

curvature is a major factor, although no tests with straight radial blades are reported for comparison. Past examples of fairly conventional regenerative machines whose only difference was radial versus curved blading, and which exhibited similar efficiencies (Singer-Diehl, Vortex Blowers models 915 and 925, designed by this writer) suggest that the efficiency improvement determined by Sixsmith and Altmann may be due either to the stationary guide ring which serves to improve the “chaotic diffusion process” in the side channel of the machine, or to improvements in inlet and exit porting, or both.

It is to be hoped in any event that the authors’ results will stimulate interest in the possibilities of performance improvement of this somewhat neglected turbomachine of significant promise.

## Author’s Closure

As pointed out by Dr. Hollenberg most of the improvement in the efficiency is almost certainly due to a more ordered and less chaotic flow pattern in the side channels. The losses associated with the transfer of momentum from the impeller to the fluid are reduced by the core which guides the fluid through “retroflow” blading which is designed to deflect the fluid through an angle of approximately 90 deg. The rate of flow through the blading is approximately proportional to the square root of the peripheral pressure gradient and the fluid leaves the impeller with a forward velocity which is greater than the peripheral velocity of the impeller. When the pressure gradient is increased by throttling the delivery, both the forward velocity at the blade exits and the number of impulses received by each particle of fluid between inlet and delivery is increased. The analysis shows how the flow velocities may be calculated. With the aid of this knowledge the inlet ports can be designed to ensure smooth entry to the blading and the exit port can be designed to match the velocity of the fluid at the outlet ends of the side channels. The exit port is followed by a conical diffuser to recover some of the exit kinetic energy thereby effecting a further small gain in efficiency.

Only two compressors have been constructed and further development work should result in significant gains in the efficiency.

## Cable Kinking Analysis and Prevention<sup>1</sup>

**F. Rosenthal.** It seems a shame that Mr. Ross’s Paper (76-WA/DE-4: “Cable Kinking Analysis and Prevention”) and mine (76-WA/APM-31: “The Application of Greenhill’s Formula to Cable Hockling”) presented simultaneously but a few doors apart at the ASME Winter Annual Meeting, could not have been presented together so as to permit an on the spot sharing of views. Regarding the Ross paper I would like to make these comments:

1 Mr. Ross’s finding that tensioned paper tape can hockle or kink is not as he suggests in conflict with my finding that circular steel rods cannot do so unless the tension is very low, on the order of .027 percent or less of the tensile strength. The essential difference between tapes and circular rods is that the former are highly flexible in one direction; the latter are not.

2 I agree with Mr. Ross that loops can be formed in cables under conditions with  $T^2/PEI < 4$ . Indeed, loops are formed for  $T^2/PEI = 0$  (force only, zero torque), when a column is subjected to a thrust exceeding 2.18 times the Euler buckling force. The complete locus in the torque-force load space of points for which a loaded rod or cable possesses a self-intersection is described in my paper 76-WA/APM-31 and depicted there in Fig. 1.

3 As shown in my paper, the actual configuration of a cable de-

pends on the loading history, not merely on the final load. In particular, for long cables which are initially straight and in tension and which are then twisted, Greenhill’s formula  $T^2/PEI = 4$  is the correct hockling criterion.

4 I do not think that the value of  $T^2/PEI = 2$  introduced by Mr. Ross has any special significance as a lower bound for loop formation, and I would have reservations about the assumptions (circular loop shape, for example) made by Mr. Ross to derive it. The quantity  $T^2/PEI$  is denoted in my Fig. 1 by  $F$ . The  $F = 4$  curve is asymptotic to the Greenhill tension curve  $\gamma = 0$  for large values of cable length  $\ell$ , and represents Greenhill’s formula for long cables in tension. The  $F = 2$  curve, also shown in the Figure, does not appear to possess any such special property, and as has been discussed in paragraph 2 above, loops can be formed for values of  $F$  well below 2 under appropriate conditions.

## Author’s Closure

The author appreciates the comments made by Rosenthal, not only for his interest, but also in that they help illuminate some mathematical and design aspects of the problem and also point out some points of my paper that might not have been presented as clearly as they should have been. It was gratifying to note that Rosenthal did not criticize, in particular, the mathematical technique presented in the paper except to note his reservations about the assumptions used in the energy method, specifically, the circular loop mode shape assumption.

The author feels however that it has been generally accepted that small errors in assumed deflections result in even “smaller” errors in the loads at which they occur and thereby felt justified in making the simplifying mode shape assumption. It should be noted that the author did however recommend in the paper that a future development need was the consideration of the actual loop formed during the “buckling” of the elastica. This could most easily be carried out using a numerical procedure.

Since Rosenthal’s paper (76-WA/APM-31) was a computer-based numerical solution it is very difficult to follow the details of his solution—even as given in his original paper [8].<sup>3</sup> All of Rosenthal’s quantitative discussion rely on comparison of the author’s paper with his own.

The author did not intend to imply that his results were in conflict with those of Rosenthal. In this regard the author would re-emphasize the statement made in his paper that he feels that both analyses are correct. The author is of the belief that the Greenhill Formula as utilized by both Liu [3] and Rosenthal gives the proper eigenvalue for the cable hockling. The author however, believes that there is a smaller value of torque at which a loop may form—at a lower energy value—perhaps as a result of imperfect (non-straight) cables, wave induced lateral loadings, etc., much the same as occurs for “snap-through” buckling of various structures, e.g., shallow spherical caps.

The author believes that a complete analysis of an actual cable would result in a solution similar to that shown qualitatively in the accompanying Fig. 1. The (upper) eigenvalue torque corresponding to the Greenhill formula is labeled  $T_U$ . The lower value,  $T_L$ , refers to a loop that can be caused to form at a lower energy level corresponding to the author’s analysis. Also depending on cable or load eccentricities, or the presence of other energy sources, the torque could follow the path of the dashed lines and a loop would then form at a much lower ( $\sim 1/2$ ) torque than that given by Rosenthal. This explanation is, of course, still hypothetical and given to explain the reason for, in the authors belief, there existing two “valid” solutions to the problem with, however, the lower (author’s) solution being preferable for design application.

In this regard the author considers Rosenthal’s comments regarding

<sup>1</sup> By A. L. Ross, JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, pp. 112–115.

<sup>2</sup> Naval Research Laboratory, Washington, D. C.

<sup>3</sup> Numbers in brackets refer to references given in the author’s paper.

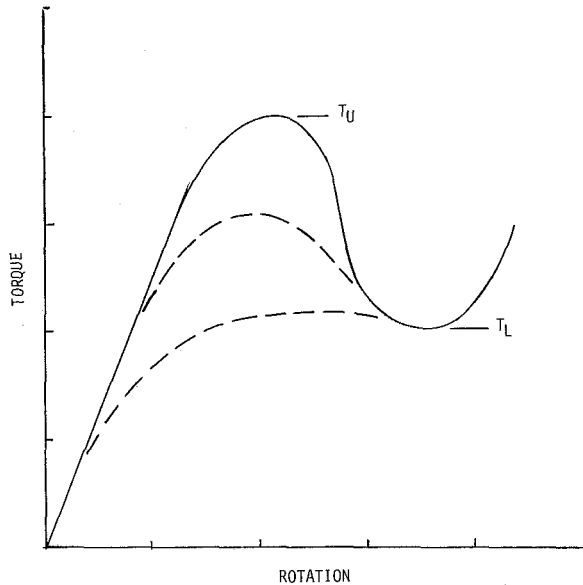


Fig. 1.

the authors prediction of loops forming at values of torque less than that given by  $T^2 < 4 PEI$  as rhetorical or possibly facetious, in that it was intended that the author's conclusions be restricted to cables in tension. In the author's opinion, loop formation in a flexible cable under an axial compressive load is a well known phenomenon (buckling) and for design purposes the condition would obviously be avoided if at all possible.

## On The Doubly Regenerative Stability of a Grinder: The Combined Effect of Wheel and Workpiece Speed<sup>1</sup>

**R. S. Hahn.**<sup>2</sup> The problem of grinding wheel chatter is one of the factors that limit the productivity of many production grinding operations. The development of chatter on the grinding wheel makes it necessary to interrupt the grinding process and dress the wheel. The time to dress the wheel frequently has to be added to the production cycle time. Frequent dressing tends to consume the grinding wheel and finally leads to a shut down of the machine for a wheel change operation. Consequently a better knowledge of grinding chatter may lead to improvements in grinding machine performance. The author is to be complimented on his analysis of regenerative chatter and encouraged toward further work on this subject.

In discussing this paper there are several points that should be considered in regard to practical grinding operations. It has been shown that the stock removal and wheel wear "laws"<sup>3</sup> for plunge grinding can be expressed by plotting the volumetric removal rate versus the interface normal force per unit width of wheel-work contact. For many commercial materials, the stock removal rate  $Z_W$ , can be expressed with reasonable accuracy as a linear function of normal force  $F_n$ :

$$Z_W = \Lambda_W F_n (\text{mm}^3/\text{s}) \quad (1)$$

where  $\Lambda_W (\text{mm}^3/\text{s} \cdot N)$  is the Metal Removal Parameter. In order to write the "linear cutting compliance"  $K_W (m/N)$  one can write equation (1) as

$$\pi D_W W \bar{v}_W = \Lambda_W F_n$$

where  $\bar{v}_W$  is the time rate of change of the workpiece radius,  $\omega$  is the width of grind. Then dividing both sides by the work speed  $N_W$  to get the wheel depth of cut  $h$  (mm) gives

$$h = \frac{\Lambda_W F_n}{\pi D_W N_W \omega}$$

and

$$K_W = \frac{h}{F_n} = \frac{\Lambda_W}{\pi D_W N_W \omega} \left( \frac{m}{N} \right) \quad (2)$$

for the "linear cutting compliance." This equation shows that  $K_W$  is inversely proportional to  $N_W$ . Therefore  $K_W$  cannot be considered in the paper as a constant, but would indeed vary as  $T_W$  is varied.

In single point machining operations the "cutting stiffness"  $K_c$  is generally independent of the cutting speed and is generally small compared to the machine stiffness  $k$ . In grinding the cutting stiffness is usually very large compared to the machine stiffness. Consequently as one increases workspeed, the wheel depth of cut  $h$  reduces and the stock removal rate  $Z_W$  remains almost constant as workspeed is increased. This means the cutting stiffness increases linearly with workspeed.

The permissibility of using a linearized wheel wear cutting compliance  $K_g$  is also dubious for practical grinding operations. The wheel wear curves in (\*) indicate that wheel wear increases linearly with interface force only over a small region near the origin. Consequently in practice violent chatter tends to occur much more rapidly under heavy forces than under low forces.

Finally, most production grinding operations take place in the unstable regime and so the question of stability limit is only partly involved. Rather the question is how much stock can be removed before the chatter amplitude reaches an unsatisfactory level.

## Author's Closure

The author wishes to thank Dr. Hahn for his very pertinent discussion of the workpiece speed dependence of the work wear coefficient,  $K_W$ . This effect is described and referenced in his discussion and is also documented elsewhere in his reports [3-4].<sup>4</sup> It was overlooked by the author in the main body of this paper, but in spite of this shortcoming, the result was useful in that the assumption of constant wear coefficients made possible the comparison of the single and doubly regenerative models.

In response to Dr. Hahn's point as well as in the interest of a more realistic and complete stability analysis, the question of speed dependent coefficients was taken up. The results are reported in what follows.

The grinding stability theory presented in the main body of this paper was based on the steady state response of a doubly regenerative system. From certain known characteristics of the steady response the boundaries separating unstable and stable operating regions were determined.

For the example problem (section 4) the wear coefficients were assumed constant and the stability boundaries were built-up by using  $\Omega_G T_G$  as an independent variable which was swept through its range  $-2\pi < \Omega_G T_G < 0$ .

Consider now the case of workpiece speed dependent wear and/or

<sup>1</sup> By R. A. Thompson, JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, No. 3, Aug. 1977.

<sup>2</sup> Manager of Research, Cincinnati, Mikeron-Heald Machine Division.

<sup>3</sup> Principles of Grinding-Part I, II, III, IV, V Machinery July, Aug., Sept., Oct., Nov., 1971.

<sup>4</sup> Numbers in brackets designate References at end of closure. Reference and Fig. numbers are continued from original paper.