used his technique of minimizing a stress-energy density integral to optimize the material properties of an orthotropic plate with an opening [1].

Reference

1 Dhir, S. K., "Optimization of Openings in Plates," AIAA/ASME/ASCE/AHS 23rd SDM Conference, New Orleans, May 1982.

Edge Effects in Laminated Composite Plates[1]

C. O. Horgan.[2] The authors have presented an interesting finite element approach to the analysis of edge and interlaminar effects in laminated composites. Their results demonstrate that numerical methods involving self-equilibrated eigenfunctions for such problems are most efficiently carried out using finite elements.

However, it should be noted that a purely numerical scheme such as that given by the authors does not reveal the explicit dependence of the *rate of decay* of edge effects on material and geometric parameters. Such results can be obtained from the qualitative analyses of Saint-Venant's principle [1–6] cited by the authors. (See also [1, 2] of this discussion for a summary of results in the anisotropic case.) For example, for plane strain of an homogeneous rectangular strip, transversely isotropic about the axial direction, it is shown by I. Choi and the discusser in [5] that the exponential stress decay rate has the asymptotic characterization

$$\gamma \sim 2\pi (G_{LT}/E_L)^{1/2}/b \quad \text{as} \quad G_{LT}/E_L \to 0, \tag{1}$$

for highly anisotropic materials. Here b is the strip width and the remaining notation is that of the paper under discussion. For the graphite/epoxy composite considered by the authors and in [5] (for which $G_{LT}/E_L = 0.03$), equation (1) yields the result $\gamma \sim 1.088/b$, which is in excellent agreement with the exact value $\gamma = 1.128/b$. We advocate use of the formula (1) for design purposes for fiber-reinforced composites (in both plane strain and generalized plane stress). Another situation considered in [6] is concerned with plane strain of a sandwich strip, composed of identical homogeneous *isotropic* face materials occupying two layers of equal thickness enclosing a dissimilar homogeneous *isotropic* core. The result analogous to (1) has the form (see equation (4.1) of [6])

$$\gamma\,(\text{total strip width}) \sim 2\left[\frac{2(f^2-3f+3)(1-\nu_f^2)E_c}{f^3(1-f)(1+\nu_c)E_f}\right]^{1/2}, \quad \text{as} \quad E_c/E_f \to 0. \tag{2}$$

Here f denotes the volume fraction of face material and E, and ν are the respective Young's moduli and Poisson ratios. It would be of interest to extend (2) to the case of laminates with anisotropic layers, as treated by the authors.

Finally, we concur with the authors' concluding remarks regarding the need for consideration of stress singularities in any discussion of edge effects in laminated plates.

References

1 Horgan, C. O., "Saint-Venant's Principle in Anisotropic Elasticity Theory," in: *Mechanical Behavior of Anisotropic Solids*, J. P. Boehler, ed., Proc. of Euromech Colloquium 115, Villard de Lans, 1979; Editions Scientifiques du CNRS, Paris, France, 1982, pp. 855–870.

2 Horgan, C. O., "Saint-Venant End Effects in Composites," *Journal of Composite Materials*, Vol. 12, 1982, in press.

Authors' Closure

There is no question regarding the usefulness and desirability of formulas describing the fundamental exponential decay rate of self-equilibrated edge effects in terms of material and geometrical parameters. However, such formulas, in the authors' opinion, are possible for a limited number of composite plates only, such as those cited by the discusser and perhaps for cross-ply and angle-ply plates with equal thickness laminates. Beyond these, the large number of parameters that must be reflected in meaningful formulas will preclude their derivations, simply due to algebraic difficulties.

The numerical analysis outlined in our paper enables an intuitive understanding of a large class of composite plate to be easily gained. The relatively low computational effort must be emphasized. Also, a great deal of information is revealed in a straightforward manner, e.g., the fundamental decay rate as well as the higher order ones and their displacement and stress distributions over the plate's cross section. Although not as elegant as a formula, the numerical technique is equally effective in sorting out the governing parameters on decay rates.

It would be a nice outcome if the numerical studies could lead to relatively simple formulas containing the essential nature of the rate of exponential decay.

A Simple and Effective Pipe Elbow Element – Interaction Effects[1]

J. F. Whatham.[2] I would like to point out that while the experimental points in Figs. 9–13 were taken from Whatham [1] and suitably acknowledged, no mention was made of the analytic solution in that paper. The results from that solution were closer to the experimental points than were the curves presented, particularly for circumferential stress; references [2–5] describe the method more fully.

The Bathe-Almeida model with only two boundary conditions cannot adequately describe the true situation in a curved pipe at a rigid flange where four strain components must vanish.

A computer program package BENDPAC written in FORTRAN IV and ASSEMBLER for an IBM 3031 computer is available from the Australian Atomic Energy Commission,

[1] By S. B. Dong and D. B. Goetschel, and published in the March, 1982, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, pp. 129–135.

[2] Professor of Mechanics, Department of Metallurgy, Mechanics and Materials Science, Michigan State University, East Lansing, Mich. 48824.

[1] By K. J. Bathe and C. A. Almeida, and published in the March, 1982 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, 1982, pp. 165–171.

[2] Australian Atomic Energy Commission, Lucas Heights Research Laboratories, New Ittawarra Road, Lucas Heights, NSW Australia.