar mesons.} We shall, therefore, take an approach in which all higher-order effects in the chiral perturbation theory can be reproduced by the introduction of vector and scalar mesons.\footnote{We shall, therefore, take an approach in which all higher-order effects in the chiral perturbation theory can be reproduced by the introduction of vector and scalar mesons.}

2.2. Model

We construct the Lagrangian including the pseudoscalar mesons as well as scalar and vector mesons. The scalar field $\Sigma$ has already obtained the VEV, and as a consequence the chiral symmetry are spontaneously broken down to the vector
symmetry subgroup \([U(3)_v]_{\text{global}}\) as in the case of linear \(\sigma\) model in § 2.1. The pseudoscalar mesons are non-linearly realized

\[
\xi(\Pi) = \exp(i\Pi),
\]

\[
\Pi = \sum_{a=0}^8 T^a \pi^a.
\]  

(2.7)

Hidden local symmetry approach treats the vector mesons as a gauge boson of the symmetry group \([U(3)_v]_{\text{local}}\). Our model assumes that the hidden local symmetry has already been broken by the VEV of scalar field \(\Sigma\). Thus additional scalar nonet \(\sigma\) as in the conventional hidden local symmetry approach has to be introduced. The \(\sigma\) is unphysical degrees of freedom which will be absorbed by the vector mesons. In the following we construct such a Lagrangian, the origin of the decay constants of pseudoscalars and the masses of vector mesons can be traced to the non-zero VEV of scalar \(\Sigma\).

The Lagrangian of our model has the structure*)

\[
\mathcal{L}_{\text{chiral}} = \mathcal{L}_2 - \frac{1}{2g_v^2} \text{Tr}(F_{\mu
u} F^{\mu\nu}),
\]  

(2.8)

where \(F_{\mu
u}\) is the field strength corresponding to vector meson \(V_\mu\)

\[
F^\mu_\nu = \partial^\mu V_\nu - \partial^\nu V_\mu - i [V^\mu, V^\nu],
\]

and \(g_v\) is the hidden local gauge coupling constant. \(F^{\mu\nu}_{\mu}\) is the field strength corresponding to vector meson \(V_\mu\)

\[
F^\nu_{\mu} = \partial^\nu V^\mu - \partial^\mu V^\nu - i [V^\nu, V^\mu],
\]

and \(g_v\) is the hidden local gauge coupling constant. \(\mathcal{L}_2\) consists of the terms which contain two powers of derivatives, the explicit forms of which will be shown in § 3. For the equipment of that purpose, the fundamental building blocks will be clarified in the rest of this section.

The symmetry that should be possessed by the Lagrangian is \(G = [U(3)_v]_{\text{local}} \times [U(3)_L \times U(3)_R]_{\text{global}}\). We denote the corresponding group elements as \(h, g_L\) and \(g_R\). As usual, \(\xi(\Pi)\) transforms as

\[
\xi(\Pi) \rightarrow \xi(\Pi') = h(\Pi, g_L, g_R) \xi(\Pi) g_R^L
\]

\[
= g_L \xi(\Pi) h^\dagger(\Pi, g_L, g_R),
\]  

(2.9)

where \(h^\dagger(\Pi, g_L, g_R)\) is an element of the unbroken symmetry \([U(3)_v]_{\text{global}}\). Likewise the transformation of \(\Sigma\) is

\[
\Sigma \rightarrow \Sigma' = h(\Pi, g_L, g_R) \Sigma h^\dagger(\Pi, g_L, g_R).
\]  

(2.10)

They are singlets under the hidden local symmetry transformation while \(\xi(\sigma)\) transforms as

*) When we consider the process which occurs through the existence of chiral anomalies, we add the appropriate anomaly terms.
For constructing the Lagrangian, it is useful to define the new fields
\[
S = \xi(\sigma) \Sigma^* (\sigma), \\
\xi_n = \xi(\sigma) \xi_n (\sigma), \\
\xi_L = \xi(\sigma) \xi_L (\sigma). \tag{2.11}
\]

The transformation properties of these fields are found from Eqs. (2.9) to be
\[
\xi_L \rightarrow \xi_L = h \xi_L g^*_L, \\
\xi_n \rightarrow \xi_n = h \xi_n g^*_n, \\
S \rightarrow S' = h S h^*. \tag{2.12}
\]

Accordingly the derivatives of these fields which are covariantly transforms under the hidden local symmetry and for the position dependent element of \[U(3)_L \times U(3)_R\] \[\mathcal{M}\] the quark mass terms are
\[
-\left( \bar{q}_L \mathcal{M} q_R + \bar{q}_R \mathcal{M}^* q_L \right). \tag{2.14}
\]

Since quark transforms as \[q_L \rightarrow g_L q_L, q_R \rightarrow g_R q_R, \mathcal{M}\] is considered to transform as
\[
\mathcal{M} \rightarrow g_L \mathcal{M} g_R^*. \tag{2.15}
\]

Note that, from the Dashen formula\(^{13}\) relating pion masses to the quark masses, \[\mathcal{M}\] appears in the hadronic description with some dimensionful constant \[1/G_1\]. Thus, as will be summarized in the beginning of the next section, \[(1/G_1) \mathcal{M}\] should be considered as \[\mathcal{O}(p^2)\] for the counting rule of momentum order.

Now the building blocks are the following quantities, each of which transforms as \[A \rightarrow h A h^*\] under \[G\]:
\[
S, \quad a_{\mu}, \quad a_{\mu}, \quad D_{\mu} S, \quad F^\mu, \quad \frac{1}{G_1} \xi_L \mathcal{M} \xi^*_L. \tag{2.16}
\]

Here \[a_{\mu}\] and \[a_{\mu}\] are
\[
a_{\mu} = \frac{D_{\mu} \xi^*_L + D_{\mu} \xi^*_R}{2i},
\]
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\[ a_{\mu\nu} = \frac{D_{\mu} \xi_{\nu} - D_{\nu} \xi_{\mu}}{2i}. \]

It will become indispensable to know the charge conjugation property of various quantities. Especially the operation of charge conjugation (C) on \( a_{\mu\nu} \) is\(^*\)

\[ C: a_{\mu} \rightarrow (a_{\mu})^T, \quad a_{\nu} \rightarrow -(a_{\nu})^T. \]  

\((2\cdot16)\)

Hence the terms such as \( \text{Tr}(a_{\mu\nu}(S, D_{\mu}S)) \) are forbidden by \( C \) symmetry.

\section*{§ 3. \( \mathcal{O}(p^2) \) terms}

Here we discuss the \( \mathcal{O}(p^2) \) terms with respect to the momentum expansion. The counting rule for momentum order is

- \( a_{\mu}^R \) and \( a_{\nu}^I \) are \( \mathcal{O}(p) \),
- \( V_\mu, L_\mu \) and \( R_\mu \) are \( \mathcal{O}(p) \), \( F_{\mu\nu} \) is \( \mathcal{O}(p^3) \),
- \( S \) is \( \mathcal{O}(p^0) \),
- \( \frac{1}{G_1} \mathcal{M} \) is \( \mathcal{O}(p^0) \).

We remark on the counting rule assigned to the scalar field \( S \). Recall that in the original hidden local symmetry approach\(^9\) the mass of vector meson is generated by the term

\[ a F_\pi^2 \text{Tr}(a_{\mu}^R a_{\nu}^I), \]  

\((3\cdot1)\)

which is called \( \mathcal{O}(p^3) \) term. In our present scheme the decay constants of pseudoscalars are essentially the nonzero VEV's of scalar \( \Sigma \). Since \( F_\pi \) is \( \mathcal{O}(p^0) \), consistency will demand that \( S \) is \( \mathcal{O}(p^0) \). Then \( \mathcal{O}(p^2) \) terms, such as

\[ \text{Tr}(\{S, a_{\mu}^R\}S, a_{\mu}^I)), \]  

\((3\cdot2)\)

contribute to the masses for vector meson.

According to the above counting rule, the fields which can participate in \( \mathcal{O}(p^3) \) terms are \( a_{\mu}^R, a_{\nu}^I, (1/G_1)\mathcal{M} \) and \( S \). The operator with dimension less than 4 requires compensation by the multiplication of constant(s) with mass dimension when involved in the Lagrangian. In our scheme we consider the constants with mass dimension available to us are only the VEV of \( \Sigma \) and \( (1/G_1)\mathcal{M} \) as \( F_\pi \) is equivalent to \( \langle \Sigma \rangle \). That means that the operators like \( \text{Tr}(\langle \Sigma \rangle^2)\text{Tr}(a_{\mu}^R a_{\nu}^I) \) must be considered to arise from \( \text{Tr}(S^2)\text{Tr}(a_{\mu}^R a_{\nu}^I) \). However, only from the symmetry consideration, the terms like \( \mu^2 \text{Tr}(a_{\mu}^R a_{\nu}^I) \) (\( \mu \) is some dimensionful constant) are allowed and can play a significant role. But it will be found that there is no such a possibility as will be shown in § 5.

As mentioned in the beginning of § 2.1, we imagine such a situation that the chiral symmetry is spontaneously broken by the non-zero VEV of \( \Sigma \) although the potential part triggering symmetry breaking is not clarified. Furthermore, by analogy with the last term of Eq. \((2\cdot1)\), the operator

\(^*\) We thank K. Yamawaki for pointing out this \( C \) transformation properties.
\[
\frac{1}{4 G_1} \text{Tr}(S(\xi_\mu \mathcal{M} \xi^\mu + \xi_\nu \mathcal{M}^\nu \xi^\nu)), \tag{3.3}
\]
will induce the splitting among the VEV's of \( \Sigma \) combined with the other implicitly present terms of the potential. This term also gives the pion masses as in the case of linear \( \sigma \) model. Hereafter we ignore the isospin breaking effect from the quark masses, i.e., \( \mathcal{M} = \text{diag}(m_{1}^{(0)}, m_{1}^{(0)}, m_{3}^{(0)}) \). Thus, the VEV \( \langle \Sigma^{(0)} \rangle \) of \( \Sigma \) takes the form
\[
\Sigma^{(0)} = \begin{pmatrix} \Sigma_1^{(0)} \\ \Sigma_1^{(0)} \\ \Sigma_3^{(0)} \end{pmatrix}.
\]

The operator with dimension \((=4+d)\) more than 4 has a coefficient of the form \((a/\Lambda^d)\) where \(a\) is a constant of order 1, and \(\Lambda\) is the cutoff under which our effective chiral Lagrangian is expected to describe the low energy behaviour of QCD. A problematic thing is that higher dimensional operators, such as
\[
-\frac{a}{\Lambda^d} \text{Tr}(S^d \mathcal{O}^{(4)}), \tag{3.4}
\]
where \(\mathcal{O}^{(4)}\) is some four-dimensional and \(\mathcal{O}(p^2)\) operator, can induce four-dimensional operators by taking the VEV's of scalars. To see that we do not need to add these contributions, write the operator in Eq. (3.4) as
\[
a\left(\frac{\Sigma_1^{(0)}}{\Lambda}\right)^d \text{Tr} \mathcal{O}^{(4)} + \left(\left(\frac{\Sigma_3^{(0)}}{\Lambda}\right)^d - \left(\frac{\Sigma_1^{(0)}}{\Lambda}\right)^d\right) \mathcal{O}^{(4)}. \tag{3.5}
\]

The first term in this expression can be absorbed by the renormalization of the operator \(\text{Tr} \mathcal{O}^{(4)}\). In the context where \(\Lambda \approx 4\pi F_{\pi}^{(0)}, \Sigma_1^{(0)} \approx F_{\pi}/2\). As will be shown later, the splitting in the VEV's of \( S \) has the form: \( \Sigma_3^{(0)} = (M_0/M_\rho) \Sigma_1^{(0)} \). Hence the second term in Eq. (3.5) is of order
\[
\left(\frac{\Sigma_3^{(0)}}{\Lambda}\right)^d - \left(\frac{\Sigma_1^{(0)}}{\Lambda}\right)^d \sim 0.013, \quad (d=1)
\]
\[
\sim 0.0012, \quad (d=2) \tag{3.6}
\]
which is smaller for larger \(d\). Hence, to order 1\% accuracy, this second term can be dropped. Since we will not consider any processes with scalar particles on the external lines, this type of higher dimensional operators is not important.

Note that the above argument assumes that there appear no terms which are singular at \(S=0\). For example, the dimension-four operator
\[
\text{Tr}\left[\ln\left(\frac{S}{\Lambda}\right) \mathcal{O}^{(4)}\right] \tag{3.7}
\]
with no \(S\) in the dimension-four operator \(\mathcal{O}^{(4)}\) transforming as \(\mathcal{O}^{(4)} \rightarrow h \mathcal{O}^{(4)} h^\dagger\) is singular at \(S=0\); the analyticity assumption implies the absence of this kind of operators. Then the Lagrangian can be expanded with respect to \(S/\Lambda\) and the above argument is justified.\(^*\) In conclusion, we can concentrate ourselves with the dimension-four operators which are polynomials with respect to \(a_\alpha^\mu, a_\alpha^\nu\) and \(S\).

\(^*\) For example, the term \(\text{Tr}[\ln(S/\Lambda)(S, a_\alpha)(S, a_\alpha^\mu)]\) which is regular at \(S=0\) will be induced from QCD, but we consider by expanding this in powers of \(S/\Lambda\) and apply the above argument.
Below we list those operators that are Lorentz, $C$, $P$ (Parity operation) invariant and have chiral and hidden local symmetries:

\[
\begin{align*}
\text{(Category 1)} & & & \\
& = & & \\
& = & & \\
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& = & & \\
& = & & \\
& = & & \\
& = & & \\
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& = & & \\
& = & & \\
& = & & \\
& = & & \\
& = & & \\
\end{align*}
\]  

We divided operators into classes according to the flavour $SU(3)$ breaking structure which will be generated by the splitting among the VEV's of scalar $S$. We first concentrate our attention on the following reduced Lagrangian which consists of the operators belonging to Category-1, plus the $\mathcal{O}(p^2)$ explicit breaking term which appeared in § 3:

\[
\mathcal{L}_2 = \text{Tr}(\{S, a_{111}'\} [S, a_{11}] + f \text{Tr}(\{S, a_{11}'\} [S, a_{111}] + a \text{Tr}(\{S, a_{11}'\} [S, a_{11}] + b \text{Tr}(\{S, a_{111}'\} [S, a_{111}] + d \text{Tr}(D^n SD_p S) + 2c i \text{Tr}(a_{111}' [S, D_p S]) + \frac{1}{4 G_1} \text{Tr}(S \xi_k M^i \xi^k + \bar{\xi}_k M^i \xi_k)).
\]  

\[\text{(3.15)}\]
The effects of operators in the other categories will be discussed later.

We first express fundamental quantities in the pseudoscalar sector. Since isospin breaking is ignored here, isospin nonsinglet mesons are always in their mass eigenstates. We did not concern \( \eta \) and \( \eta' \) explicitly here. So it is convenient to work on the nonet basis

\[
\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\pi^0 + \eta}{\sqrt{2} F_\pi} & \frac{\pi^+}{F_\pi} & K^+ \\
\frac{\pi^-}{F_\pi} & -\frac{\pi^0 + \eta}{\sqrt{2} F_\pi} & K^0 \\
\frac{K^-}{F_K} & \frac{K^0}{F_K} & \frac{\eta}{F_{\eta}}
\end{pmatrix}
\]

Then we have

\[
F_\pi = 2 \Sigma_1^{(0)},
\]

\[
F_{\eta} = 2 \Sigma_3^{(0)},
\]

\[
F_K^2 = \frac{1}{4} \left( (F_\pi + F_{\eta})^2 - f(F_{\eta} - F_\pi)^2 \right). \tag{3.16}
\]

The masses of \( \pi \) and \( K \) can be read off from the last term in Eq. (3.15)

\[
M_{\pi}^2 = \frac{m_1^{(0)}(2 \Sigma_1^{(0)})}{2 G_1 F_\pi^2} = \frac{m_1^{(0)}}{2 G_1 F_\pi},
\]

\[
M_K^2 = \frac{(m_1^{(0)} + m_3^{(0)})(\Sigma_1^{(0)} + \Sigma_3^{(0)}) - (m_1^{(0)} + m_3^{(0)})(F_\pi + F_{\eta})}{4 G_1 F_K^2} = \frac{(m_1^{(0)} + m_3^{(0)})(F_\pi + F_{\eta})}{8 G_1 F_K^2}. \tag{3.17}
\]

Next we turn our attention to the scalar-vector sector. There are transitions among vector meson \( V_\mu \), scalar \( \sigma \) and \( \Sigma' \) where the field \( \Sigma \) is defined as

\[
\Sigma = \xi^\dagger(\sigma) S \xi(\sigma) = \Sigma^{(0)} + \frac{1}{\sqrt{d}} \Sigma'.
\]

We define the unitary gauge as follows. First we set \( \sigma = 0 \) and redefine the fields \( V_\mu \) and \( \Sigma' \) as

\[
V_\mu \rightarrow V_\mu + i \frac{1}{\sqrt{2}} \frac{C}{\sqrt{1 - C^2}} \frac{g_\mu}{M_K} \partial_\mu K^s,
\]

\[
\Sigma_i^\dagger \rightarrow \frac{1}{\sqrt{1 - C^2}} \Sigma_i^\dagger \text{ for } (i, j) = (1, 3), (2, 3), (3, 1), (3, 2). \tag{3.18}
\]

In these expressions \( M_K \) is the mass of \( K^* \) (see Eq. (3.22)), and \( K^s \) consists of scalar components \( \kappa \) each of which has strangeness
Chiral Lagrangian with Higher Resonances and Flavour SU(3) Breaking

\[ K^s = \begin{pmatrix}
0 & 0 & -\kappa^+ \\
0 & 0 & -\kappa^0 \\
\kappa^- & \bar{\kappa}^0 & 0
\end{pmatrix} \tag{3\cdot19} \]

and

\[ C = \frac{d+c}{\sqrt{d}} g_V \frac{\Sigma_3^{(0)} - \Sigma_1^{(0)}}{M_{K^*}}. \tag{3\cdot20} \]

If we denote the vector meson matrix \( V_\nu \) as

\[ V_\nu = \frac{g_V}{\sqrt{2}} \begin{pmatrix}
\frac{1}{\sqrt{2}} (\rho^0 + \omega)_\mu & \rho^\mu_+ & K^{*+} \\
\rho^- & \frac{1}{\sqrt{2}} (-\rho^0 + \omega)_\mu & K^{*0} \\
K^{*-} & \bar{K}^{*0} & \phi_\mu
\end{pmatrix}, \tag{3\cdot21} \]

the masses of vector mesons are given by

\[ M_\rho^2 = a g_V^2 F_\pi^2 = M_\omega^2, \]
\[ M_\phi^2 = a g_V^2 F_3^2, \]
\[ M_{K^*}^2 = \frac{a g_V^2}{4} \left( (F_{33} + F_\pi)^2 + \frac{d + 2c - b}{a} (F_{33} - F_\pi)^2 \right). \tag{3\cdot22} \]

We can see from Eq. (3\cdot18) that \( C^2 < 1 \) is necessary for our model to become meaningful. Eq. (3\cdot20) tells us that there is no mixing between \( V_\nu \) and \( \Sigma' \) when \( c = -d. \) Note that if \( (d + 2c - b)/a \) is order unity, from Eq. (3\cdot22), we have

\[ M_{K^*} = \frac{1}{2} (M_\rho + M_\phi), \tag{3\cdot23} \]

since

\[ \frac{(F_{33} - F_\pi)^2}{(F_{33} + F_\pi)^2} = \frac{(M_\phi - M_\rho)^2}{(M_\phi + M_\rho)^2} \approx 0.02. \tag{3\cdot24} \]

The \( f \) term in the expression for \( F_\pi \) cannot be neglected: in fact, by considering \( f = O(1) \) and neglecting the \( f \) term in Eq. (3\cdot16), we obtain the relation

\[ \frac{F_{K^*}}{F_\pi} = \frac{1}{2} \left( 1 + \frac{M_\phi}{M_\rho} \right) \approx 1.16, \tag{3\cdot25} \]

which is off by 6%.

We close this section by quoting the mass \( M_k \) of \( \kappa \) since \( \kappa \) will contribute to the \( K_{33} \) form factor. \( M_k \) is given by

\[ M_k^2 = \frac{1}{d} \left( 1 - \frac{1}{C^2} \right) \frac{1}{\Sigma_3^{(0)} - \Sigma_1^{(0)}} \frac{1}{\Sigma_3^{(0)} + \Sigma_1^{(0)}} \left( \frac{M_{K^*}^2 F_{K^*}^2}{\Sigma_3^{(0)} - \Sigma_1^{(0)}} - \frac{M_k^2 F_{\pi}^2}{2 \Sigma_1^{(0)}} \right). \tag{3\cdot26} \]

Using Eq. (3\cdot16), it can also be expressed as
§ 4. Electromagnetic form factors and \( K_{es} \) form factor

We now explore the electromagnetic form factors and \( K_{es} \) form factor based on our chiral Lagrangian. In the standard model the external gauge fields \( CV' = R' + L' \) and \( A' = R' - L' \) take the form

\[
CV'_\mu = \frac{2}{3} e \gamma'_\mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\]

\[
+ \frac{g}{c_w} Z'_\mu \begin{pmatrix} \frac{1}{2} - \frac{4}{3} s_w^2 & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{2}{3} s_w^2 & 0 \\ 0 & 0 & -\frac{1}{2} + \frac{2}{3} s_w^2 \end{pmatrix}
\]

\[
+ \frac{g}{\sqrt{2}} W'_\mu \begin{pmatrix} 0 & c_1 & -s_1 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{g}{\sqrt{2}} W'_\mu \begin{pmatrix} 0 & 0 & 0 \\ c_1 & 0 & 0 \\ 0 & -s_1 c_3 & 0 \end{pmatrix},
\]

\[
A'_\mu = -\frac{g}{2c_w} Z'_\mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\]

\[
-\frac{g}{\sqrt{2}} W'_\mu \begin{pmatrix} 0 & c_1 & -s_1 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{g}{\sqrt{2}} W'_\mu \begin{pmatrix} 0 & 0 & 0 \\ c_1 & 0 & 0 \\ 0 & -s_1 c_3 & 0 \end{pmatrix},
\]

where \( c_w = \cos \theta_w \) with the Weinberg angle \( \theta_w \), and \( g \) is the \( SU(2) \) coupling constant. \( c_i = \cos \theta_i, \ s_i = \sin \theta_i \) and \( \theta_i's \) are the angles parameterizing the Kobayashi-Maskawa matrix.\(^{14} \)

First we see that photon \( \gamma'_\mu \) and neutral vector mesons mix

\[
L_{\gamma'_\nu} = -e \gamma'_\mu [g_\rho \partial_\rho \nu + g_\omega \omega_\mu - g_\phi \phi_\mu],
\]

where\(^9 \)

\[
g_\rho = a g_\gamma F^2 = \frac{1}{g_\gamma} M^2,
\]

\[
g_\omega = \frac{1}{3} g_\rho,
\]

\[
g_\phi = \frac{\sqrt{2}}{3} a g_\gamma F^2 = \frac{\sqrt{2}}{3} M^2 = \frac{\sqrt{2}}{3} M^2 g_\rho.
\]

\( (4.2) \)
The $W$-$V$ mixing is given by
\[
\mathcal{L}_{WV}^{\text{mix}} = S_1 c_a g_{K^*} W^- \pi^+ K^{*+}.
\] (4.3)

Here the constant $g_{K^*}$ is calculated to be
\[
g_{K^*} = \frac{g_{\psi}}{8} \left\{ (F_{33} + F_\pi)^2 - \frac{c + b}{a} (F_{33} - F_\pi)^2 \right\}
= \frac{1}{2g_\psi} \left( 1 - \frac{C^2}{1 + \bar{c}} \right) M_{K^*}^2 = \frac{g_{\psi}}{2} \frac{M_{K^*}^2}{M_\rho^2} \left( 1 - \frac{C^2}{1 + \bar{c}} \right)
\] (4.4)

with $\bar{c} = \frac{c}{d}$.

The $V$-$PP$ coupling ($P$ denotes pseudoscalar) takes the form
\[
\mathcal{L}_V^{PP} = -i \phi^0 \left( g_{\rho KK} \partial_\mu K^{-} + g_{\omega KK} \partial_\mu K^{-} - g_{\phi KK} (\partial_\mu K^0 \partial_\mu \bar{K}^0) \right)
- i g_{\omega KK} \omega^\mu K^{-} \partial_\mu K^{-} + i g_{\rho KK} \phi^0 (K^{-} \partial_\mu K^{-} + K^0 \partial_\mu \bar{K}^0)
- i g_{K^* KK} K^{*+} \partial_\mu K^- - i g_{K^* KK} K^{*0} \left( K^{-} \partial_\mu K^- + \frac{1}{\sqrt{2}} K^0 \partial_\mu \bar{K}^0 \right) + \cdots,
\] (4.5)

and each coefficient is given by KSRF(I) relation
\[
g_{\rho KK} = \frac{g_\rho}{2F_\rho^2},
g_{\omega KK} = \frac{g_\omega}{4F_\rho^2},
g_{\phi KK} = \frac{3g_\phi}{2\sqrt{2}F_\rho^2},
g_{K^* KK} = \frac{g_{K^*}}{F_\rho F_\pi},
\] (4.6)

We explore the numerical values of various quantities by using the measured values of $F_\pi$, $F_K$, $M_\rho$, $M_\phi$, $M_{K^*}$ and $g_{\rho KK}$. The results are shown in Tables I and II. There, experimental uncertainties $^{19}$ $\sqrt{2}F_\pi = 130.8 \pm 0.3$ MeV, $\sqrt{2}F_K = 159.8 \pm 1.4$ MeV and $5.9 \leq g_{\rho KK} \leq 6.1$ were taken into account. For the values of $M_\rho$ and $M_{K^*}$, there is some theoretical uncertainties due to our approximation. Here we used $(M_\rho)_{\text{exp}} \leq M_\rho$

<table>
<thead>
<tr>
<th>Table I. $g_V$ ($V = \rho, \omega, \phi$) coupling with $g_{\rho KK}$ used as input through KSRF(I) relation (in unit [GeV$^2$]).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prediction</strong></td>
</tr>
<tr>
<td>$g_\rho$</td>
</tr>
<tr>
<td>$g_\omega$</td>
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<tr>
<td>$g_\phi$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II. $g_{\rho \pi}$ coupling with $g_{\rho KK}$ used as input.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prediction</strong></td>
</tr>
<tr>
<td>$g_{\rho \pi}$</td>
</tr>
<tr>
<td>$g_{\omega \pi}$</td>
</tr>
<tr>
<td>$g_{K^* \pi}$</td>
</tr>
</tbody>
</table>
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\( (M_\omega)_{\text{exp}} \leq M_\rho \) and \( (M_{K^*})_{\text{exp}} \leq (M_{K^*})_{\text{exp}} \) for \( M_{K^*} \). The value of \( g_{K^*K^*} \) shown in Table II was obtained under the assumption that \( d + c \) is at most of order unity: in fact, we have from Eq. (3.20)

\[
C \approx \frac{d + c}{\sqrt{d}} \times 0.1,
\]

so that the quantity dependent on \( (d + c) \) in the parenthesis in Eq. (4.4) becomes \( (d + c) \times 0.01 \), and this correction is within the experimental errors accompanied with it.

The direct \( \gamma - PP \) and \( W - PP \) coupling are

\[
L_{\gamma - PP} = -ie\gamma^\mu \left( (1 - r_\rho) (\pi^+ \partial_\mu \pi^- - \partial_\mu \pi^+ \pi^-) + (1 - r_\phi) (K^+ \partial_\mu K^- - \partial_\mu K^+ K^-) - r_\rho \frac{F_\pi^2 - F_K^2}{3F_R^2} (\partial_\mu K^0 - \partial_\mu K^0 K^0) \right),
\]

\[
L_{W - PP} = -i\frac{e}{c_5} \frac{g}{4} W^{-\mu} \left\{ \left( \frac{F_\pi}{F_K} - r_{K^*} \right) (K^+ \partial_\mu K^0 + \sqrt{2} \partial_\mu \pi^+ K^0) - \left( \frac{F_K}{F_K} - r_{K^*} \right) (\partial_\mu K^+ \pi^0 + \sqrt{2} \pi^+ \partial_\mu K^0) \right\},
\]

where \( r_\rho \) and \( r_\phi \) characterize the vector dominance and are given by

\[
r_\rho = \frac{a}{2} = \frac{2g_{\rhoKK}^2 F_K^2}{M_\rho^2},
\]

\[
r_\phi = \frac{2g_{\phiKK}^2 F_K^2}{M_\phi^2} \left( \frac{F_\pi^2 + 2F_\rho^2}{3F_\pi^2} \right),
\]

\[
r_{K^*} = \frac{a}{2F_\pi F_K} \frac{1}{4} \left\{ (F_\rho + F_\pi)^2 - \frac{b}{a} (F_\rho - F_\pi)^2 \right\}
\]

\[
= \frac{2g_{K^*KK}^2 F_\pi F_K}{M_{K^*}^2} \frac{1 - \frac{1 + 2 \bar{c}}{(1 + \bar{c})^2} C^2}{\left( 1 - \frac{C^2}{1 + \bar{c}} \right)^2}.
\]

The expression for \( r_\rho \) has been obtained before. This originates from the fact that when ignoring the \( SU(3) \) breaking, the first term and \( a \) term in Eq. (3.15) induces the same terms as in the original hidden local Lagrangian. However the expression for \( r_\phi \) is a peculiar one arising from our model. Table III shows the numerical consequences for these quantities in our model.

With these preparations, the electromagnetic form factors can be obtained:

\[
F_\pi(s) = 1 + r_\rho \frac{s}{M_\rho^2 - s},
\]

\[
F_K(s) = 1 + \frac{1}{3} \left\{ 2r_\rho \left( \frac{F_\pi}{F_K} \right)^2 \frac{s}{M_\rho^2 - s} + \bar{r}_\phi \frac{s}{M_\rho^2 - s} \right\},
\]

\[
\bar{r}_\phi = 1.069 \pm 0.048.
\]
Chiral Lagrangian with Higher Resonances and Flavour SU(3) Breaking

\[ F_{K^0}(s) = -\frac{1}{3} \left\{ r_s \left( \frac{F_\pi}{F_K} \right)^2 \frac{s}{M_\phi^2 - s} \right. \]
\[ - \left. \bar{\rho} \left( \frac{1}{s} \frac{s}{M_\phi^2 - s} \right) \right\}, \quad (4.8) \]

where \( s \) is the invariant mass squared of virtual photon, and
\[ \bar{\rho} = \frac{2g_{KK}^2 F_K^2}{M_\phi^2}. \]

We can deduce the charge radius for each particle
\[ \langle r^2 \rangle_{\pi^+} = 6 \frac{r_s}{M_\rho^2}, \]
\[ \langle r^2 \rangle_{K^+} = 6 \frac{\bar{\rho}}{M_\phi^2} = \left( \frac{F_K}{F_\pi} \right)^2 \langle r^2 \rangle_{\pi^+}, \]
\[ \langle r^2 \rangle_{K^0} = 0. \quad (4.9) \]

In deriving these expressions we used the relation
\[ \bar{\rho} = r_s \left( \frac{M_\phi}{M_\rho} \right)^2 \left( \frac{F_\pi}{F_K} \right)^2. \]

The numerical results for the charge radii are summarized in Table IV. The SU(3) breaking in the charge radii appears in the simple form
\[ \frac{\langle r^2 \rangle_{K^+}}{\langle r^2 \rangle_{\pi^+}} = \left( \frac{F_\pi}{F_K} \right)^2, \]
the value of which is about 0.67 (its experimental value is 0.64) so that in this aspect our model is not inconsistent with the experiment.

Table IV shows that our model gives slightly smaller values for both \( \langle r^2 \rangle_{\pi^+} \) and \( \langle r^2 \rangle_{K^+} \). Also, as shown in Eq. (4.9), the charge radius \( \langle r^2 \rangle_{K^0} \) for \( K^0 \) becomes exactly 0, in contrast to its being experimentally small but having a finite value \((-0.054 \pm 0.026 \text{ fm}^2\)). This seems to result from the simple relation between the ratio of \( M_\phi \) to \( M_\rho \) and the ratio of \( F_{33} \) to \( F_\pi \), obtained from Eq. (3.22),
\[ \frac{M_\rho}{M_\phi} = \frac{F_{33}}{F_\pi}, \quad (4.10) \]

which is a consequence of the fact that the SU(3)$_v$ breaking effects both in vector meson masses and the decay constants are induced in the same manner by the VEV's of \( \Sigma \) in our model.

We will reexamine in § 5 whether there is some modification to these results by the addition of \( 1/N_c \) non-leading but \( \mathcal{O}(p^3) \) terms, which are not included in the reduced Lagrangian in Eq. (3.15).

To calculate the \( K_{33} \) form factor, we further need the transition term between \( \Sigma' \) and \( W_\rho \), and the interaction among \( \kappa^- \), \( \pi^0 \) and \( K^+ \).
\[ \mathcal{L}_{s-w} = -i\sqrt{d(1-C^2)}(F_{33}-F_\pi)s_1c_3 \frac{\rho}{4} W_\mu^\rho \partial^\mu \kappa^- + (\text{h.c.}), \]

\[ \mathcal{L}_{K^+K^-} = \frac{1}{\sqrt{d}} \frac{v_\sigma}{F_K^{-}} \partial^\mu \pi^0 \partial_\mu K^- - \frac{\lambda}{\sqrt{d}} \frac{g\pi\kappa}{M_{K^*}} \partial^\mu \kappa^-(K^+ \partial^\mu \pi^0), \]

where the constants \( v_\sigma \) and \( \lambda \) are

\[ v_\sigma = \frac{1}{2\sqrt{1-C^2}} \left( 1 + \frac{F_{33} + F_\pi - f}{2F_\pi} \right), \]

\[ \lambda = \frac{C\sqrt{1-C^2}}{1+C-C^2}. \]

The \( K_{e3} \) form factors \( F_\pm(s) \) \((s=(p_K-p_\pi)^2)\) is defined by

\[ \langle \pi^0(p_\pi)|\bar{s} \gamma_\mu(1-\gamma_5)u|K^+(p_K)\rangle = -\frac{1}{\sqrt{2}} [F_+(s)(p_K+p_\pi)_\mu + F_-(s)(p_K-p_\pi)_\mu]. \]

Direct calculation shows that \( F_+(s) \) and \( F_-(s) \) are given in our model by

\[ F_+(s) = \frac{1}{2} \left( \frac{F_K}{F_\pi} + \frac{F_\pi}{F_K} \right) - \frac{1}{2} \left( \frac{F_K}{F_\pi} - \frac{F_\pi}{F_K} \right) \frac{s}{M_{K^*}^2 - s}, \]

\[ F_-(s) = \frac{1}{2} \left( \frac{F_K}{F_\pi} - \frac{F_\pi}{F_K} \right) - \frac{1}{2} \left( \frac{F_K}{F_\pi} + \frac{F_\pi}{F_K} \right) \frac{M_{K^*}^2 - M_\pi^2}{M_{K^*}^2 - s} \]

\[ + \frac{1}{M_{K^*}^2 - s} \left( \frac{1}{2} \left( \frac{F_K}{F_\pi} - \frac{F_\pi}{F_K} \right) (M_{K^*}^2 - M_\pi^2 - s) + (\gamma_K^* - \bar{\gamma}_K^*) (M_{K^*}^2 - M_\pi^2) \right). \]

Here \( \bar{\gamma}_K^* \) is

\[ \bar{\gamma}_K^* = \frac{2g^{3}K^*_\pi F_\pi F_K}{M_{K^*}^2}. \]

Hence the linear slope \( \lambda_{e3} \) defined by

\[ F_+(s) = F_+(0) \left( 1 + \lambda_{e3} \frac{s}{M_\pi^2} \right) + \mathcal{O}\left( \left( \frac{s}{M_\pi^2} \right)^2 \right), \]

takes the form

\[ \lambda_{e3} = \bar{\gamma}_K^* \frac{M_\pi^2}{M_{K^*}^2 F_+(0)}, \]

where

\[ F_+(0) = \frac{1}{2} \left( \frac{F_K}{F_\pi} + \frac{F_\pi}{F_K} \right) - (\gamma_K^* - \bar{\gamma}_K^*), \]

\[ \gamma_K^* - \bar{\gamma}_K^* = \frac{C(1-C^2)}{1+C-C^2} \approx d_\gamma K^* \times 0.01 \approx d \times 0.01. \]
In obtaining the first approximate relation in Eq. (4.15), we made the same assumption \( d + c \leq O(1) \) as used for evaluating the numerical value of \( g_{K^*K} \). The second one follows by using the explicit value of \( \bar{r}_{K^*} \) in Table III.

There is still an ambiguity due to the presence of factor containing \( d \). The value of \( d \) would be determined if we knew the mass of strange scalar \( \kappa \) because from Eq. (3.27)

\[
M_{\kappa} \simeq \sqrt{\frac{2}{d}} \times 984 \text{ MeV}.
\]

However we only know that the strange scalar may be rather heavy. We require that the mass of \( \kappa \) be greater than 980 MeV. Also, following the spirit of the effective Lagrangian approach, the mass of the particles contained explicitly in the Lagrangian should be less than the cutoff \( \Lambda \) of the theory (\( \Lambda \approx 4\pi F_{\pi} \approx 1.2 \text{ GeV} \)). These considerations lead to 980 MeV \( \ll M_{\kappa} < \Lambda \) (\( \approx 1.2 \text{ GeV} \)). This constraint to \( M_{\kappa} \) yields \( \bar{r}_{K^*} - \bar{r}_{K^*} \approx 0.02 \) through Eqs. (4.15) and (4.16). Now we have

\[
F_{\kappa}(0) \approx \frac{1}{2} \left( \frac{F_{K}}{F_{\pi}} + \frac{F_{\pi}}{F_{K}} \right)
\]

to the 2\% level. Then the value of \( \lambda_{e3} \) is determined which is shown in Table IV.

§ 5. Effects of extra terms

In this section, we examine the effects of the operators in Eqs. (3.8) \~ (3.14) to the results obtained in the previous two sections. We make a few remarks before adding these operators to the reduced Lagrangian in Eq. (3.15) as extra terms.

First note that only operators in Category 1 are allowed at the leading order of \( 1/N_c \) expansion (\( N_c \) is the number of colours) if it is admitted as a proper approximation. This is because a trace in flavour space closes the flow of flavour so that we have one quark loop corresponding to one trace, while the number of quark loops must be one at the leading order of \( 1/N_c \). Hence at this order only terms with just one trace are allowed. In this respect, the extra terms are non-leading.

The terms in which two \( a_{\|} \)'s are contained in separate traces will induce the deviation from \( SU(3) \) nonet basis for vector mesons. Since we know that the ideal mixing of the octet and singlet vector mesons is fairly good experimentally, we do not consider such operators any more. On the other hand, \( \text{Tr}(D_{\mu}S)\text{Tr}(D_{\mu}S) = \partial_{\mu}\text{Tr}(S) \delta^{\mu\nu}\text{Tr}(S) \) and the operators which contain two \( a_{\|} \)'s in different traces such as \( \text{Tr}(Sa_{\|}) \times \text{Tr}(S_{a_{\|}}) \) do not affect our interested quantities. Hence these operators will not be explicitly included in the renewed Lagrangian.

Now we discuss the possible effects to the charge radii of pseudoscalars and the linear slope of \( K_{e3} \) form factor, by the addition of the following Lagrangian to the reduced Lagrangian in Eq. (3.15):

\[
\mathcal{L}_{\text{extra}} = \delta_{\|}^{(0)} \text{Tr}(Sa_{\|}a_{\|}) \text{Tr}(S) + \{ \delta_{\|}^{(1)} \text{Tr}(S^2) + A_{\|}^{(1)} \text{Tr}(S) \text{Tr}(S) \} \text{Tr}(a_{\|}a_{\|}) \\
+ \delta_{\|}^{(0)} \text{Tr}(Sa_{\|}a_{\|}) \text{Tr}(S) + \{ \delta_{\|}^{(1)} \text{Tr}(S^2) + A_{\|}^{(1)} \text{Tr}(S) \text{Tr}(S) \} \text{Tr}(a_{\|}a_{\|}) .
\]

(5.1)
Each coefficient denoted by $\delta$ includes a possible suppression factor associated with $1/N_c$ expansion. A similar remark is also made for $\delta$, but with a doubled suppression factor.

From the modified Lagrangian, Eq. (5·1),

$$F_\pi^2 = 4(\Sigma_1^{(0)})^2 + \delta_{\perp}^{(0)} \operatorname{Tr}(\Sigma^{(0)}) \cdot \Sigma_1^{(0)} + T_\perp,$$

$$F_\kappa^2 = (\Sigma_1^{(0)} + \Sigma_3^{(0)})^2 - f(\Sigma_3^{(0)} - \Sigma_1^{(0)})^2 + \frac{1}{2} \delta_{\perp}^{(0)} \operatorname{Tr}(\Sigma^{(0)})(\Sigma_1^{(0)} + \Sigma_3^{(0)}) + T_\perp,$$  \hspace{1cm} (5·2)

where $T_\perp$ is defined as

$$T_\perp \equiv \delta_{\perp}^{(1)} \operatorname{Tr}(\Sigma^{(0)}) + \delta_{\parallel}^{(1)} (\operatorname{Tr}(\Sigma^{(0)})^2). \hspace{1cm} (5·3)$$

The vector meson masses are also calculated to give

$$M_\rho^2 = M_\omega^2 = g_\rho^2 \left( 4a(\Sigma_1^{(0)})^2 + \delta_{\parallel}^{(0)} \operatorname{Tr}(\Sigma^{(0)}) \cdot \Sigma_1^{(0)} + T_{\parallel} \right),$$

$$M_\kappa^2 = g_\kappa^2 \left( a(\Sigma_3^{(0)} + \Sigma_1^{(0)})^2 + (d + 2c - b)(\Sigma_3^{(0)} - \Sigma_1^{(0)})^2 + \delta_{\parallel}^{(0)} \operatorname{Tr}(\Sigma^{(0)}) \cdot \Sigma_1^{(0)} + T_{\parallel} \right),$$

$$M_\phi^2 = g_\phi^2 \left( 4a(\Sigma_3^{(0)})^2 + \delta_{\parallel}^{(0)} \operatorname{Tr}(\Sigma^{(0)}) \cdot \Sigma_3^{(0)} + T_{\parallel} \right). \hspace{1cm} (5·4)$$

where

$$T_{\parallel} \equiv \delta_{\parallel}^{(1)} \operatorname{Tr}(\Sigma^{(0)}) + \delta_{\parallel}^{(1)} (\operatorname{Tr}(\Sigma^{(0)})^2). \hspace{1cm} (5·5)$$

Since the order of VEV of $S$ does not change in this extension, the discussion which asserts that higher dimensional operators are less important than four-dimensional operators in § 3 makes sense even in this case.

Note that the effects from the operators with dimension lower than four, e.g., $\mu_1^2 \operatorname{Tr}(a_{1\mu}a_{1}^{\mu})$, $\mu_2^2 \operatorname{Tr}(a_{1\mu}a_{1}^{\mu})$, tentatively ignored in § 3, can also be included by obvious redefinition of $T_\perp$ and $T_{\parallel}$.

By using these expressions we can directly check that, even with the presence of the terms in Eq. (5·1), the relations

$$g_\rho = \frac{1}{g_\pi} M_\rho^2, \quad g_\omega = \frac{1}{3} g_\rho, \quad g_\phi = \frac{\sqrt{2}}{3} \frac{M_\phi^2}{M_\rho^2} g_\rho,$$  \hspace{1cm} (5·6)

appearing in Eq. (4·2),

$$g_\kappa = \frac{g_\pi}{2} \frac{M_\kappa^2}{M_\rho^2} \left( 1 - \frac{C^2}{1 + C} \right), \hspace{1cm} (5·7)$$

in Eq. (4·4), and all KSRF(I) relations in Eq. (4·6) remain valid. As a consequence, the expression of $r_{\rho}$ in Eq. (4·7),

$$r_{\rho} = \frac{2 g_{\rho\pi\pi} F_\pi^2}{M_\rho^2}$$  \hspace{1cm} (5·8)

and the expressions for various electromagnetic form factors in Eq. (4·8) does not
change with the same definition of \( r_\varepsilon \) as before. Therefore the charge radii are the same as those obtained according to the reduced Lagrangian. We can also calculate the \( K_{\varepsilon 3} \) form factor and find that the result is the same as in Eq. (4.12). Hence Eqs. (4.14) and (4.15) for the normalization and the linear slope of \( F_+(s) \) remain true. However, in this extended model, we cannot express \( \kappa \) mass in the form of Eqs. (3.27) and (4.16) (but Eq. (3.26) is true). Since Eq. (4.15) holds even in this case, \( F_+(0) \approx 1 \) if we further assume that \( d \) is at most of order unity.

§ 6. Discussion and summary

We have proposed a chiral Lagrangian (Eqs. (2.8) and (3.15)) with higher resonances (scalars and vectors), paying close attention to the flavour SU(3) breaking structure which shall be crucial for the description of \( K \) decay. From the observation of Eq. (3.23) the resulting SU(3) breaking structure in vector meson sector seems to be well incorporated into our Lagrangian. Our model constructed here includes not only \( \mathcal{O}(\phi^2) \) operators but also the kinetic term of vector meson which is \( \mathcal{O}(\phi^4) \) in the lowest order Lagrangian. Hence the quantities which requires higher-order terms can be calculated. Our challenge with the use of our model is to ask whether it can give sufficient predictions consistent with the experimental facts only by taking the flavour SU(3) breaking structure into account, with only such an \( \mathcal{O}(\phi^4) \) term.

The first test of our model was performed by confronting its prediction for charge radii of pseudoscalars \( \pi^+, K^+ \) and \( K^0 \) with their experimentally obtained values. As a result, the predicted value for \( \langle r^2 \rangle_{\pi^+} \) is found to be slightly smaller. Also the charge radius of \( K^0 \) becomes exactly zero.

These consequences does not change even if we add the extra terms in Eq. (5.1) as was shown in § 5. From this fact we can say that the SU(3) breaking structure in the pseudoscalar-vector sector in our model is determined only by the chiral and hidden local symmetries.

We finally remark on the application of our Lagrangian to the actual calculation of \( K \) decays. The most familiar framework for it will be the effective Hamiltonian method\(^{(27)}\) with the factorization hypothesis. There the long distance contribution from QCD is considered to reside in the hadronic matrix elements of four-Fermi operators and is expected to be calculated by using our chiral Lagrangian. The unreliability to the results obtained by following this approach comes from the strong dependence of them on the renormalization point which is conceptually the matching scale between short and long distance physics.\(^{(3)}\) Hence we must at first reexamine this point in order to give definite prediction from our chiral Lagrangian.

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