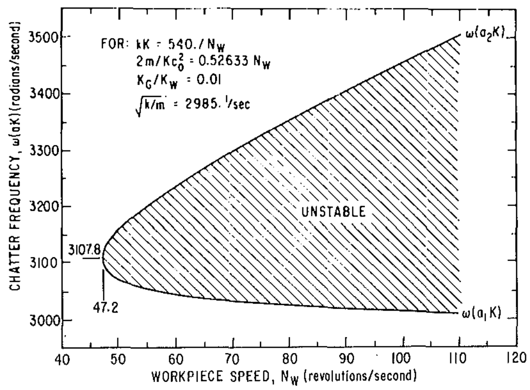

Fig. 4 Admissable chatter frequencies for the grinder illustrated by Fig. 3 i.e. $K_W \alpha 1/N_W$.

analysis of grinding systems. It tips the scale away from using high workpiece speeds as a deterrent to chatter. But, Fig. 3 does not give the complete story either, because K_W increases, or a softening effect generally accompanies increased wheel speed (see Fig. 6 of reference [3]). Thus, in reality, K_W is a function of both N_W and N_G.

The double dependence of the work wear coefficient can be analyzed by the doubly regenerative stability theory of this paper, but a much more complicated iterative computer program is needed. The development of such a program is beyond the scope of this paper, but without actually doing the work, one can still glean the result.

Since increased wheel speed is generally a linear softening effect while work speed is a linear stiffening effect the workpiece wear coefficient is in reality of the form:

$$K_W \propto N_G/N_W$$

Given this equation if N_G and N_W were varied such that $N_G \propto N_W$, one would move diagonally across the stability chart and K_W would remain constant. Under these conditions, Fig. 2 would be valid. Therefore, if one considers only the diagonal, Fig. 2 probably gives a good picture of stability. On the other hand, Fig. 3 is probably valid at low wheel speeds, while at higher wheel speeds, Fig. 3 should have much smaller unstable zones.

Finally, with regard to the physical significance of the destabilizing effect shown by Fig. 3, one might ask, what was the key observable factor underlying the large effect on stability of the variable wear coefficient? The answer appears to be the chatter frequence.

Fig. 4 shows the chatter frequencies, $\omega(a_1K)$ and $\omega(a_2K)$, versus workpiece speed at the limits of stability. These frequencies were constant for the constant coefficient analysis, having values given by equations (32) and (33), or conversely, from Fig. 4 at N_W = 60 rps. For the variable coefficient analysis however, the difference between $\omega(a_1K)$ and $\omega(a_2K)$ increased considerably with increasing work speed. This increased difference between $\omega(a_1K)$ and $\omega(a_2K)$ admitted a wider band of wheel and workpiece speeds where chatter could occur, consequently, the destabilizing effect. The reason for it was that $\omega(a_2K)$ became larger due to the stiffening effect while $\omega(a_1K)$ decreased. The decrease of $\omega(a_1K)$ with increased cutting stiffness runs counter to intuition. It was caused by the diminished relative effect of damping (i.e. the growth in equation (13) of the term $2m/Kc_0{}^2$ which tended to drive aK and with it $\omega(aK)$ toward the limits given by equation (42).

In fact, this approach of the maximum and minimum chatter frequencies to the limits of equation (42) appears to be a good indicator of the overall stability of the cutting system. For example, at N_W = 110 rps, the highest speed analyzed, the chatter frequencies $\omega(a_1K)$ and $\omega(a_2K)$ were less than 1 percent removed from their limiting values as given by equation (42) whereas for the constant coefficient analysis they were within only 20 percent of their limiting values. To the other end of the spectrum, according to Fig. 4 unconditional stability ensued below 47.2 rps. At this point $\omega(a_1K)$ equalled $\omega(a_2K)$ and their value was 3107.8 rad/s, 40 percent removed from the limits given by equation (42).

The analysis for a variable work wear coefficient as given by Fig. 3 answers the key point of Dr. Hahn's discussion. A brief response to Dr. Hahn's other points follows.

In the latter part of his discussion, Dr. Hahn's questions the use in the stability analysis of a linearized wheel wear coefficient (i.e., constant K_G) In order to answer this question, while leaving all other things unchanged, the computer program was run for various values of the ratio K_G/K_W. The result was that for all values $K_G/K_W < .01$ Fig. 3 was, for practical purposes, unchanged. In the other direction for $K_G/K_W > .01$ up to $K_G/K_W = .1$ there was only a slight change in the stability regions, the increasing value of K_G/K_W tending to round off the corners of the unstable zones in Fig. 3.

The implication is that in the practical range of the ratio K_G/K_W the magnitude of K_G has little effect on grinding stability. Consequently, as regards Dr. Hahn's point, it can be said with reasonable certainty that the particular form or function of K_G has little effect on the overall stability of a grinding system. This does not mean to imply, though, that the magnitude of K_G is unimportant; because under a given set of unstable conditions, although K_G has little to do with the instability itself, if it is doubled, wheel lobes will develop twice as quickly. Therefore, the most favorable case for K_G is to make it as small as possible consistent with good cutting properties.

Of course, if the situation required, K_G could be treated as a variable quantity just as K_W was earlier in this discussion.

Finally, in response to Dr. Hahn's last point, if conditions are unavoidably unstable, the growth rate of chatter can best be impeded by choosing cutting speeds which put the system as close as possible to stable regions of the stability chart. In these regions, less energy is available to increase the chatter amplitude and the magnitude of the instability is minimized.

References

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4 Hahn, R. S., and Lindsay, R. P., "The Influence of Process Variables on Material Removal, Surface Integrity, Surface Finish, and Vibration in Grinding," *Proceedings, 10th International M.T.D.R. Conference*, University of Birmingham, September, 1969.

Influence of Nonlinear Wheel/Rail Contact Geometry on Stability of Rail Vehicles[1]

G. C. Martin,[2] V. K. Garg,[3] Y. H. Tse.[4] The authors are to be congratulated for presenting this study of the effect of nonlinear wheel/rail contact geometry on stability of rail vehicles. Although, nonlinear characteristics of wheel/rail interaction are presently being studied by various investigators, in writers' opinion this paper is a significant published work in this area.

The model presented in the paper has the motion of a wheel-set described by two parameters-lateral displacement and yaw rotation.

[1] By R. Hull and N. K. Cooperrider, published in the Feb. 1977 issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, No. 1 pp. 172–185.

[2] Director, Dynamics Research Division, Association of American Railroads, Research & Test Center, Chicago, Illinois.

[3] Manager, Dynamics Research Division, Association of American Railroads, Research & Test Center, Chicago, Illinois.

[4] Research Engineer, Association of American Railroads, Research & Test Center, Chicago, Illinois.

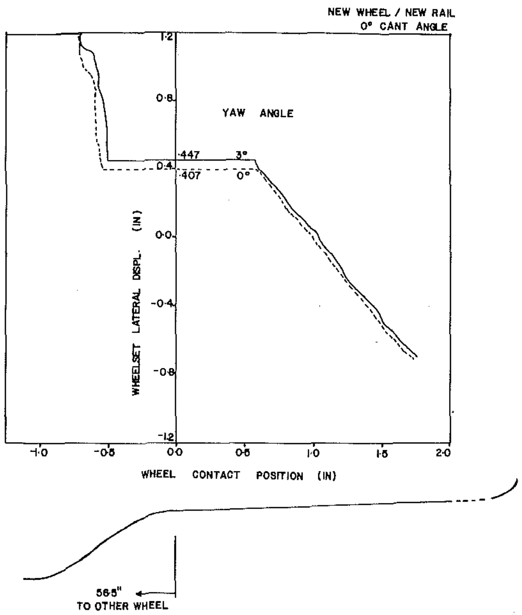

Fig. 1

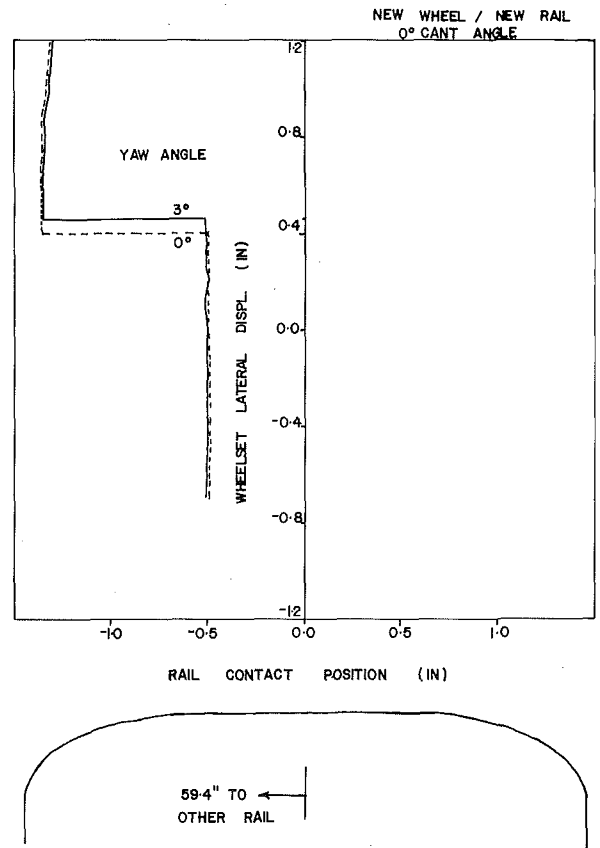

Fig. 2

The roll and vertical motion are determined by geometric constraint. This representation may not be adequate for the determination of an unstable hunting region, since it does not allow the severe hunting condition in which the wheel flange impacts the rail with a significant force. Under this condition, the roll and vertical motion of the wheel are no longer constrained by lateral movement alone. This inadequacy is obvious when wheel-lift occurs. Consequently, the graphs obtained using these constraints for the geometrical characteristics may not be valid.

The authors assume that the wheel set yaw is a second order effect and can be neglected to find the contact positions and the constraint characteristics between wheel and rail. The nonlinear hunting model developed at the AAR has the capability to incorporate the yaw angle in finding these characteristics. Figs. 1 and 2 show that there is significant influence of yaw angle on wheel/rail contact position. The yaw angles in these figures are intentionally fixed, and the wheelset is gradually displaced laterally. The results indicate the wheel flange contact occurs earlier for 0 deg yaw angle. Also, after flange contact the wheelset contact position between the two yaw angles behaves differently. This can be attributed to the fact that a wheelset yaw angle gives more clearance for flange contact. Thus, it is felt that the wheelset yaw should be taken into account to find the contact position between wheel and rail, particularly for large amplitude limit cycle oscillations. Thus because of these two assumptions, the results for large amplitudes are questionable.

The authors have mentioned that at large amplitudes, friction in suspension elements causes hunting to occur at lower speeds. These results contradict the experimental results reported by Love.[5] Love found that constant-contact side bearings, which have the effect of increasing frictional resistance to yaw motion of the truck relative to the carbody, act to reduce the hunting tendency of a freight car. The writers are not aware of any published results indicating decrease in critical speed with applied friction particularly centerplate friction applied to conventional North American freight car equipment.

In light of the many significant differences between the results predicted by authors' model and those reported elsewhere, the validity of the model appears to be questionable.

D. J. Reynolds.[6] The authors of this paper show the introduction of dry friction into their model car, and they find that "at low amplitudes, the stability is drastically reduced." They account for this by pointing out that the friction tends to lock the parts together and make the whole model behave as a rigid body.

It is worth examining the values of friction that they have selected. The bowl torque of 606 ft. lbs. is a reasonable value for a partially lubricated bowl. But, it must be kept in mind that in a hunting freight car, the bowl will often be worn and dry; and the vertical edge of the bowl will always be rubbing heavily against the vertical bolster rim; considerations which would justify a higher value of bowl torque.

They have taken a value of 4271 ft. lbs. for warp torque (i.e., the resistance of the truck to unsquaring). This seems very high. Byrne and Andresen [1][7] show a car negotiating various curves, at least one third of the truck rotation being tramming rather than swivelling. This suggests strongly that resistance to one motion is not greatly more than resistance to the other. The writer [2] has estimated that the warp resistance is only 500 ft. lb. more than the bowl resistance in a moderately worn three-piece truck. In effect, the authors' model appear to be almost a rigid truck, and the frequencies reported, a little lower than experienced in the field, tend also to confirm this.

The writer believes that the phenomenon reported, limit cycles at small amplitudes, is real and can be observed in the field under suitable conditions. But, it is not what is ordinarily named "instability" and "hunting." The word used is "lively"—"The car is lively." It must be kept in mind that an oscillation of less than ¼-inch at normal frequencies is not an important matter in the present freight car, in

[5] Love, R. B. "Improved Suspension for 100-Ton cars on Rough Track; Effect of Heavy Axle Loads on Track," Presented at the 12th Annual Railroad Engineering, FRA Conf., Pueblo, Colorado, Oct. 1975.

[6] Asst. Manager, Research and Tests, Southern Railway Company, Mem. ASME.

[7] Numbers in brackets designate References at end of discussion.

which random oscillations normally produce an 0.2 G level (which would not be acceptable in a passenger car, of course).

I would urge further study of this condition, because it may cause wear on wheel and rail and truck parts, and work on it will undoubtedly give further insight on the full hunting condition. But, I would urge that other terminology be considered, because the use of hunting terminology leads to statements that will be misunderstood, such as the conclusion (p. 14), "that dry friction . . . . . will lower the critical speeds." One would consider this a plain prediction of derailment if one was not aware that a stable condition awaits, at a higher amplitude. Or consider the statement, also in the conclusions, that "This suggests that the stabilizing effects of friction found in the single wheel set analysis may not be of practical importance." The considerable industry that supplies and uses constant contact bearings for control of hunting may find this suggestion quite academic.

References

1 Byrne, R., and Andresen, J. A., "Performance Characteristies of Freight-Car Trucks Determined Through Road Testing," JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99 pp. 196–205 (Fig. 9.).

2 Reynolds, D. J., 74-RT-2, pg. 9.

Author's Closure

We wish to thank the discussers for their interest in our paper. We were particularly pleased to learn that Reynolds' had observed in the field the small amplitude limit cycles predicted in our analysis. We were somewhat surprised to find these limit cycles during our analysis, and doubted whether they could be actually detected in freight car operation. Reynolds' opinion that this phenomenon is real and observable is encouraging. We certainly did not intend to imply that these small oscillations led to derailment. Reynolds' suggestion is well taken that terminology such as "lively" should be used to describe these low amplitude limit cycles.

Apparently some readers suffered some confusion concerning the objectives of the analysis presented in our paper, and the intended purpose of the paper itself. The objective of the analysis presented in the paper is to predict the the influence of the rail, wheel and vehicle parameters on the occurance of limit cycle behavior, rather than to accurately simulate the actual limit cycle, or hunting behavior. Thus, our first concern is to accurately predict the speed when hunting begins, and the factors that influence that critical speed. Although we also believe our model and analysis produce a reasonable first estimate of the limit cycle amplitudes and force levels we do not expect our prediction of these quantities to be extremely precise.

Because the dynamic behavior, including the stability limits, of a freight car depends strongly on nonlinear behavior, and because the brute force method of solving such nonlinear problems by numerical integration is quite expensive and time consuming, our study, sponsored by the U.S. DOT Transportation Systems Center, focused on the possibility of using quasi-linearization techniques to solve this nonlinear problem. This technique proved both feasible and efficient. A comparison of quasi-linear results with analog integration of nonlinear wheelset equations of motion demonstrated the close agreement between the two methods [15]. Although precise run time comparisons are difficult to make, we estimate that the quasi-linearization technique utilizes, at the very most, 1/15 of the time needed to obtain stability results by numerical integration. For more complex models, the ratio may be more like 1/1000.

Thus, the primary purpose of our paper was to demonstrate how these quasi-linear techniques could be applied to a realistic model of a rail car, and to illustrate the ease of varying the various system parameters. The results presented should be interpreted in terms of actual freight car operation, but the study presented in the paper is not extensive enough to extrapolate the results to a wide variety of types of freight vehicles and components. We intend to publish such a study at a later date.

Martin, Garg and Tse question the validity of the results we presented because our model does not include the possibility of wheel lift and does not account for the influence of yaw angle on the wheel/rail geometric constraints. We believe that wheel-lift will only occur in the later stages of a severe hunting situation. Our model predicts the onset of the hunting situation, severe or otherwise. An accurate model for several hunting situations where wheel lift occurs should include rail flexibility in several directions, three dimensional wheel/rail geometric constraints, large contact angles, and a nonlinear creep theory that includes the effects of spin creep.

The influence of yaw on the wheel/rail geometric constraints as described by Martin, et. al., is to reduce the effective lateral spacing of the wheels. This shortening effect allows a slightly larger displacement of the wheelset before flange contact occurs when the wheelset is yawed relative to the track. However, this effect is a second order effect because the lateral movement of the wheel depends on the yaw angle to the second power, i.e.,

$$\delta = a(1 - \cos\theta) \simeq a\theta^2$$

where:

a—one half the rail gauge

θ—wheelset yaw angle.

For small angles the yaw angle has a very small influence on the position where flange contact occurs. Martin and his associates show that a 3 deg yaw angle changes the lateral position of the wheelset at flange contact by 1 mm (0.04 in). A more realistic yaw angle is 0.50 deg, the wheelset yaw amplitude found during hunting in the field tests conducted for us by the AAR Research Center and the Union Pacific Railroad. At this yaw angle the wheelset position at flange contact is shifted only 0.029 mm (0.001 in) from its lateral position when the yaw angle is zero. Both these displacements are less than the variations we expect due to rail gauge variation, rail head profile differences and rail deflections.

As mentioned above, the significant influence of yaw on the wheel/rail geometry is not the shift of lateral wheel position at flange contact, but the fact that yaw, or angle of attack, causes the wheel to contact the rail ahead or behind the axle centerline. This mechanism is in operation during wheel climb. This difficult three dimensional geometry problem has been recently solved by a group of West German researchers. Their report, [1],[8] compares and finds agreement between their results and results from our approach (reference [12] of original paper). A model intended to predict large amplitude motion and derailment should include this three dimensional geometry as well as the fact that small angle approximations for the contact angles are no longer valid.

Our choice of numerical values for dry friction represented in our analysis seems to have been a point of concern for all the discussers and perhaps many other readers. The specific values used in the paper were obtained from tests conducted by ASF and Martin-Marietta on the actual 70 ton truck that was used in the AAR Field Tests. This truck was in a nearly new condition, but we do not know whether this specific truck is representative of the majority of freight trucks in service. Certainly different values would be expected from an older vehicle. The warp torque was also taken from the Martin-Marietta test results.

We should have stated more clearly in the paper that when we varied the friction levels in the truck, we simultaneously varied friction at the sideframe/bearing adapter, the pedestal friction shoe, and the centerplate surfaces. Thus our observations pertain to a situation where all these friction levels vary and not a situation such as occurs with constant contact side bearings, where one of these friction levels is increased. In fact, subsequent parameter studies showed that the friction levels at the sideframe surfaces are the contributing factors to the decrease in stability observed in our study. As borne out by the AAR Field Tests as well as other tests, constant contact side bearings do have a stabilizing effect. The tests cited by Martin, et.al., do not

[1] Numbers in brackets designate References at end of closure.

contradict our analysis because those results pertain to a very different situation than that reported in the paper.

We are surprised to see Martin, et.al., dismiss the validity of our model and analysis at this early date on the basis of one limited parameter study. As these discussers are well aware, from the numerous presentations we have made to them, we have developed several models representing differing levels of detail in the representation of the vehicle dynamics. We are in the process of comparing results from these models and analyses with field test data recently provided us by the AAR Research Center. For those interested in details of this process, certain of these other analyses are described in a companion paper (reference [15] of original paper), and in a Federal Railroad Administration Report [2] now being printed. The field tests are described in [3] and our validation procedure is documented in [4]. Limited quantities of these documents are available from the authors.

In our opinion, the model presented in this paper quite adequately represents the dynamic behavior of typical freight cars with three piece trucks on tangent track. Improvement of the model for studying situations such as derailment mechanics should include representation of rail flexibility, nonlinear creep force laws and possibly three dimensional rail/wheel geometry effects. The first two of these effects are included in an analysis that we have developed for studying rail vehicle curving behavior. The wheel lift and wheelset yaw influence on wheel-geometry effects cited by Martin, et. al., are, in our opinion, relatively unimportant in accurately modeling rail vehicle dynamic behavior.

To our knowledge, ours is the most complete attempt yet to correlate theoretical and experimental results for rail vehicle dynamics. We hope that most people will withhold judgement on the validity of our models until we complete our validation study.

References

1 Hauschild, W., "Die Kinematik des Rad-Schiene Systems," Institut für Mechanik, Technische Universitat Berlin, Mar. 1977.

2 Law, E. H., Hadden, J. A., and N. K. Cooperrider, "General Models for Lateral Stability Analyses of Railway Freight Vehicles," FRA OR&D Report, Dec. 1976.

3 Cooperrider, N. K. and E. H. Law, "Program Plan for Field Test Validation of Lateral Freight Car Dynamic Analysis," Arizona State and Clemson Universities, July 1974.

4 Cooperrider, N. K. and E. H. Law, "Validation Plan for Freight Car Dynamic Analysis," Arizona State and Clemson Universities, June 1976.

Influence of Axial Load, Track Gage, and Wheel Profile on Rail Vehicle Hunting[1]

D. J. Reynolds.[2] This paper describes analyses conducted on simplified models of wheelsets. It is excellent that this work should be done, and it is evident that the authors' linearization of hunting conditions by the describing-function method gives us all new insights into this aspect of vehicle behavior. However, the title of the paper and the summary, mentioning and discussing "Rail-Vehicle Hunting," seem to extrapolate this computer analysis to the point of seeking to give immediate practical guidance. For instance, in the summary we read, "For freight car applications, coulomb friction in the suspension (e.g., constant contact side bearings) may act to increase the range of speeds over which hunting will not occur and may permit operation at higher speeds for extremely straight track."

[1]By D. N. Hannebrink, H. S. H. Lee, H. Weinstock and J. K. Hedrick JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 99, Feb. 1977, 99, 77, pp. 186–195.

[2] Asst. Mgr. Research and Tests, Southern Railway Company, Mem. ASME.

Noting that by "extremely straight track" the authors mean "track extremely free from irregularities," one asks why the "mays" in this statement. It is common knowledge in the industry that dry friction so acts, and quite apart from the products on the market and in daily use, the action has been published many times [1, 2].[3]

Turning to the text to find the support for this statement about constant contact side bearers, Table 1 shows that the only breakout torsional friction, considered for a light car, is 1.916 × 0.00122, × 2, or 4675 ft.lbs. This is the appropriate figure for a moderately lubricated center plate alone [3]. Any effective side bearer would add at least twice as much resistance again. It would appear that the statement regarding constant contact side bearers may be a complete extrapolation.

In Fig. 12, it would appear that instability occurs at 125 ft./s. over a wide range of axle loads stated as 8E, 6E, 4E, and 2E. This is contrary to field experience where it almost is axiomatic that increased loading raises the critical speed, as is confirmed by the authors' Fig. 16(*b*), and this casts doubt on the usefulness of the linear-suspension model initially used. It also appears that for the first three loads, vibration will increase to an amplitude of 0.48-inch approximately, where it remains limited by flange action, but for the load of 2.00 E, there is no stability whatever. This is also contrary to field experience; unless a wheel is completely weightless or completely flangeless, it is bound to receive some restraint when the flange touches the rail. Possibly relevant to these points are the substitutions Kg^+ and $f_L{}^1$, given at the top of page 4. A check on the preceding equations 12 and 14 shows that values of $Kg^+ = Kg(1 + K_y/K_r)$ and $f_L{}^1 = f_L(1 + K_y/K_r)$ would be more rational than the values given.

I find the statement on page 3, "For the cases of interest here, the creep force is small compared to the contact force," rather puzzling. In the first place, for small amplitudes, without flange contact, the peak creep force calculates to be at least twice the peak lateral contact force, even for worn wheels, even for low values of the creep coefficient. Secondly, these forces are sinusoidal and approximately 90 deg out of phase with each other. Thirdly, the resultant approximation, $y_r = K_g/K_r \cdot y$ is not further used in the text.

For the more sophisticated model, a lateral suspension and a yaw suspension with a spring and a friction element in series was used. This contrasts with the car model of the companion paper, 76-WA/RT-2, where the yaw suspension was modeled by friction only, which would seem to be the condition obtained on any car with a center bowl, and I have submitted discussion on the very interesting results obtained there.

"Derailment" is mentioned several times, and it would be helpful if the authors could state what criterion is used to indicate this. All the graphs seem to contemplate side-to-side motion of up to two inches continuously maintainable provided the forward speed is below a critical velocity. For instance, in Fig. 12, the worst case is permitted

[3] Numbers in brackets designate References at end of discussion.

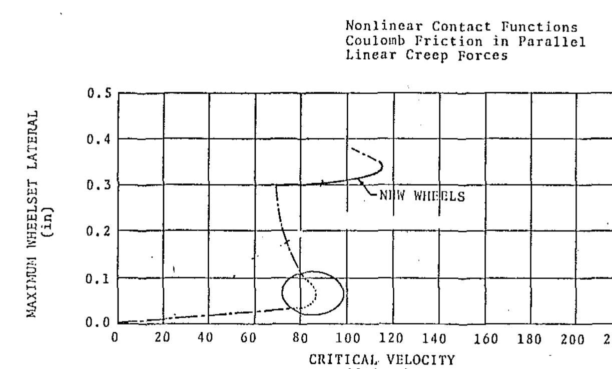

Fig. 1 Freight vehicle limit cycle amplitudes: nonlinear suspension