Journal of Heat Transfer

Discussion

Discussion: "An Integral Equation for the Dual-Lag Model of Heat Transfer" (Kulish, V. V., and Novozhilov, V. B., 2004, ASME J. Heat Transfer, 126, pp. 805–808)

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In this paper, the authors claim in the abstract and the conclusion that the "solution with dual phase lag depends only on the difference between the two lags." Then, they proceed to derive an integral solution for the temperature history.

I show first that this conclusion is erroneous and second that the Laplace transform method they use is incorrect.

The authors claim that the system of the dual lag Eqs. (1) and (3) in their paper is equivalent to the new system of Eqs. (6) and (7) in their paper. This is quite incorrect for the following reasons:

1. To obtain Eq. (6) from Eq. (1), the authors perform the shift in time $t_{\text{old}}+\tau_q \Rightarrow t_{\text{new}}$, where t_{old} corresponds to the time in Eq. (1) and t_{new} is the time in Eq. (6).
   With this shift Eq. (3) becomes using the new time
   $$\frac{\partial T}{\partial t}(x,t-\tau_q) = -\frac{1}{\rho c_p}\frac{\partial q''(x,t-\tau_q)}{\partial x}$$
   whereas the authors assume that Eq. (3) does not change with the transformation of time applied to obtain Eq. (6) from Eq. (1).
2. It follows that the derivation of Eq. (7) is wrong. Equation (7) is correct only if $\tau_q=0$.

Thus, their conclusion that the "solution with dual phase lag depends only on the difference between the two lags" is wrong.

The solution that follows Eq. (7) can be applicable only when $\tau_q=0$ as pointed out in the previous discussion. In this case, Eq. (8) is fine but its Laplace transform Eq. (10) is not.

To find the Laplace of Eq. (8), we multiply both sides by e^{-st} and then integrate over time from $\Delta\tau$ to infinity following the notation in the subject paper.

The result is not Eq. (10) as the authors claim but the following equation

$$\frac{d^2\Theta}{dx^2} = se^{s\Delta\tau}\int_{\Delta\tau}^{\infty}\theta(x,t)e^{-st}\,dt - \theta(x,\Delta\tau).$$

Here the Laplace transform of the temperature rise is defined as

$$\Theta(x,s) = \int_0^{\infty}\theta(x,t)e^{-st}\,dt$$

I suggest that the subject paper be retracted.

Contributed by the Heat Transfer Division of ASME for publication in the Journal of Heat Transfer. Manuscript received May 26, 2005; final manuscript received January 30, 2006. Review conducted by Yogesh Jaluria.

