Closure to "Discussion of 'An Integral Equation for the Dual-Lag Model of Heat Transfer' (Milov, D., 2007, ASME J. Heat Transfer, 129, p. 927)"

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Both comments from our opponent are incorrect.

Comment 1.

For the purpose of mathematical model derivation, relations (1) and (3) are identities which hold for any $-\infty < t < +\infty$. They are invariant with respect to any shift of independent variable and those shifts can be made completely independently in those relations without compromising their correctness for any $-\infty < t < +\infty$.

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For example, one can write this pair of relations in the form

$$q''(x,t+\tau_q) = -k\frac{\partial T}{\partial x}(x,t+\tau_T) \qquad (*)$$

$$\frac{\partial T}{\partial t}(x,t+\tau_q) = -\frac{1}{\rho c_p}\cdot\frac{\partial q''(x,t+\tau_q)}{\partial x} \qquad (**)$$

both are true for any $-\infty < t < +\infty$.

Now one can substitute (*) into (**), make transformation of variable $t+\tau_q=\tilde{t}$ and get (6). This is just another way to derive Eq. (6). Of course, the original derivation in the paper is absolutely correct.

Comment 2.

Laplace transform of Eq. (8) is its integration from 0 to ∞ (as it should be), and not from $\Delta\tau$ to ∞. The way we make it is clear from the initial condition (9) which is set at $t=0$. This initial condition leads to transform equation in the form (10).

Note that for real materials $\Delta\tau < 0$ (see comments on p. 807 of the paper).

The way we specify domain in (8), i.e., $t \in [\Delta\tau, \infty[$ is just to emphasize that Eq. (6) also holds true for $t \in [\Delta\tau, 0]$. In fact, for the boundary conditions that we are considering the both sides of (8) are identically zero at this interval.

In order to see this, analyse the final integral equation (18). (We have requested to remove (*) from the formulas (17–19) at the proof stage, but it is still there. These equations should contain t only, not t^*.) Considering $t \in [\Delta\tau, \infty[$ one would see that for imposed fluxes that are identically zero for $t < 0$ (we are considering only such fluxes) the integral term turns into zero in the interval $t \in [\Delta\tau, 0[$.

