Pionic Decay of Hypernucleus $^5\text{He}$

--- Green's Function Method ---

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The pionic decay of $^5\text{He}$ is investigated to obtain information on the $\Lambda$-nucleus interaction. Two types of $\alpha$-$\Lambda$ potentials are used for the purpose. One is a potential with central repulsion (Isle) and the other is a single Gaussian potential with no repulsion (SG). The obtained $\pi^-$ decay rate is 0.35 $T_\alpha$ for Isle and 0.27 $T_\alpha$ for SG, $T_\alpha$ being the total decay rate of a free $\Lambda$ particle. The former is in better agreement than the latter with the experimental value of 0.44 ± 0.11 $T_\alpha$. This result suggests that the $\alpha$-$\Lambda$ potential has central repulsion. In order to discuss final-state nuclear resonance effects Green's function method is introduced in the calculation of the continuum $\pi^-$ decay spectrum. Both the sharp $p_{\alpha\pi}$ and the broad $\gamma_{\alpha\pi}$ $\alpha$-$\pi$ resonances play roles to build up a peak structure in the spectrum. It is observed that the low-energy nuclear resonances enhance the partial decay rates.

§1. Introduction

Three aspects of hypernuclear study are production, structure and decay of systems with one or more hyperon(s). It is timely at present to develop a treatment of the weak decay of hypernuclei because of the following trends of experimental developments. Data of both mesonic and nonmesonic decay modes were obtained in the cases of $^3\Lambda\text{He}$ and $^3\Lambda\text{C}$ at BNL in 1986. The lifetime of $^4\Lambda\text{H}$ was measured in 1991 at KEK. Recently the new spectrometer TOROIDAL was installed there, which is expected to provide $\pi^-$ spectrum data from the weak decay of $\Lambda$ hypernuclei with good resolution of 1~2 MeV/c.

The hypernucleus $^5\Lambda\text{He}$ is an appropriate object to study. The first reason is that much experimental information is available from $\pi^-$ energy spectrum of emulsion data and from $\pi^-$, $\pi^0$ decay rates of BNL. The second reason is motivated by theoretical interests. In 1984 Kurihara et al. proposed an $\alpha$-$\Lambda$ potential with central repulsion, named Isle, on the basis of the multiple scattering theory. The central repulsion in very light $\Lambda$ hypernuclei, however, has not been well established yet. Therefore, it is important to discuss effects of the central repulsion through observed quantities. The pionic decay of $^5\Lambda\text{He}$ is expected to give serious information on the $\alpha$-$\Lambda$ interaction, since the final-state nuclear structure of the decay is well-known. Indeed Kurihara et al. discussed the effect of the central repulsion on the decay rate of $^5\Lambda\text{He}$. Their results, however, are qualitative because the calculations were done with pion plane wave in a closure approximation. Recently, Motoba et al. obtained quantitative results of the decay rate and the decay spectrum with pion distorted wave. Their decay rate is in good agreement with the experimental data, but the fitting of their result to the spectrum is not satisfactory due to a simple final nuclear potential used. A proper final nuclear potential should be employed to evaluate the
The dependence of decay rates on the initial $\Lambda$ wave function.

The purpose of this work is to investigate the dependence of the pionic decay rates on the initial $\Lambda$-nucleus potentials. We use two types of $\alpha-\Lambda$ potentials: One is the Isle potential, and the other is a single Gaussian (SG) potential which consists of an attractive part only. Both the potentials reproduce the binding energy of $^6\text{He}$. Such investigation has been already done in Ref. 9. We, however, pay attention to the influence of final nuclear potentials on the pionic decay spectrum to extract precisely the effect of the initial $\Lambda$-nucleus potential.

Final states of the $^6\text{He}$ decay are mainly of three bodies, $\alpha+p+\pi^-$ and $\alpha+n+\pi^0$. In order to treat the continuum final states we introduce Green's function method, which was applied to $\Sigma$ hypernuclear problems by Morimatsu and Yazaki.\(^{10}\) The method has the advantage of treating not only continuum states but also bound ones in the same framework. Then it has a wide applicability to such cases as $^6\text{H}$ where both continuum and discrete spectra appear in the weak decay. The Green's function method is useful even for the cases of broad resonance or non-resonant scattering states, and is conveniently applied to the pionic decay of $^6\text{He}$ since it decays into three-body continuum final states with the broad $p_{1/2}$ $\alpha-N$ resonance.

The pionic decay has another importance: It can provide a means of detecting the production of light double-$\Lambda$ hypernuclei as discussed in the case of $^6\Lambda\text{He}$ by Motoba et al.\(^9\) The present work is intended as a starting point toward the weak decay spectroscopy of identifying double-$\Lambda$ hypernuclear systems.\(^{11}\)

§ 2. Formulation

2.1. Decay rate

First we derive the PWIA formula for the pionic decay of a $\Lambda$ hypernucleus. The $p\pi^-$ decay mode of $\Lambda$ is explicitly treated in this section: The $n\pi^0$ decay mode can be done in a similar way. The decay process is

$$^2\Lambda(Z) (\text{core nucleus } c+\Lambda) \rightarrow A^{-1}Z+p+\pi^-,$$

and the transition matrix is given by

$$T_n = \frac{\hbar}{\sqrt{2E_\pi}} \langle \Phi_{pc}, K'; k_\pi | T | 0, \Phi_{Ac} \rangle,$$  \hspace{1cm} (2.2)

where $\Phi_{Ac}$ and 0 are the relative wave function and the center-of-mass momentum of the initial $\Lambda$-$c$ system, $\Phi_{pc}$ and $K'$ are those of the final $p$-$c$ system, and $k_\pi$ is the momentum of an emitted pion. The factor $1/\sqrt{2E_\pi}$ comes from the normalization of the pion wave function. By considering a box with a size of $L^3$, Eq. (2.2) is written as

$$T_n = \frac{\hbar}{\sqrt{2E_\pi}} \left( \frac{L}{2\pi} \right)^2 \int dq_c dq_{c'} dq_{\pi} dq_{\pi'} \langle \Phi_{pc} | q' \rangle \langle K' | q_c' + q_{\pi'} \rangle \langle q_{c'} | q_c \rangle \langle q_{\pi'} | q_\pi + k_\pi \rangle \langle k' | l_{p\pi-A} | \Lambda \rangle \langle q_c + q_{\pi} | 0 \rangle \langle q | \Phi_{Ac} \rangle ,$$

$$k' = \frac{M_p k_\pi - m_\pi q_\pi}{M_p + m_\pi},$$  \hspace{1cm} (2.3)
where $q$ and $q'$ are relative momenta of the initial $\Lambda$-c and the final $p$-c systems, respectively. The notations are explained in Fig. 1.

Now we assume that the weak decay is a spectator process. Equation (2·3) is reduced to

$$T_n = \frac{\hbar c}{\sqrt{2}E_\pi} L^{-21/2} \delta(K' + k_\pi) \int dq'\rho(k' | t_{p\pi-\Lambda} | \Lambda \rangle \langle \Phi_{pc} | q' \rangle \langle q | \Phi_{\Lambda c} \rangle,$$

$$q = q' + k_\pi, \quad q' = q' + \frac{M_p}{M_c + M_p} k_\pi,$$

where a round bracket $|q\rangle$ means $L^{3/2}|q\rangle$ and its coordinate representation $\langle r | q \rangle$ is $\exp(i q \cdot r)$.

The partial decay rate $\mathcal{R}_n$ to a given $p$-c state is calculated by

$$\mathcal{R}_n = \frac{2\pi}{\hbar} \int \delta(E_f - E_i) \left( \frac{L}{2\pi} \right)^3 dK' \left( \frac{L}{2\pi} \right)^3 dk_\pi |T_n|^2,$$

where $\delta(E_f - E_i)$ means the energy conservation. We safely regard the $t$-matrix as independent of the integration variable, because the weak interaction is mediated by the very heavy gauge boson. Thus we put $\langle k' | t_{p\pi-\Lambda} | \Lambda \rangle$ to be a constant $g_\Lambda$, and the $t$-matrix is factorized out of the integral. Then we get

$$\mathcal{R}_n = |g_\Lambda|^2 \frac{(\hbar c)^2}{\hbar(2\pi)^3} \int dk_\pi \delta(E_f - E_i) \frac{1}{2E_\pi} \left| \int dr \Phi_{pc}^*(r) e^{-i k_\pi \cdot r} \Phi_{\Lambda c}(r) \right|^2,$$

$$\beta = 1 - \frac{M_p}{M_c + M_p},$$

where $\beta$ is introduced to take into account a recoil of the core nucleus in the final state.

The decay rate $\mathcal{R}_n$ is expressed by using a final state $|n_f\rangle$ of the $p$-c system. The total $\pi^-$ decay rate $\mathcal{R}$ is obtained after the summation over all final states $|n_f\rangle$. 

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**Fig. 1.** Schematic pictures of the pionic decay process, explaining notations used in the text.
This PWIA formula is extended to Green's function one in the next subsection.

2.2. Green's function method

Final states of the pionic decay of $^{3}$He are in continuum spectrum. We have to perform the summation of infinite number of states to get the decay rate of Eq. (2.7). Motoba et al. treated continuum states by using the Kapur-Peierls method. $^{12}$ To treat the continuum without approximation we introduce Green's function method into the calculation.

The initial-state and the final-state Hamiltonians are given as follows:

$$\mathcal{H}_i = \mathcal{H}_{Ac} + MAc^2 + Mc^2,$$

$$\mathcal{H}_f = \mathcal{H}_{pc} + Mpc^2 + T_K + E_{\pi} + Mc^2,$$  \hspace{1cm} (2.8)

where $T_K$ is the center-of-mass kinetic energy of the final $p$-$c$ system. The initial state $|i\rangle$ and the final state $|f\rangle$ are eigenstates of the Hamiltonians,

$$\mathcal{H}_i |i\rangle = (EAc + MAc^2 + Mc^2)|i\rangle = E_i |i\rangle,$$

$$\mathcal{H}_f |f\rangle = (Epc + Mpc^2 + T_K + E_{\pi} + Mc^2)|f\rangle = E_f |f\rangle.$$  \hspace{1cm} (2.9)

The energy conservation part of Eq. (2.7) is expressed with the relative energy $E_{pc}$ of the final $p$-$c$ system,

$$\delta(E_f - E_i) = \delta(\bar{E} - E_{pc}), \quad \text{where} \quad \bar{E} = E_{Ac} + MAc^2 - Mpc^2 - T_K - E_{\pi}.$$  \hspace{1cm} (2.10)

We intend to treat by Green's function method both two-body and three-body decay modes in the same framework. In the case of two-body decay, the kinetic energy $T_K$ depends on $E_{pc}$ through the total mass of the $p$-$c$ system. Thus, we introduce an auxiliary variable $E_{pc}$ and rewrite Eq. (2.10) as

$$\delta(\bar{E} - E_{pc}) = \int dE_{pc} \delta(E - E_{pc}) \delta(E_{pc} - E_{pc}).$$  \hspace{1cm} (2.11)

The quantity $E$ is defined by replacing $T_K$ in $\bar{E}$ with $T_K(\bar{E})$,

$$T_K(E_{pc}) = \frac{\hbar^2 k_s^2}{2(M_p + M_c + w \frac{E_{pc}}{c^2})},$$  \hspace{1cm} (2.12)

where $w$ is 1 for two-body decay and 0 for three-body decay. It is noted that $E$ is independent of the final state $|n_f\rangle$. By using $E_{pc}(\bar{E}) = \langle n_f | \mathcal{H}_{pc} | n_f \rangle$ we get

$$\delta(E_f - E_i) = \int dE_{pc} \delta(E - E_{pc}) \langle n_f | \delta(\mathcal{H}_{pc} - E_{pc}) | n_f \rangle.$$  \hspace{1cm} (2.13)

Some part of Eq. (2.7) is rewritten as follows:

$$\sum_{n_f} \delta(E_f - E_i) \int dr \Phi_{pc,n_f}^*(r) e^{-i\mathcal{H}_{pc}} \Phi_{Ac}(r).$$
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\[ \begin{align*}
\rho & = \int \delta(E - E_{pc}) \int dr dr' \sum_{n_f} \langle r' | n_f \rangle \langle n_f | \delta(H_{pc} - E_{pc}) | n_f \rangle \langle r | f^{*}(r') f(r), \tag{2.14} \\
& = \int \delta(E - E_{pc}) \int dr dr' \langle r' | \delta(H_{pc} - E_{pc}) | r \rangle f^{*}(r') f(r),
\end{align*} \]

where

\[ f(r) = e^{-i\hbar c r} \Phi_{Ac}(r). \]

By considering the relation

\[ \delta(E - \mathcal{H}) = \frac{1}{i \pi} \text{Im} \frac{1}{E - \mathcal{H} + i \varepsilon}; \quad \varepsilon \to 0, \tag{2.15} \]

we give the following Green's function formula for the total $\pi^-$ decay rate,

\[ \begin{align*}
\mathcal{R} & = |g_{\lambda}|^2 \left( \frac{\hbar c}{2(2\pi)^2} \right)^2 \int \frac{d\mathbf{k} e}{2E_{\pi}} \left( -\frac{1}{\pi} \right) \int dE_{pc} \delta(E - E_{pc}) \\
& \times \text{Im} \left[ \int dr dr' f^{*}(r') G_{Epc}(r', r) f(r) \right], \\
G_{Epc}(r', r) & = \langle r' \left| \frac{1}{E_{pc} - \mathcal{H}_{pc} + i \frac{I}{2}} \right| r \rangle,
\end{align*} \tag{2.16} \]

where $\Phi_{Ac}(r)$ is the relative wave function of the initial $\Lambda$-$c$ system, which reflects a certain property of the $\Lambda$-$c$ potential. $\mathcal{H}_{pc}$ in $G_{Epc}(r', r)$ is the Hamiltonian of the final $p$-$c$ system, where $T$ and $U_{pc}$ are the kinetic energy and the potential energy operators, respectively. It should be noted that $\varepsilon$ is put to be a finite value $\frac{I}{2}$ in Eq. (2.16). As shown in Appendix A, this $I$ is a smearing parameter, with which we can take into account the resolution of a detector. In the present calculation, we put it as 0.6 MeV considering the TOROIDAL spectrometer at KEK. In Eq. (2.16) the pion is treated with a plane wave as seen from $e^{-i\hbar c r}$ in $f(r)$. We can obtain the DWIA formula, replacing the plane wave by a distorted wave.

By performing the partial-wave expansion shown in Appendix B, we obtain the following expression,

\[ \begin{align*}
\int dr dr' f^{*}(r') G_{Epc}(r', r) f(r) & = \frac{2\mu \infty}{\hbar^2} \sum_{l=0}^{\infty} (2l + 1) \int dr dr' r^2 \phi_{Ac}(r) \phi_{Ac}^{*}(r') \\
& \times j_l(\beta_{kr} r) j_l(\beta_{kr'} r') \frac{u_{l+0}(r_1) u_{l+1}(r_2)}{W(u_{l+0}, u_{l+1})}, \tag{2.17} \\
\end{align*} \]

where $\mu$ is the reduced mass of the final $p$-$c$ system, $j_l(\beta_{kr} r)$ is the spherical Bessel function (or a corresponding distorted radial function in the case of DWIA). $\phi_{Ac}(r)$ is the radial wave function of the initial $\Lambda$-$c$ system. $W(u_{l+0}, u_{l+1})$ is the Wronskian of the functions $u_{l+0}(r)$ and $u_{l+1}(r)$. $u_{l+0}(r)$ and $u_{l+1}(r)$ are the solutions of the following
equation, of which boundary conditions are \( u_i(0) = 0 \) and \( u_i^{(+)}(r) \approx k_0 r t_i^{(+)}(k_0 r) \) in the asymptotic region, respectively,

\[
\begin{aligned}
&k_0^2 + \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \tilde{U}_p c(u_i(r) = 0,
&k_0 = \sqrt{\frac{2\mu}{\hbar^2}} (E_{pc} + \frac{i}{2}), \quad \tilde{U}_p c = \frac{2\mu}{\hbar^2} U_{pc}.
\end{aligned}
\tag{2.18}
\]

By using Eq. (2.17), the total \( \pi^- \) decay rate is written as follows:

\[
R = |g_A|^2 \left( \frac{\hbar c}{2\pi}\frac{1}{dE_{\pi}^*} \left( -\frac{1}{\pi}\int \frac{dE_{pc}\delta(E-E_{pc})}{2E_{\pi}} \right) \right)
\times \text{Im} \left[ \frac{2\mu}{\hbar^2} \sum_{l=0}^{\infty} (2l+1) \int dr dr' \phi_{AC}(r) \phi_{AC}^*(r') j_i(\beta_k r) j_i(\beta_k r') \right]
\times \frac{u_i^{(0)}(r_\pi) u_i^{(+)}(r_\pi)}{W(u_i^{(0)}, u_i^{(+)})}.
\tag{2.19}
\]

We make a change of the integration variable from the momentum to the energy of \( \pi^- \) as

\[
\int dk_{\pi} = \int d\Omega \int k_{\pi}^2 dk_{\pi} = \frac{4\pi}{\hbar c^3} \int E_{\pi} \sqrt{E_{\pi}^2 - m_{\pi}^2 c^4} dE_{\pi}.
\tag{2.20}
\]

The \( E_{pc} \) integration with \( \delta(E-E_{pc}) \) gives the following relation for three-body decay:

\[
E_{pc} = a_1 E_{\pi}^2 - E_{\pi} + a_2,
\]

\[
a_1 = \frac{1}{2(M_p + M_c)c^2}, \quad a_2 = E_{AC} + M_A c^2 - M_p c^2 - a_1 m_{\pi}^2 c^4.
\tag{2.21}
\]

The strength of the coupling constant \( g_A \) can be related to the experimental data of the free \( \Lambda \) decay. The expression for the total width \( \Gamma_\Lambda \) is derived in a similar way,

\[
b_{\pi^-} \Gamma_\Lambda = \frac{k_{\pi}^{(0)}}{2\pi} \left( \frac{E_{\pi}^{(0)}}{M_{pc}^2} \right) |g_A|^2,
\tag{2.22}
\]

where \( b_{\pi^-} \) and \( k_{\pi}^{(0)} \) \( (E_{\pi}^{(0)}) \) are the branching ratio and the pion momentum (energy) of the free \( \Lambda \to p + \pi^- \) decay mode.

With the aid of Eq. (2.22) the \( \pi^- \) decay rate of the \( \Lambda \) hypernucleus is finally given by

\[
\Gamma_{\pi^-} = \hbar R = \frac{1}{\pi} \left( b_{\pi^-} \Gamma_\Lambda \right) \frac{1}{\hbar c} \frac{1 + M_{pc}^2}{k_{\pi}^{(0)}} \int dE_{\pi} \sqrt{E_{\pi}^2 - m_{\pi}^2 c^4}
\times \text{Im} \left[ \frac{2\mu}{\hbar^2} \sum_{l=0}^{\infty} (2l+1) \int dr dr' \phi_{AC}(r) \phi_{AC}^*(r') j_i(\beta_k r) j_i(\beta_k r') \right]
\times \frac{u_i^{(0)}(r_\pi) u_i^{(+)}(r_\pi)}{W(u_i^{(0)}, u_i^{(+)})}.
\tag{2.23}
\]
Spin degrees of freedom can be incorporated by the following replacements in the case of a spinless core nucleus,
\[
\Sigma (2l+1) \rightarrow \frac{\Sigma (2j+1)}{2}, \quad u_i \rightarrow u_{ij}.
\]
This formula is used to investigate the \( ^9\text{He} \) decay property in the next section.

**§ 3. Results and discussion**

The pionic decay modes of the hypernucleus \( ^9\text{He} \) are mainly of three-body decay as
\[
^9\text{He} \rightarrow \alpha + p + \pi^- \\
\rightarrow \alpha + n + \pi^0.
\]
In the emulsion data accumulated so far the \( \alpha + p + \pi^- \) decay events are 2780, the \( ^8\text{He} + d + \pi^- \) ones are 15 and the other four-body events are negligible.\(^3\) Therefore, we consider only the decay modes given in Eq. (3·1).

### 3.1. Initial \( \alpha-\Lambda \) potential and final \( \alpha-p \) potential

Kurihara et al. presented the \( \alpha-\Lambda \) potential named Isle which has the repulsive core of about 50 MeV height at short distances.\(^7\) The potential is obtained from realistic \( \Lambda N \) interaction on the basis of the multiple scattering theory by Kerman, McManus and Thaler.\(^13\) We use the Isle and the SG potentials as the initial \( \alpha-\Lambda \) potential,
\[
U_{\alpha\Lambda} = V_c \exp\left(-\left(\frac{r}{b_c}\right)^2\right) - V_a \exp\left(-\left(\frac{r}{b_a}\right)^2\right),
\]
\[
V_c = 450.4 \text{ MeV}, \quad V_a = 404.9 \text{ MeV}, \quad b_c = 1.25 \text{ fm}, \quad b_a = 1.41 \text{ fm} \quad \text{for Isle},
\]
\[
V_c = 0, \quad V_a = 43.92 \text{ MeV}, \quad b_a = 1.566 \text{ fm} \quad \text{for SG}.
\]
Both of them reproduce the binding energy, 3.12 MeV of \( ^9\text{He} \). Figure 2 shows the potential shapes and the wave functions between \( \alpha \) and \( \Lambda \). Motoba et al. have done such comparison by using their potentials.\(^9\)

As the final nuclear potential we use Kanada et al.’s \( \alpha-p \) potential,\(^14\) which is derived from a microscopic calculation with the resonating-group method (RGM), and the Coulomb potential:
\[
U_{p\alpha} = \sum_{i=1}^{2} V_{i}^{C} \exp\left(\frac{-\mu_{i}^{C} r^2}{2}\right) + \left(-1\right)^{i} \sum_{i=1}^{3} V_{i}^{g} \exp\left(\frac{-\mu_{i}^{g} r^2}{2}\right)
\]
\[
+ (l \cdot s)\left( V_{18}^{ig} \exp\left(-\mu_{18}^{ig} r^2\right) + \left[1 + 0.3(-1)^{i-1}\right] \sum_{i=1}^{3} V_{i}^{gs} \exp\left(-\mu_{i}^{gs} r^2\right)\right) + V_{\text{coul}},
\]
\[
(3·3)
\]
where \( V_{i}^{C}[\text{MeV}] = (-96.3, 77.0), \quad \mu_{i}^{C}[\text{fm}^2] = (0.36, 0.9), \quad V_{i}^{g}[\text{MeV}] = (34.0, -85.0, 51.0), \quad \mu_{i}^{g}[\text{fm}^2] = (0.20, 0.53, 2.50), \quad V_{18}^{ig}[\text{MeV}] = -16.8, \quad \mu_{18}^{ig}[\text{fm}^2] = 0.52, \quad V_{i}^{gs}[\text{MeV}] = (-20.0, 20.0) \) and \( \mu_{i}^{gs}[\text{fm}^2] = (0.396, 2.2) \). The potential reproduces empirical phase shifts in
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Fig. 2. (a) $\alpha$-$\Lambda$ potentials and (b) $\alpha$-$\Lambda$ wave functions. The solid lines are for the Isle potential and the dashed lines are for the SG potential.

low energies of $\alpha$-$p$ scattering with the $p_{3/2}$ and $p_{1/2}$ resonances very well.

3.2. Pionic decay rates

We derive the pion distorted wave by solving the Klein-Gordon equation with a modified Kisslinger optical potential, whose parameters were phenomenologically determined by Seki et al.\textsuperscript{19} The pion wave function is used in the calculation of Eq. (2.17) instead of the spherical Bessel function $j_i(\beta k_{\pi} r)$. The calculated decay rates are summarized in Table I together with corresponding experimental data. PWIA results are given in the parentheses. The distortion of pion waves in final states
enhances the decay rate of $^8$He as discussed in Ref. 9).

In DWIA, the $\pi^-$ decay rate is obtained to be $0.35 \Gamma_A$ for Isle and $0.27 \Gamma_A$ for SG. The result of the Isle case is in better agreement with the experimental value of $0.44 \pm 0.11 \Gamma_A$ than that of the SG case. It denotes that the potential with central repulsion like Isle is more reliable as a description of the $\alpha$-$\Lambda$ interaction, though the error of the data is rather large. The $\pi^0$ decay rate is also a little larger for Isle than for SG, but no conclusion is derived in this case because of the large experimental error. If the data of the pionic decay rate of $^8$He are much improved, the pionic decay rate will give an experimental evidence of the central repulsion.

As seen in Table I, the pionic decay rates for Isle are generally larger than those for SG. The reason is explained from the overlapping of the initial $\alpha$-$\Lambda$ wave function with final $\alpha$-$p$ continuum states. The $\alpha$-$\Lambda$ wave functions of Isle and SG are compared with each other in Fig. 2(b). The distribution of $\Lambda$ extends more outside for Isle than for SG owing to the central repulsion. The pionic decay of $^8$He is mainly determined by the $p_{1/2}$ and $p_{3/2}$ resonances of the final $\alpha$-$p$ system as is discussed in the next subsection. The initial $\alpha$-$\Lambda$ wave function has a larger overlap with the $p$-wave resonances of final $\alpha$-$p$ states for Isle than for SG. On the contrary, if a bound state could exist in the $\alpha$-$p$ system, the decay rate would become larger in the SG case compared to the Isle one.

The total pionic decay rate of $^8$He is $0.55 \Gamma_A$ for Isle and $0.43 \Gamma_A$ for SG as shown in Table I. It means that the total pionic decay rate is suppressed from the free $\Lambda$ one. The suppression is due to the Pauli principle which excludes the bound state of the final $\alpha$-$p$ system. If we switch off the Pauli principle in the calculation, we get $0.47 \Gamma_A$ to the bound state for the Isle case. Thus, if the Pauli principle did not work, the total pionic decay rate could be $1.02 \Gamma_A$, i.e., essentially the free $\Lambda$ decay value.

### 3.3. Energy spectrum of weak decay pion

The energy spectrum of $\pi^-$ is shown in Fig. 3(a) for the Isle case together with the experimental data.

The spectrum shape is similar also in the SG case. A good agreement is obtained between the theoretical result and the data. Although the spectrum is of continuum due to the Pauli principle in the final $\alpha$-$p$ system, a smearing is done with $I=0.6$ MeV in the Green's function calculation. This is needed to compare the result with the experimental spectrum which was obtained by counting emulsion events with the energy interval of 0.5 MeV. In some cases non-smearing results were compared to the spectrum. In Fig. 3(b) we can see the smearing effect by making a comparison between the $I=0$ spectrum (dashed line) and the $I=0.6$ MeV one (solid line). The peak height of the spectrum is reduced by about 30% due to the smearing.
The decay $\pi^-$ spectrum for each partial wave is presented in Fig. 3(c) together with its partial decay rate. Important ingredients for the spectrum shape are contributions from the $p_{3/2}$ and the $p_{1/2}$ resonances of the final $\alpha-p$ system. The dash-dotted line and the dotted line in Fig. 3(c) are for the sharp $p_{3/2}$ resonance and for the broad $p_{1/2}$ one, respectively. The sharp $p_{3/2}$ resonance forms the spectrum peak and gives the largest contribution to the total decay rate. The broad $p_{1/2}$ resonance brings about a change of the shape in the total-energy $E_\pi=160\sim170$ MeV region of $\pi^-$. Thus, not only the $p_{3/2}$ but also the $p_{1/2}$ resonances play important roles to build up the peak structure of the spectrum.

The pionic decay rate and the $\pi^-$ spectrum of $^3$He have been intensively inves-
Fig. 3. (a) The $\pi^{-}$ energy spectrum of the $^8\text{He}$ decay calculated with DWIA for the $I=0$ case. The histogram shows the experimental data.

(b) Same as (a). The solid line is the case of the $I=0.6$ MeV smearing, and dashed line is the non-smearing case.

(c) Same as (a). The solid line is the total $\pi^{-}$ spectrum. The dash-dotted and the dotted lines are the contributions from the $p_{3/2}$ and the $p_{1/2}$ resonances of the final $\alpha-p$ system, respectively. The partial-wave contributions to the $\pi^{-}$ decay rate are shown in the figure.

In their calculation, however, the effect of the $p_{1/2}$ resonance is underestimated. In order to see it we make a calculation with the $\alpha-p$ potential $U_{\alpha-p}=-43.0$ MeV $\cdot \exp(-r/2.236 \text{ fm})^2-27.5$ MeV $\cdot \exp(-r/2.375 \text{ fm})^2 \cdot (l \cdot s) + V_{\text{out}}$ used by them. The resultant $\pi^{-}$ spectrum is compared with the original one in Fig. 4(a). The spectrum shows a deviation at around $E_x=168$ MeV. Their underestimation is owing to the use of an improper potential which does not reproduce the broad $p_{1/2}$ resonance. We should use a proper nuclear potential chosen carefully even for the broad resonance when we discuss the dependence of decay rates on the initial $\alpha-L$ potentials quantitatively. The decay spectrum shape is sensitive to the final-state nuclear interaction.

Figure 4(b) compares partial-decay spectra of the $p_{3/2}$ and the $p_{1/2}$ final states between the cases of no final-nuclear potential and of Kanada et al.'s potential. The $p_{3/2}$ partial-decay rate is 0.11 $\Gamma_L$ for the former case and 0.20 $\Gamma_L$ for the latter case: The $p_{3/2}$ resonance enhances the partial decay rate by a factor 1.8. The $p_{1/2}$ resonance also brings about an enhancement factor of 1.4. Thus, the low-energy nuclear resonances increase appreciably the pionic decay rate. It is noted that the pionic decay rate of $\Lambda$ in a nucleus is largely affected by low-energy nuclear resonances in addition to the Pauli principle and the pion distortion discussed in the previous subsection.

The $\alpha-p$ potential given by Kanada et al. has an $s$-wave bound state, whose contribution to the total decay rate must be removed because the bound state is forbidden by the Pauli principle. Phenomenologically a repulsive core is sometimes
Fig. 4. (a) Same as Fig. 3(a). The solid line is the total spectrum and the dashed line is the $p_{1/2}$ contribution for Kanada et al.'s $\alpha$-$p$ potential. The dash-dotted line and the dotted line are the total spectrum and the $p_{1/2}$ contribution for the $\alpha$-$p$ potential used in Ref. 9).

(b) Same as Fig. 3(a). The solid (dashed) and the long-dashed (dotted) lines are the $p_{3/2}$ ($p_{1/2}$) contributions for Kanada et al.'s $\alpha$-$p$ potential and for no $\alpha$-$p$ potential, respectively. The partial decay rates are also shown in the figure.

used as a substitute for the Pauli principle. In order to see an effect of this prescription, we derive a phase-shift equivalent local potential with no bound state from the $s$-wave attractive potential $U_{dp}^{(s)}$ of Eq. (3·3) on the basis of the super-symmetry theory.
Fig. 5. (a) $\alpha$-$p$ s-wave potentials and (b) scattering wave functions ($u_1(r)$) at an energy corresponding to $E_{\pi}=160$ MeV. The solid lines are for the original Kanada et al.'s potential and the dashed lines are for the derived repulsive potential.

Fig. 6. Same as Fig. 3(a). The solid line is the total spectrum and the dotted line is the $s_{12}$ contribution for the derived repulsive $\alpha$-$p$ potential. The dash-dotted line is the $s_{12}$ contribution for the original attractive $\alpha$-$p$ potential. The s-wave contributions ($S_{\text{sat.}}, S_{\text{rep.}}$) to the \(\pi^\pm\) decay rate are also shown in the figure.

\[
\bar{U}_{\alpha p}^{(s)} = U_{\alpha p}^{(s)} - \frac{\hbar^2}{\mu} \frac{d^2}{dr^2} \log \left[ 1 + \int_0^\infty dt (\phi_{\pi^\pm}(t))^2 \right]. \tag{3.4}
\]

In Fig. 5 the potential shape and the scattering wave function are compared between the derived and the original potentials. We get a large difference of the decay rate...
Table II. Comparison with other results. Figures in the parentheses are the values obtained by a closure approximation before averaging them.10 (unit in \( \Gamma_\pi \))

<table>
<thead>
<tr>
<th>Decay rate</th>
<th>Ours</th>
<th>Motoba et al.9</th>
<th>Straub et al.10</th>
<th>Exp.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^- )</td>
<td>0.35</td>
<td>0.393 (0.402, 0.459)</td>
<td>0.431</td>
<td>0.44±0.11</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>0.20</td>
<td>0.215 (0.227, 0.250)</td>
<td>0.239</td>
<td>0.18±0.20</td>
</tr>
<tr>
<td>Total</td>
<td>0.55</td>
<td>0.608 (0.628, 0.709)</td>
<td>0.670</td>
<td>0.59±0.01</td>
</tr>
</tbody>
</table>

between the two s-wave potentials. In the case of the derived repulsive potential the s-wave contribution to the total \( \pi^- \) decay rate is no longer suppressed and increases to 0.15 \( \Gamma_\pi \) from 0.04 \( \Gamma_\pi \). Figure 6 shows the total spectrum (solid line) and the s-wave contribution (dotted line) for the repulsive \( a-p \) potential together with the s-wave contribution (dash-dotted line) for the original attractive \( a-p \) potential. The s-wave contribution for the repulsive case is about four times as large as that for the attractive case at \( E_\pi = 160 \) MeV. The reason is easily seen from the scattering wave functions in Fig. 5(b). In the repulsive case no cancellation occurs up to 6 fm in the overlap integral of Eq. (2·7) since the scattering wave function does not change its sign, while in the attractive case a large cancellation takes place in that region. The spectrum shape for the repulsive potential deviates from the experimental one due to the large s-wave contribution. Thus we should employ the attractive potential microscopically founded on RGM. It is noted that the pionic decay spectrum also contains rich information on the s-wave final nuclear potential.

3.4. **Comparison with other results**

The pionic decay rate of the \( ^5\text{He} \) hypernucleus was investigated in distorted wave calculations by several authors.9,10,18 We compare our results with theirs in Table II. Two comments are given to compare our results with those of Motoba et al.9 The first is about the branching and the lifetime of the elementary process. A free \( \Lambda \) particle decays as

\[
\Lambda \rightarrow p + \pi^- (0.641) \text{ and } n + \pi^0 (0.357), \quad \tau_\Lambda = 2.631 \times 10^{-10} \text{ sec (3·5)}
\]

with the branching ratios denoted in the parentheses. The ratios are slightly different from 2/3 and 1/3 of the \( \Delta I = 1/2 \) rule. The empirical branching ratios of Eq. (3·5) are employed in our calculation, while an interaction Hamiltonian satisfying the \( \Delta I = 1/2 \) rule which gives \( \tau_\Lambda = 2.45 \times 10^{-10} \text{ sec} \) is used by them. To take into account this difference, the absolute values of their results should be multiplied by a factor 0.90 (0.97) in the case of the \( \pi^- (\pi^0) \) decay. The second comment is on the spin-flip amplitude in the elementary process. Motoba et al. properly treated the spin-flip term. Since the spin-flip term is known to be about 10% contribution, we deal with it in a good approximation by using the experimental data of the free \( \Lambda \) decay in Eq. (2·23). The error to the total decay rate is about 2% in the case of...
Three important ingredients of the pionic decay are the initial $\alpha$-$\Lambda$ interaction, the pion distortion and the final $\alpha$-$p$ interaction. Straub et al. obtained an $\alpha$-$\Lambda$ potential with repulsion at short distances on the basis of a nonrelativistic quark-cluster model, and discussed the importance of the pion distortion with a great care. However, since they used a closure approximation to treat continuum final states, they could discuss neither the $\pi^-$ spectrum shape nor the $\alpha$-$p$ potential effect. The final nuclear potential brings the large contributions from the $p_{3/2}$ and the $p_{1/2}$ resonances as is mentioned in the previous subsection, but these features are not reflected in the closure approximation. Thus their decay rates obtained by averaging the closure-approximation values are rather crude, though it is interesting that they showed the appearance of the short-range repulsion by the quark-cluster model.

§ 4. Summary and conclusion

The Green's function method is introduced in the calculation of the pionic decay spectrum of $\Lambda$ hypernuclei. It provides a simple means to treat continuum final nuclear states, and is useful even for the cases of broad resonance or non-resonant scattering states. Thus the method is conveniently applied to the pionic decay of $\Lambda^+$He which decays into three-body continuum final states with also the broad $p_{1/2}$ $\alpha$-$N$ resonance. In the Green's function method the spectrometer resolution is easily incorporated into the spectrum with any Lorentzian weight. Such incorporation is necessary to compare the result with the experimental data.

In the present calculation we use two types of $\alpha$-$\Lambda$ potentials, Isle and SG, in order to obtain information about the initial $\Lambda$ wave function. The effect of final nuclear potentials is investigated in detail to know precisely the dependence of the decay rate on the initial $\alpha$-$\Lambda$ potentials. The obtained $\pi^-$ decay rate is $0.35 \Gamma_{\alpha}$ for Isle and $0.27 \Gamma_{\alpha}$ for SG. The former is larger than the latter due to well-overlapping between the initial and the final wave functions. The experimental value is $0.44 \pm 0.11 \Gamma_{\alpha}$. Thus the Isle potential seems to be a better description of the $\alpha$-$\Lambda$ interaction than the SG potential in accordance with Motoba et al.'s conclusion, though the experimental error is rather large. If the experimental data of the pionic decay rate of $\Lambda^+$He is much improved, it will give an evidence of the central repulsion.

The pionic decay spectrum is carefully investigated in relation to the influence of final-state nuclear potentials. As for the $s$-wave nuclear potential, the spectrum shows that the $\alpha$-$p$ interaction is attractive as is founded on the microscopic resonating-group theory. The peak in the $\pi^-$ spectrum is mainly determined by the sharp $p_{3/2}$ resonance of the $\alpha$-$p$ system. The $p_{1/2}$ resonance gives an appreciable contribution to the spectrum in the $160$~$170$ MeV total-energy region of $\pi^-$. Thus both the resonances play roles to build up the peak structure of the spectrum. It is also observed that the $p_{3/2}$ and the $p_{1/2}$ resonances enhance the respective partial decay rates by 1.8 and by 1.4 compared to no nuclear potential cases. It should be remarked that the pionic decay rate of $\Lambda$-hypernucleus is largely affected not only by the Pauli principle and the pion distortion but also by low-energy nuclear resonances.
Appendix A

We consider the smearing of the pionic decay spectrum in the Green's function method. From Eqs. (2.14), (2.16) and (2.20) the decay rate is expressed as

\[ R = \int dE_{\pi} G(E_{\pi}), \quad (A\cdot1) \]

\[ G(E_{\pi}) = \int dE_{pc} \sum_{n_f} \delta(E(E_{\pi}) - E_{pc}) F(k_{\pi}, n_f) \delta(\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E_{pc}), \quad (A\cdot2) \]

in the case \( \epsilon \to 0 \). The function \( G(E_{\pi}) \) is the non-smearing pion spectrum. To put it simply, we neglect the recoil term \( T_{\pi} \) in Eq. (2.10),

\[ E(E_{\pi}) = E_{\pi} + M \alpha c^2 - M_{\pi} c^2 - E_{\pi}. \quad (A\cdot3) \]

By carrying out the integration \( \int dE_{pc} \) with \( \delta(E(E_{\pi}) - E_{pc}) \) in Eq. (A.2), we obtain

\[ G(E_{\pi}) = \sum_{n_f} \delta(\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E(E_{\pi})) F(k_{\pi}, n_f). \quad (A\cdot4) \]

Now, let us consider the case \( \epsilon \to I/2 \). The pion spectrum is

\[ \bar{G}(E_{\pi}) = \int dE_{pc} \sum_{n_f} \delta(E(E_{\pi}) - E_{pc}) F(k_{\pi}, n_f) \]

\[ \times \left( -\frac{1}{\pi} \right) \text{Im} \left( \frac{1}{\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E_{pc} + i \frac{I}{2}} \right). \quad (A\cdot5) \]

The integration \( \int dE_{pc} \) with \( \delta(E(E_{\pi}) - E_{pc}) \) gives

\[ \bar{G}(E_{\pi}) = \sum_{n_f} F(k_{\pi}, n_f) \left( -\frac{1}{\pi} \right) \text{Im} \left( \frac{1}{\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E(E_{\pi}) + i \frac{I}{2}} \right). \quad (A\cdot6) \]

Here, we introduce an auxiliary integration with respect to \( E'_{\pi} \) with \( \delta(\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E(E'_{\pi})) \) and rewrite Eq. (A.6) as

\[ \bar{G}(E_{\pi}) = \int dE'_{\pi} \left( \sum_{n_f} \delta(\langle n_f | \mathcal{H}_{pc} | n_f \rangle - E(E'_{\pi})) F(k_{\pi}, n_f) \right) \]

\[ \times \left( -\frac{1}{\pi} \right) \text{Im} \left( \frac{1}{E(E'_{\pi}) - E(E_{\pi}) + i \frac{I}{2}} \right). \quad (A\cdot7) \]

Comparing the bracket \{ \} to Eq. (A.4), we regard the quantity \{ \} as the pion spectrum \( G(E'_{\pi}) \). But this is an approximate treatment because \( k_{\pi} \) is determined from not \( E'_{\pi} \) but \( E_{\pi} \). Therefore we should consider the case of an interval \( I \) in which \( F(k_{\pi}, n_f) \) does not depend on \( k_{\pi} \) so much. Since \( E(E'_{\pi}) - E(E_{\pi}) = E_{\pi} - E_{\pi} \) from Eq. (A.3), the following relation is obtained:
\( \mathcal{G}(E_z) = \int dE_z G(E_z) W(E_\pi - E_z) \),

\[
W(E_\pi - E_z) = \left( -\frac{1}{\pi} \right) \text{Im} \left( \frac{1}{E_\pi - E_z + i \frac{I}{2}} \right) = \frac{1}{\pi} \frac{2}{I} \frac{1}{1 + \left\{ \frac{2}{I}(E_\pi - E_z) \right\}^2},
\]

(A·8)

where \( W \) is the Lorentzian weight function normalized to the unity. Equation (A·8) shows that \( \mathcal{G}(E_z) \) of Eq. (A·5) is the pion spectrum averaged over the \( I \) region. Thus, the Green’s function method gives the smeared pion spectrum.

**Appendix B**

The Green’s function operator \( \mathcal{G}^{(+)} \) is defined by

\[
\mathcal{G}^{(+)} = \frac{1}{E_{pc} + \mathcal{H}_{pc} + i \frac{I}{2}}.
\]

(B·1)

The \( r \)-representation of \( \mathcal{G}^{(+)} \) satisfies the following equation,

\[
\left\{ E_{pc} + i \frac{I}{2} - \mathcal{H}_{pc}(r) \right\} G^{(+)}(r, r') = \delta(r - r').
\]

(B·2)

Considering the expansion of the delta function on the spherical harmonics,

\[
\delta(r - r') = \frac{\delta(r - r')}{r^2} \sum_{L=0}^\infty \sum_{M=-L}^L Y_{LM}(\hat{r}) Y_{LM}^*(\hat{r}'),
\]

(B·3)

we express Green’s function \( G^{(+)}(r, r') \) as follows:

\[
G^{(+)}(r, r') = \sum_{L=0}^\infty \sum_{M=-L}^L Y_{LM}(\hat{r}) \frac{G^{(+)}(r, r')}{rr'} Y_{LM}^*(\hat{r}').
\]

(B·4)

Then, Eq. (B·2) is reduced to the radial equation,

\[
\left\{ k_0^2 + \frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} - \bar{U}_{pc}(r) \right\} G^{(+)}(r, r') = \frac{2\mu}{h^2} \delta(r - r'),
\]

(B·5)

\[
k_0 = \sqrt{\frac{2\mu}{h^2} \left( E_{pc} + i \frac{I}{2} \right)} , \quad \bar{U}_{pc} = \frac{2\mu}{h^2} U_{pc}.
\]

We calculate two solutions \( u_L^{(0)}(r) \) and \( u_L^{(+)}(r) \) of the homogeneous equation with the respective boundary conditions, \( u_L^{(0)}(0) = 0 \) and \( u_L^{(+)}(r) \approx k_0 r h_L^{(+)}(k_0 r) \quad (r \to \infty) \), \( h_L^{(+)}(k_0 r) \) being the spherical Hankel function. With the aid of the two solutions we obtain the \( r \)-representation of Green’s function,

\[
G^{(+)}(r, r') = \frac{2\mu}{h^2} \sum_{L=0}^\infty \sum_{M=-L}^L Y_{LM}(\hat{r}) \frac{u_L^{(0)}(r) u_L^{(+)}(r')}{rr'} \frac{W(u_L^{(0)}, u_L^{(+)})}{W(u_L^{(0)}, u_L^{(+)})},
\]

(B·6)

where \( r_\prec \) and \( r_\succ \) correspond to the smaller and the larger of \( r \) and \( r' \) respectively. \( W(u_L^{(0)}, u_L^{(+)})) \) is the Wronskian of \( u_L^{(0)} \) and \( u_L^{(+)} \).

The information of the initial wave function and the final pion wave function is
included in $f(r)$. The pion wave $e^{-i\beta k_r r}$ with the recoil effect of the core nucleus is expanded as

$$e^{-i\beta k_r r} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} 4\pi (-i)^l j_l(\beta k_r r) Y_{lm}(\hat{r}) Y_{lm}^{\ast}(\hat{k}_r). \quad (B\cdot 7)$$

Now we obtain Eq. (2.17) by carrying out the angular integration of $f dr dr' f^{\ast}(r') \times G_{Ep}(r', \mathbf{r}) f(r)$ with the aid of Eqs. (B.6)~(B.7) and the orthogonality property of the spherical harmonics.

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