



Optimization in a Class of Hole Shapes in Plate Structures¹

K. Rajaiah² and N. K. Naik². The determination of optimum hole shapes to achieve minimum stress concentration is an important engineering problem. The author's analytical contribution is a welcome addition to this area. In this context, the discussers would like to draw attention to their recent work on optimum hole-shape determination in infinite orthotropic plates under inplane loading by conformal transformation [1]. An extension of this approach to quasi-rectangular holes has also been reported separately [2]. In this approach, following the Lekhnitskii's method of analysis [3], the hole-shape parameter ϵ and hence the hole shape itself was varied until the maximum tangential stress on the hole boundary became a minimum. For this optimum condition, it is found that the tangential stress distribution becomes nearly uniform on the tensile segment of the boundary. A comparison of the results for isotropic plates reported in reference [1] with those given by Dhir, is given in Table 1, which shows very close correlation. Table 1 also contains additional information for the case of orthotropic plates.

An interesting point to observe is the stress distribution achieved which is not absolutely uniform even along the tensile segment of the hole boundary in Dhir's as well as our work [1] for the case of uniaxial loading condition. On the compressive segment, the distribution is still worse and minimum stress values are not achieved. Whereas, with our approach [1], one can independently minimize the stress concentrations on the tensile and compressive segments of the hole boundary by choosing different optimum values of ϵ for the two segments, this does not appear to be possible by Dhir's approach even though he has followed a minimization principle. Clearly, when the applied load becomes compressive (reversal of loading), the importance of the respective segments gets reversed and in that case, the numerically smaller tensile stress becomes important from a design point of view. Perhaps another possibility would be to choose a superior transformation function to optimize both the segments simultaneously. However as mentioned by Dhir in his Example II, there can be considerable complexity in the algebraic manipulation in such a case. With the experimental approach, one has the advantage of obtaining real optimized shape on both the segments and dealing with finite size plates [4], although one has to conduct a series of experiments to cover a large range of parameters.

¹By S. K. Dhir, and published in the December, 1981, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 48, pp. 905-908.

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Table 1

Loading	Stress concentration factor (s.c.f.)			
	Optimized hole		Square hole	Circular hole
	[1]	[Dhir]		
Isotropic				
Uniaxial	2.47	2.47	2.85	3.00
Biaxial $\sigma_0 : \sigma_0$	2.00	2.00	4.00	2.00
Biaxial $\sigma_0 : -\sigma_0$	3.06	3.07	3.10	4.00
Orthotropic				
Uniaxial	3.02	—	3.21	4.60
$E_1/E_2 = 3.41$				
Uniaxial	2.38	—	2.68	2.95
$E_1/E_2 = 0.293$				
Biaxial, $\sigma_0 : \sigma_0$	2.10	—	3.37	2.75
$E_1/E_2 = 3.41$				
Biaxial, $\sigma_0 : -\sigma_0$	4.23	—	4.25	6.45
$E_1/E_2 = 3.41$				

References

- 1 Rajaiah, K., and Naik, N. K., "Optimum Hole Shape Determination in Infinite Orthotropic Plates Under Inplane Loading by Conformal Transformation," to appear in *Mechanics Research Communications*.
- 2 Rajaiah, K., and Naik, N. K., "Quasi-Rectangular Holes for Minimizing Stress Concentrations in Infinite Plates Under Inplane Loading," Technical Report No. AE TR 811201, I.I.T. Bombay, India, 1981.
- 3 Lekhnitskii, S. G., *Anisotropic Plates*, Gordon and Breach, New York, 1968.
- 4 Durelli, A. J., and Rajaiah, K., "Optimum Hole Shapes in Finite Plates Under Uniaxial Load," ASME JOURNAL OF APPLIED MECHANICS, Vol. 46, 1979, pp. 691-695.

Author's Closure

The discussion by Rajaiah and Naik is very much appreciated. It is somewhat unfortunate that both the references [1] (not yet published) and [2] mentioned by Rajaiah and Naik in their discussion are presently not available to the author. As a result it is difficult to comment on the advantages and/or disadvantages of their approach. However, it seems that independent minimization of the stresses on tensile and compressive segments of the opening would not necessarily lead to a truly optimized hole; especially because both compressive and tensile segments of the opening are not considered simultaneously. As was suggested in the original paper, the use of Lagrange multipliers would perhaps also be a viable possibility to accomplish the simultaneous optimization of both segments of the opening. For the information of the discussers the author has also successfully

used his technique of minimizing a stress-energy density integral to optimize the material properties of an orthotropic plate with an opening [1].

Reference

¹ Dhir, S. K., "Optimization of Openings in Plates," AIAA/ASME/ASCE/AHS 23rd SDM Conference, New Orleans, May 1982.

Edge Effects in Laminated Composite Plates¹

C. O. Horgan.² The authors have presented an interesting finite element approach to the analysis of edge and interlaminar effects in laminated composites. Their results demonstrate that numerical methods involving self-equilibrated eigenfunctions for such problems are most efficiently carried out using finite elements.

However, it should be noted that a purely numerical scheme such as that given by the authors does not reveal the explicit dependence of the *rate of decay* of edge effects on material and geometric parameters. Such results can be obtained from the qualitative analyses of Saint-Venant's principle [1-6] cited by the authors. (See also [1, 2] of this discussion for a summary of results in the anisotropic case.) For example, for plane strain of an homogeneous rectangular strip, transversely isotropic about the axial direction, it is shown by I. Choi and the discussor in [5] that the exponential stress decay rate has the asymptotic characterization

$$\gamma \sim 2\pi(G_{LT}/E_L)^{1/2}/b \text{ as } G_{LT}/E_L \rightarrow 0, \quad (1)$$

for highly anisotropic materials. Here b is the strip width and the remaining notation is that of the paper under discussion. For the graphite/epoxy composite considered by the authors and in [5] (for which $G_{LT}/E_L = 0.03$), equation (1) yields the result $\gamma \sim 1.088/b$, which is in excellent agreement with the exact value $\gamma = 1.128/b$. We advocate use of the formula (1) for design purposes for fiber-reinforced composites (in both plane strain and generalized plane stress). Another situation considered in [6] is concerned with plane strain of a sandwich strip, composed of identical homogeneous *isotropic* face materials occupying two layers of equal thickness enclosing a dissimilar homogeneous *isotropic* core. The result analogous to (1) has the form (see equation (4.1) of [6])

γ (total strip width)

$$\sim 2 \left[\frac{2(f^2 - 3f + 3)(1 - \nu_f^2)E_c}{f^3(1-f)(1+\nu_c)E_f} \right]^{1/2}, \quad (2)$$

as $E_c/E_f \rightarrow 0$.

Here f denotes the volume fraction of face material and E , and ν are the respective Young's moduli and Poisson ratios. It would be of interest to extend (2) to the case of laminates with anisotropic layers, as treated by the authors.

Finally, we concur with the authors' concluding remarks regarding the need for consideration of stress singularities in any discussion of edge effects in laminated plates.

¹ By S. B. Dong and D. B. Goetschel, and published in the March, 1982, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, pp. 129-135.

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References

- Horgan, C. O., "Saint-Venant's Principle in Anisotropic Elasticity Theory," in: *Mechanical Behavior of Anisotropic Solids*, J. P. Boehler, ed., Proc. of EuroMech Colloquium 115, Villard de Lans, 1979; Editions Scientifiques du CNRS, Paris, France, 1982, pp. 855-870.
- Horgan, C. O., "Saint-Venant End Effects in Composites," *Journal of Composite Materials*, Vol. 12, 1982, in press.

Authors' Closure

There is no question regarding the usefulness and desirability of formulas describing the fundamental exponential decay rate of self-equilibrated edge effects in terms of material and geometrical parameters. However, such formulas, in the authors' opinion, are possible for a limited number of composite plates only, such as those cited by the discussor and perhaps for cross-ply and angle-ply plates with equal thickness laminates. Beyond these, the large number of parameters that must be reflected in meaningful formulas will preclude their derivations, simply due to algebraic difficulties.

The numerical analysis outlined in our paper enables an intuitive understanding of a large class of composite plate to be easily gained. The relatively low computational effort must be emphasized. Also, a great deal of information is revealed in a straightforward manner, e.g., the fundamental decay rate as well as the higher order ones and their displacement and stress distributions over the plate's cross section. Although not as elegant as a formula, the numerical technique is equally effective in sorting out the governing parameters on decay rates.

It would be a nice outcome if the numerical studies could lead to relatively simple formulas containing the essential nature of the rate of exponential decay.

A Simple and Effective Pipe Elbow Element - Interaction Effects¹

J. F. Whatham.² I would like to point out that while the experimental points in Figs. 9-13 were taken from Whatham [1] and suitably acknowledged, no mention was made of the analytic solution in that paper. The results from that solution were closer to the experimental points than were the curves presented, particularly for circumferential stress; references [2-5] describe the method more fully.

The Bathe-Almeida model with only two boundary conditions cannot adequately describe the true situation in a curved pipe at a rigid flange where four strain components must vanish.

A computer program package BENDPAC written in FORTRAN IV and ASSEMBLER for an IBM 3031 computer is available from the Australian Atomic Energy Commission,

¹ By K. J. Bathe and C. A. Almeida, and published in the March, 1982 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, 1982, pp. 165-171.

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