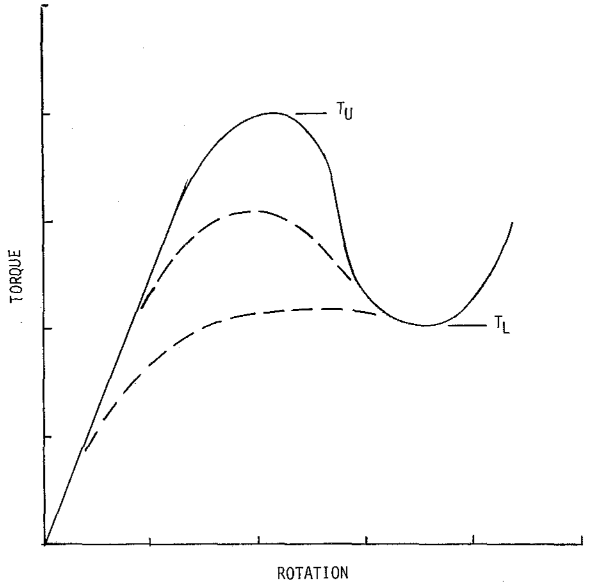

Fig. 1.

the authors prediction of loops forming at values of torque less than that given by $T^2 < 4\,PEI$ as rhetorical or possibly facetious, in that it was intended that the author's conclusions be restricted to cables in tension. In the author's opinion, loop formation in a flexible cable under an axial compressive load is a well known phenomenon (buckling) and for design purposes the condition would obviously be avoided if at all possible.

On The Doubly Regenerative Stability of a Grinder: The Combined Effect of Wheel and Workpiece Speed[1]

R. S. Hahn.[2] The problem of grinding wheel chatter is one of the factors that limit the productivity of many production grinding operations. The development of chatter on the grinding wheel makes it necessary to interrupt the grinding process and dress the wheel. The time to dress the wheel frequently has to be added to the production cycle time. Frequent dressing tends to consume the grinding wheel and finally leads to a shut down of the machine for a wheel change operation. Consequently a better knowledge of grinding chatter may lead to improvements in grinding machine performance. The author is to be complimented on his analysis of regenerative chatter and encouraged toward further work on this subject.

In discussing this paper there are several points that should be considered in regard to practical grinding operations. It has been shown that the stock removal and wheel wear "laws"[3] for plunge grinding can be expressed by plotting the volumetric removal rate versus the interface normal force per unit width of wheel-work contact. For many commercial materials, the stock removal rate Z_W, can be expressed with reasonable accuracy as a linear function of normal force F_n:

$$Z_W = \Lambda_W F_n (\text{mm}^3/\text{s}) \qquad (1)$$

where $\Lambda_W(\text{mm}^3/\text{s}.\ N)$ is the Metal Removal Parameter. In order to write the "linear cutting compliance" $K_W(m/N)$ one can write equation (1) as

$$\pi D_W W \bar{v}_W = \Lambda_W F_n$$

where $\bar{v}_W$ is the time rate of change of the workpiece radius, ω is the width of grind. Then dividing both sides by the work speed N_W to get the wheel depth of cut h (mm) gives

$$h = \frac{\Lambda_W F_n}{\pi D_W N_W \omega}$$

and

$$K_W = \frac{h}{F_n} = \frac{\Lambda_W}{\pi D_W N_W \omega} \left(\frac{m}{N}\right) \qquad (2)$$

for the "linear cutting compliance." This equation shows that K_W is inversely proportional to N_W. Therefore K_W cannot be considered in the paper as a constant, but would indeed vary as T_W is varied.

In single point machining operations the "cutting stiffness" K_c is generally independant of the cutting speed and is generally small compared to the machine stiffness k. In grinding the cutting stiffness is usually very large compared to the machine stiffness. Consequently as one increases workspeed, the wheel depth of cut h reduces and the stock removal rate Z_W remains almost constant as workspeed is increased. This means the cutting stiffness increases linearly with workspeed.

The permissibility of using a linearized wheel wear cutting compliance K_g is also dubious for practical grinding operations. The wheel wear curves in (*) indicate that wheel wear increases linearly with interface force only over a small region near the origin. Consequently in practice violent chatter tends to occur much more rapidly under heavy forces than under low forces.

Finally, most production grinding operations take place in the unstable regime and so the question of stability limit is only partly involved. Rather the question is how much stock can be removed before the chatter amplitude reaches an unsatisfactory level.

Author's Closure

The author wishes to thank Dr. Hahn for his very pertinent discussion of the workpiece speed dependence of the work wear coefficient, K_W. This effect is described and referenced in his discussion and is also documented elsewhere in his reports [3–4].[4] It was overlooked by the author in the main body of this paper, but in spite of this shortcoming, the result was useful in that the assumption of constant wear coefficients made possible the comparison of the single and doubly regenerative models.

In response to Dr. Hahn's point as well as in the interest of a more realistic and complete stability analysis, the question of speed dependent coefficients was taken up. The results are reported in what follows.

The grinding stability theory presented in the main body of this paper was based on the steady state response of a doubly regenerative system. From certain known characteristics of the steady response the boundaries separating unstable and stable operating regions were determined.

For the example problem (section 4) the wear coefficients were assumed constant and the stability boundaries were built-up by using $\Omega_G T_G$ as an independent variable which was swept through its range $-2\pi < \Omega_G T_G < 0$.

Consider now the case of workpiece speed dependent wear and/or

[1] By R. A. Thompson, JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, No. 3, Aug. 1977.
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[3] Principles of Grinding-Part I, II, III, IV, V Machinery July, Aug., Sept., Oct., Nov., 1971.

[4] Numbers in brackets designate References at end of closure. Reference and Fig. numbers are continued from original paper.

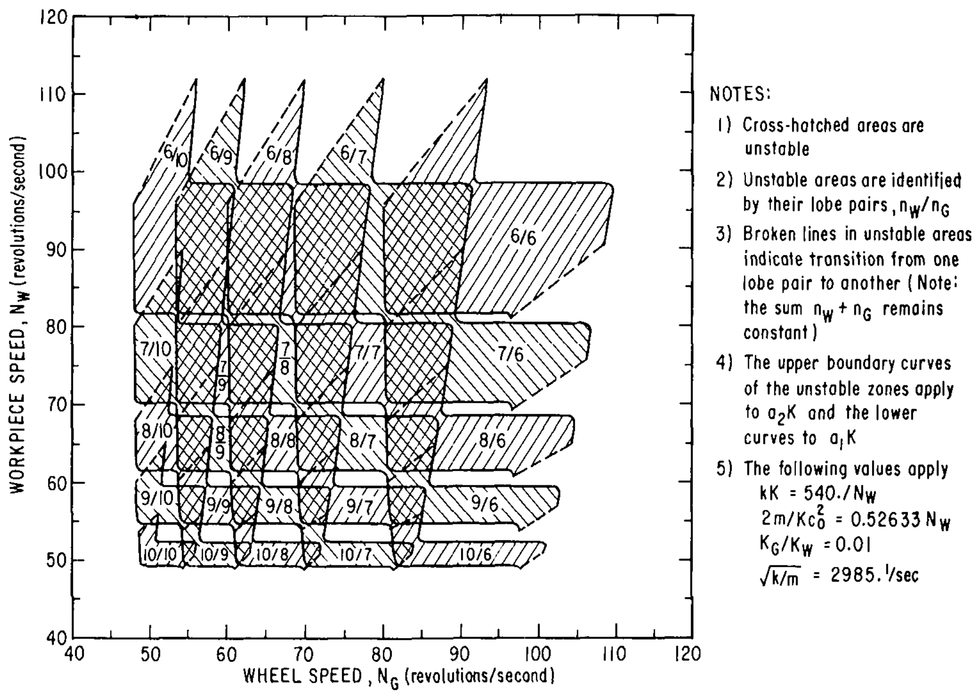

Fig. 3 The doubly regenerative, conditional stability of a plunge cylindrical grinder for which $K_W \alpha\ 1/N_W$.

machine coefficients. In this case, the doubly regenerative stability analysis can still be used so long as the steady state solution upon which it is based is valid. This requires that appropriate coefficients be expressed as functions of workpiece speed, N_W, and that N_W be chosen as the independent variable instead of $\Omega_G T_G$. To achieve this, the steps of the conditional stability analysis given in section 3 for constant coefficients must be modified to those which follow for workpiece speed dependent coefficients.

The Analysis of Conditional Stability for Work Speed Dependent Coefficients

1 Choose a workpiece speed, N_W, to be analyzed.

2 Calculate kK, $2m/Kc_0^2$ etc. as functions of N_W.

3 Calculate $a_1K\ (N_W)$ and $a_2K(N_W)$ from equation (13).

4 Calculate $\omega(a_1K)$ and $\omega(a_2K)$ using equation (12).

5 Determine the number of workpiece lobes associated with $\omega(a_1K)$ and $\omega(a_2K)$ from the equations

$$n_{W1} = \text{Integer}\,\{\omega(a_1K)/2\pi N_W\} + 1$$

$$n_{W2} = \text{Integer}\,\{\omega(a_2K)/2\pi N_W\} + 1$$

6 Determine the lobe precession angles per revolution, $\Omega_W T_W$ from the equations

$$\Omega_W T_{W1} = \omega(a_1K)/N_W - 2\pi n_{W1}$$

$$\Omega_W T_{W2} = \omega(a_2K)/N_W - 2\pi n_{W2}$$

7 Invert equations (25) and (26), solving them for $\Omega_G T_G$ in terms of aK and $\Omega_W T_W$. Determine $\Omega_G T_G(a_1K,\ \Omega_W T_{W1})$ and $\Omega_G T_G(a_2K,\ \Omega_W T_{W2})$.

8 Insert $\omega(a_1K)$ and $\Omega_G T_G(a_1K,\ \Omega_W T_{W1})$ into equation (21*a*) along with the number of lobes, n_G, to be analyzed and evaluate N_{G1}.

9 Insert $\omega(a_2K)$, $\Omega_G T_G(a_2K,\ \Omega_W T_{W2})$ and n_G into equation (22*a*) and evaluate N_{G2}.

10 For a given number of lobes n_W and n_G all speeds, N_G, between N_{G1} and N_{G2} are unstable. They represent the unstable wheel speeds associated with the work speed N_W.

11 Repeat steps 1 through 10 while sweeping N_W through some desired range. In this way, curves similar to those of Fig. 2 can be built-up for desired lobe pairs n_W/n_G. These curves give the stability boundaries for work speed dependent coefficients.

A computer program was written to perform steps (1) through (11). Along with the wheel speeds at the boundaries of stability the computer also printed out the chatter frequencies $\omega(a_1K)$ and $\omega(a_2K)$ for each workpiece speed, N_W.

The program was verified by demonstrating that Fig. 2 was reproduced exactly when the constant coefficients of equations (28) through (30) were used. The effect of a work speed dependent wear coefficient was then analyzed. The following parameters were arbitrarily chosen for the analysis.

$$kK_W \approx kK = kC/N_W = 540./N_W \quad (1A)$$

$$2m/K_W c_0^2 \approx 2m/Kc_0^2 = 2mN_W/Cc_0^2 = .52633N_W \quad (2A)$$

where, from equation (2) of Dr. Hahn's discussion $C = \Lambda_W/\pi D_W w$.

It will be noticed that for $N_W = 60$ rps kK and $2m/Kc_0^2$ reduce to the values given by equations (28) and (29). Since these were the constant values used to develop Fig. 2, one would expect the constant and work speed dependent analyses to coincide at $N_W = 60$ rps.

Using equations 1A and 2A the computer program was run for the speed range $40 \le N_W \le 120$. Fig. 3 shows the result. It is directly comparable to Fig. 2.

Since the workpiece wear coefficient, K_W, of Fig. 3 was inversely proportional to work speed there was a decrease in K_W or a stiffening effect associated with increased work speed. Increased cutting stiffness is a destabilizing effect and it is evidenced in Fig. 3 by much larger unstable areas above $N_W = 60$ rps when compared to the constant coefficient analysis of Fig. 2 (remember, the coefficients for the two analyses were equal at $N_W = 60$). At $N_W = 60$ rps, as would be expected, the stability was the same for the two analyses; while below $N_W = 60$ the variable coefficient case was more stable. In fact, so strong was this effect that unconditional stability occurred below $N_W = 47.2$ rps.

The comparison of Figs. 2 and 3 points to the importance of incorporating variable wear and machine coefficients into the stability

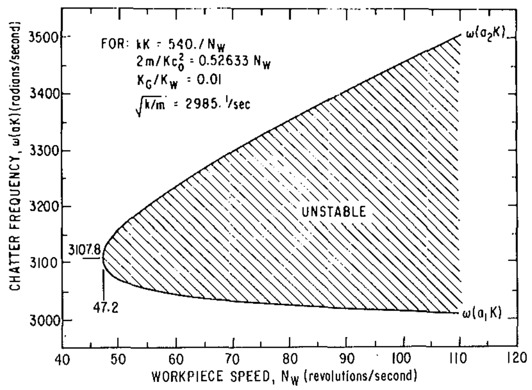

Fig. 4 Admissable chatter frequencies for the grinder illustrated by Fig. 3 i.e. $K_W \alpha 1/N_W$.

analysis of grinding systems. It tips the scale away from using high workpiece speeds as a deterrent to chatter. But, Fig. 3 does not give the complete story either, because K_W increases, or a softening effect generally accompanies increased wheel speed (see Fig. 6 of reference [3]). Thus, in reality, K_W is a function of both N_W and N_G.

The double dependence of the work wear coefficient can be analyzed by the doubly regenerative stability theory of this paper, but a much more complicated iterative computer program is needed. The development of such a program is beyond the scope of this paper, but without actually doing the work, one can still glean the result.

Since increased wheel speed is generally a linear softening effect while work speed is a linear stiffening effect the workpiece wear coefficient is in reality of the form:

$$K_W \propto N_G/N_W$$

Given this equation if N_G and N_W were varied such that $N_G \propto N_W$, one would move diagonally across the stability chart and K_W would remain constant. Under these conditions, Fig. 2 would be valid. Therefore, if one considers only the diagonal, Fig. 2 probably gives a good picture of stability. On the other hand, Fig. 3 is probably valid at low wheel speeds, while at higher wheel speeds, Fig. 3 should have much smaller unstable zones.

Finally, with regard to the physical significance of the destabilizing effect shown by Fig. 3, one might ask, what was the key observable factor underlying the large effect on stability of the variable wear coefficient? The answer appears to be the chatter frequence.

Fig. 4 shows the chatter frequencies, $\omega(a_1K)$ and $\omega(a_2K)$, versus workpiece speed at the limits of stability. These frequencies were constant for the constant coefficient analysis, having values given by equations (32) and (33), or conversely, from Fig. 4 at N_W = 60 rps. For the variable coefficient analysis however, the difference between $\omega(a_1K)$ and $\omega(a_2K)$ increased considerably with increasing work speed. This increased difference between $\omega(a_1K)$ and $\omega(a_2K)$ admitted a wider band of wheel and workpiece speeds where chatter could occur, consequently, the destabilizing effect. The reason for it was that $\omega(a_2K)$ became larger due to the stiffening effect while $\omega(a_1K)$ decreased. The decrease of $\omega(a_1K)$ with increased cutting stiffness runs counter to intuition. It was caused by the diminished relative effect of damping (i.e. the growth in equation (13) of the term $2m/Kc_0^2$ which tended to drive aK and with it $\omega(aK)$ toward the limits given by equation (42).

In fact, this approach of the maximum and minimum chatter frequencies to the limits of equation (42) appears to be a good indicator of the overall stability of the cutting system. For example, at N_W = 110 rps, the highest speed analyzed, the chatter frequencies $\omega(a_1K)$ and $\omega(a_2K)$ were less than 1 percent removed from their limiting values as given by equation (42) whereas for the constant coefficient analysis they were within only 20 percent of their limiting values. To the other end of the spectrum, according to Fig. 4 unconditional stability ensued below 47.2 rps. At this point $\omega(a_1K)$ equalled $\omega(a_2K)$ and their value was 3107.8 rad/s, 40 percent removed from the limits given by equation (42).

The analysis for a variable work wear coefficient as given by Fig. 3 answers the key point of Dr. Hahn's discussion. A brief response to Dr. Hahn's other points follows.

In the latter part of his discussion, Dr. Hahn's questions the use in the stability analysis of a linearized wheel wear coefficient (i.e., constant K_G) In order to answer this question, while leaving all other things unchanged, the computer program was run for various values of the ratio K_G/K_W. The result was that for all values $K_G/K_W < .01$ Fig. 3 was, for practical purposes, unchanged. In the other direction for $K_G/K_W > .01$ up to $K_G/K_W = .1$ there was only a slight change in the stability regions, the increasing value of K_G/K_W tending to round off the corners of the unstable zones in Fig. 3.

The implication is that in the practical range of the ratio K_G/K_W the magnitude of K_G has little effect on grinding stability. Consequently, as regards Dr. Hahn's point, it can be said with reasonable certainty that the particular form or function of K_G has little effect on the overall stability of a grinding system. This does not mean to imply, though, that the magnitude of K_G is unimportant; because under a given set of unstable conditions, although K_G has little to do with the instability itself, if it is doubled, wheel lobes will develop twice as quickly. Therefore, the most favorable case for K_G is to make it as small as possible consistent with good cutting properties.

Of course, if the situation required, K_G could be treated as a variable quantity just as K_W was earlier in this discussion.

Finally, in response to Dr. Hahn's last point, if conditions are unavoidably unstable, the growth rate of chatter can best be impeded by choosing cutting speeds which put the system as close as possible to stable regions of the stability chart. In these regions, less energy is available to increase the chatter amplitude and the magnitude of the instability is minimized.

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Influence of Nonlinear Wheel/Rail Contact Geometry on Stability of Rail Vehicles[1]

G. C. Martin,[2] V. K. Garg,[3] Y. H. Tse.[4] The authors are to be congratulated for presenting this study of the effect of nonlinear wheel/rail contact geometry on stability of rail vehicles. Although, nonlinear characteristics of wheel/rail interaction are presently being studied by various investigators, in writers' opinion this paper is a significant published work in this area.

The model presented in the paper has the motion of a wheel-set described by two parameters-lateral displacement and yaw rotation.

[1] By R. Hull and N. K. Cooperrider, published in the Feb. 1977 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, No. 1 pp. 172–185.

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