Strange Sea Polarization in the Proton
and
A Constituent Quark Model

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The valence-quark and $q\bar{q}$-sea polarizations in the polarized proton are calculated in the framework of a constituent quark model. It is pointed out that the configuration mixing of the symmetric three-quark state with the quark-diquark state gives a good fit to existing data on the strange sea polarization in the proton.

Recently, the Spin Muon Collaboration (SMC) reported on the measurement of the spin-dependent structure function of the deuteron\(^1\) and pointed out that its first moment is smaller than the prediction of the Ellis-Jaffe sum rule\(^2\) as already found by the European Muon Collaboration (EMC).\(^3\) Namely, they conclude that the quarks do not carry the spin of the nucleon and the strange sea polarization is large and negative. On the other hand, the SLAC group (E142)\(^4\) reported the opposite result from the measurement at $Q^2 = 2 \text{ GeV}^2$, i.e., the quarks carry half the nucleon spin and the strange sea polarization is consistent with the one expected by the naive parton model.\(^5\) This apparent discrepancy is an open question.

In this paper, we calculate the valence-quark and $q\bar{q}$-sea polarizations in the polarized proton in the framework of a constituent quark model.\(^6\) It is shown that the amount of the strange sea polarization is strongly dependent on the structure of constituents and the $SU_3$ symmetry breaking parameter in the $q\bar{q}$ sea polarization. It is pointed out that the configuration mixing of the simple three-quark state with the quark-diquark state gives a good fit to existing data on the strange sea polarization in the proton below $Q^2 = 12 \text{ GeV}^2$.

We allow a distinction between the valence and sea quarks in the proton with positive helicity. For the up- and down-quark polarizations, we write $\Delta u = \Delta u_v + \Delta u_s$ and $\Delta d = \Delta d_v + \Delta d_s$ where $\Delta u(\Delta d)$ denotes the polarization carried by the $u$ quark ($d$ quark). The subscripts $V$ and $S$ denote valence and sea, respectively. We introduce the valence-quark spin $\Sigma_v(= \Delta u_v + \Delta d_v)$.\(^7\) We assume the isospin invariance of the sea-quark with respect to the up- and down-quark but the broken $SU_3$ symmetry so that

$$\Delta u_s: \Delta d_s: \Delta s = \delta: \delta: 1,$$  \hspace{1cm} (1)

where $\Delta s_s$ denotes the strange sea polarization and $\delta$ is the $SU_3$ symmetry breaking
parameter.

The polarizations of the partons in the proton are constrained by the sum rules from the beta-decay of octet baryon as

\[ \Delta q_3 = (\Delta u - \Delta d), \]
\[ \Delta q_8 = \Delta u + \Delta d - 2\Delta s, \]

where the non-singlet components \( \Delta q_3 \) and \( \Delta q_8 \) are defined by the \( F/D \) parameter for the beta-decay of octet baryon as \( (F + D) \) and \( (3F - D) \), respectively and \( \Delta s = \Delta s_s \). Also, there is the equality \( (F + D) = G_A/G_V \). The weak decay amplitudes for octet baryon are the same at any scale since the axial vector currents are conserved in the chiral limit.

Let us calculate the parton polarizations in a proton in the framework of the constituent quark model. It is assumed that the nucleon is composed of constituent quarks, which are a complex object made of point-like partons. For the first moment of the polarized density of the \( u \) quark, we have

\[ \Delta u = \sum \Delta \phi^u_J \Delta H_{J\bar{p}}, \]

where \( \Delta \phi^u_J \) and \( \Delta H_{J\bar{p}} \) denote the \( u \)-quark polarization in the constituent of type \( J \) and the polarization of the \( J \) constituent in the polarized proton, respectively. The sum on the r.h.s. of Eq. (4) runs over all sorts of constituents, e.g., \( U, D, S, G, \bar{U}, \bar{D} \) and \( \bar{S} \) where \( U \) denotes the constituent quark of type \( u \) and so on.

The first moments of the valence \( u \)- and \( d \)-quark are given

\[ \Delta u_v = \Delta \phi^v \Delta U, \quad \Delta d_v = \Delta \phi^v \Delta D, \]

where \( \Delta \phi^v \) denotes the spin of the valence quark in the constituent quark and \( \Delta U(\Delta D) \) is \( \Delta H_{U\bar{p}}(\Delta H_{D\bar{p}}) \). From Eq. (2), we have

\[ \Delta q_3 = \Delta \phi^v (\Delta U - \Delta D), \]

because of the iso-spin invariance of the sea-quarks. Thus, the valence spin is given as

\[ \Sigma_v = \frac{\Delta q_3 (\Delta U + \Delta D)}{(\Delta U - \Delta D)} \]

Thus, it is independent of \( Q^2 \). Furthermore, the strange sea polarization can be estimated from Eq. (3). We have

\[ \Delta s = \frac{(\Delta q_8 - \Sigma_v)}{2(\delta - 1)}, \]

because \( \delta \neq 1 \). It is noted that the polarization \( \Delta s \) is dependent on the valence spin. The value of \( (\Delta q_8 - \Sigma_v) \) characterizes the amount of the \( SU_3 \) symmetry breaking of the \( q\bar{q} \) sea polarization.

The flavour singlet component \( \Delta \Sigma \) is given as

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s = \Delta q_8 + 3\Delta s \]

where the non-singlet components \( \Delta q_3 \) and \( \Delta q_8 \) are defined by the \( F/D \) parameter for the beta-decay of octet baryon as \( (F + D) \) and \( (3F - D) \), respectively and \( \Delta s = \Delta s_s \). Also, there is the equality \( (F + D) = G_A/G_V \). The weak decay amplitudes for octet baryon are the same at any scale since the axial vector currents are conserved in the chiral limit.
Therefore, by means of the values $\Delta q_s$, $\Delta q_s$ and $\Delta s$, the first moments of the spin dependent structure functions of the proton and the neutron are given as

$$I^p_{1t}(Q^2) = \int_0^1 dx g^p(x, Q^2) = \frac{1}{12} \Delta q_s + \frac{5}{36} \Delta q_s + \frac{1}{3} \Delta s$$

$$I^n_{1t}(Q^2) = \int_0^1 dx g^n(x, Q^2) = -\frac{1}{12} \Delta q_s + \frac{5}{36} \Delta q_s + \frac{1}{3} \Delta s$$

where $g^p(x, Q^2)$ ($g^n(x, Q^2)$) denotes the spin-dependent structure function of the proton (neutron). If $\Delta s = 0$, we have the Ellis-Jaffe result. Thus, these moments are determined by the amount of valence quarks and the $SU_f(3)$ symmetry breaking parameter.

Let us consider the nucleon models in order to calculate the valence spin. It is well known that the naive $SU(6)$ model ($L=0$) suggests $\Delta U = 4/3$ and $\Delta D = -1/3$. Thus, we have $G_A/G_V = 1$. In order to avoid the disagreement between model and experiment, we put the factor $\Delta \phi^V$ into adjustment with Eq. (5) as discussed in Ref. 6). The QCD with massless quarks suggests that the factor $\Delta \phi^V$ is equal to one owing to the helicity conservation. Therefore, we assume the following configuration mixing models:

(1) Model A

Badcock et al. considered the quark-diquark model. This idea has been applied to many physical processes and already received some support from the experimental result of the ratio $F_2^p/F_2^n$ at $x \sim 1$. The spin structure of the quark-diquark state $|Q(QQ)d\rangle$ in the proton is given as $\Delta U = 1$ and $\Delta D = 0$. For example, we consider $|U(UD)d\rangle$ where $(UD)_d$ is the spin singlet state. The $U$ constituent is parallel to the total spin. Thus, we have $\Delta U = 1$ and $\Delta D = 0$. This result is satisfied in a general case.

In order to get rid of the discrepancy between the naive $SU(6)$ model and the experimental data on the ratio $G_A/G_V$, we consider the superposed state of these states. There is no a priori reason for these states to satisfy the orthonormality conditions. However, we may consider that the overlap of their wave functions is very small since the state $|Q(QQ)_d\rangle$ contributes to the regions near $x \sim 1$ and the $SU(6)$ state $|QQ\rangle$ to the regions near $x \sim 1/3$. Therefore, we assume that they approximately satisfy the orthonormality conditions.

We consider

$$|\phi\rangle \approx \cos \theta_1 |QQQ\rangle + \sin \theta_1 |Q(QQ)_d\rangle$$

The mixing angle $\theta_1$ is determined from Eq. (5) with $\Delta \phi^V = 1$. Thus, we have $\cos^2 \theta_1 = 3(\Delta q_s - 1)/2$. When $F = 0.47$ and $D = 0.81$, $\cos^2 \theta_1 = 0.42$. The value of the valence spin $\Sigma_v$ is equal to one from Eq. (6).

(2) Model B

We assume the following mixing model:

$$|\phi\rangle = \cos \theta_2 |QQQ\rangle + \sin \theta_2 (|QQQ\rangle_G + |QQQ\rangle_S)$$
The state $(QQQ)_s$ denotes the wave function with a spin-1/2 three-constituent-quark colour-octet state and couples with a spin-1 colour-octet constituent gluon ($G$) to make a colour-singlet state with total spin $J=1/2$. The spin structure of the state $(UUD)_s$ with positive helicity is given as $\Delta U = 2/3$ and $\Delta D = 1/3$ on the analogy of the nucleon in the flavour octet. When $L=0$, the total constituent quark spin in the state $(QQQ)_s$ in the proton has a probability of $2/3$ of being antiparallel to the total spin and $1/3$ of being parallel. Thus, this mixing model suggests $L.J = -2/3$ and $L.J = -1/3$ in the proton with $J_z = 1/2$. This model was investigated by Lipkin in order to improve the ration $\mu^2/\mu^n$. From Eq. (5) with $\Delta J = 1$, the mixing angle is given as $\cos 2 \beta = (9L.J/Q_3 + 1)/16 \approx 0.78$. From Eq. (6), we have $\Sigma = (3\Delta Q_3 - 1)/4 \approx 0.71$.

Let us introduce the enhancement factor of the non-strange sea quarks $\bar{\delta}$ which is defined as $\bar{\delta} = u_s/s_s$ where $u_s$ and $s_s$ denote the first moments of the unpolarized parton distributions of the up- and strange-sea quarks, respectively. We have

$$\delta = \frac{\Delta u_s}{\Delta s_s} = \frac{\bar{\delta}}{A(u_s)/A(s_s)},$$

where $A(u_s)$ denotes the quark asymmetry namely $A(u_s) = \Delta u_s/u_s$ and so on. The underlying microscopic processes are the incoherent fragmentation of a gluon into a $q\bar{q}$ pair. Therefore, as a first approximation, we assume the flavour independence of the asymmetry, namely $A(u_s) = A(s_s)$. Thus, we get $\delta = \bar{\delta}$.

The CHARM and DFLM groups used $K = 1/4$ in the analyses of the unpolarized structure function obtained by the neutrino data at $Q^2=10 \text{GeV}^2$ where $K = 2s/(\bar{u} + \bar{d})$. The CHDS data suggest $K = 0.52 \pm 0.09$ over $10 \text{GeV}^2$. Also, the CCFR group determines $K = 0.373 \pm 0.051$ and $0.432 \pm 0.059$ with the charm quark mass $m_c = 1.31 \pm 0.25 \text{GeV}$ and $1.61 \pm 0.26 \text{GeV}$ at the region of $Q^2=23 \text{GeV}^2$ on average, respectively. Thus, we get $\bar{\delta} = 1/K = 2.5$ at $Q^2 \geq 10 \text{GeV}^2$, while the $K/\pi$ ratio in $p\bar{p}$, $\bar{p}p$ and $e^+e^-$ collisions at high energies gives us information on $K$ at low $Q^2(Q^2 \leq 2 \text{GeV}^2)$. The experimental value of the suppression factor $K$ is around $0.22 \pm 0.25$. Thus, we have $\bar{\delta} = 4 \sim 4.5$. The $Q^2$-dependence of $K$ is shown in Fig. 1. We parametrize as

$$K(Q^2) = \frac{Q^2}{(4.96 + 1.97Q^2)},$$

where $Q^2 = Q^2/S_0$ ($S_0 = 1 \text{GeV}^2$). This behaviour is natural because the gluon production increases with $Q^2$ and thus the $s\bar{s}$-pair production increases.

From Eq. (7), we can estimate the amount of $\Delta s$ without any parameter. The strange sea polarization extracted from the same $F$ and $D$ values is given as

![Fig. 1. The $Q^2$-dependence of the ratio $K = 2s/(\bar{u} + \bar{d})$. Data are taken from CHARM(•), CHDS(□) and CCFR(□). The value (○) at low $Q^2$ is estimated from the reactions of $e^+e^-$, $p\bar{p}$ and $\bar{p}p$. The solid line is given by Eq. (10).](https://academic.oup.com/ptp/article-abstract/92/4/909/1897523)
\[ \Delta s = \begin{cases} -0.16 \pm 0.08 & \text{for EMC (}Q^2 = 10.7 \text{ GeV}^2), \\ -0.21 \pm 0.11 & \text{for SMC (}Q^2 = 4.6 \text{ GeV}^2), \\ -0.01 \pm 0.06 & \text{for E142 (}Q^2 = 2 \text{ GeV}^2), \end{cases} \]

where \( F = 0.47 \pm 0.04 \) and \( D = 0.81 \pm 0.03 \).^{10}

From the assumption of \( \delta = \bar{\delta} \), the value of \( \delta \) is larger than one. Thus, \( \Delta s \) is negative, since \( \Sigma > \Delta q_s \) for both models.

If \( \delta \) is independent of \( Q^2 \) and equal to 2 \( \sim \) 2.5, we have

\[ \Delta s = \begin{cases} -0.2 \sim -0.133 & \text{for Model A}, \\ -0.055 \sim -0.037 & \text{for Model B}. \end{cases} \]

The results of Model A are in good agreement with the EMC and SMC values but inconsistent with the E142 one. While Model B is inconsistent with the EMC and SMC values but consistent with the E142 one. The difference of the magnitude of \( \Delta s \) between both the models is due to the amount of the valence spin.

If \( \delta \) is dependent on \( Q^2 \) as given by Eq. (10), the results are shown in Fig. 2. Model A is in good agreement with the data, while Model B is not. Also, our approach predicts that the \( Q^2 \) dependence of \( \Delta u \) and \( \Delta d \) shows a similar behaviour to the one of \( \Delta s \), while, with the increase of \( Q^2 \), \( \Delta \Sigma \) decreases strongly as compared with \( \Delta s \) from Eq. (8). These behaviours are supported by the data.\(^{17}\)

The spin sum rule of the proton is given as \( 1 = \Delta \Sigma + 2 \Delta L_z \) where \( \Delta L_z \) denotes the average value of the orbital angular momentum of all partons along the quantization axis. Model A suggests that \( \Delta \Sigma \) is nearly equal to zero with the increase of \( Q^2 \), since the positive spin of the \( u \) quark is canceled out by the spin of the \( d \) and \( s \) quarks. Thus, the spin of the proton is mainly carried by the orbital angular momentum of partons. On the other hand, Model B suggests that \( \Delta \Sigma \sim 0.54 \) at \( Q^2 \rightarrow \infty \). It is noted that the \( Q^2 \) dependence of \( \Delta s \) does not contribute to the Bjorken sum rule.

In conclusion, we have shown that the amount of the strange sea polarization is strongly dependent on the valence spin and the \( SU_f(3) \) symmetry breaking parameter in the \( q \bar{q} \) sea polarization because of the constraints of the sum rules of the weak decay amplitudes for octet baryon. Assuming the flavour independence of the quark asymmetry, we found that the configuration mixing of the symmetric three-quark state with the quark-diquark state gives a good fit to existing data on the strange sea polarization in the proton, while the mixing of the symmetric three-quark state with the quark-gluon state is consistent with the E142 data but inconsistent with the EMC and SMC results. Our approach improves the Ellis-Jaffe result for \( \Gamma_1^p \) and \( \Gamma_1^n \). This
is because that the value of $\delta$ is finite for our model but infinite for the Ellis-Jaffe model.

We have examined the sign and magnitude of the strange sea polarization. The $x$ dependence of the polarized structure function of the proton and the $Q^2$ dependence of the Bjorken sum rule will be discussed elsewhere.