used his technique of minimizing a stress-energy density integral to optimize the material properties of an orthotropic plate with an opening [1].

Reference

1 Dhir, S. K., "Optimization of Openings in Plates," AIAA/ASME/ASCE/AHS 23rd SDM Conference, New Orleans, May 1982.

Edge Effects in Laminated Composite Plates[1]

C. O. Horgan.[2] The authors have presented an interesting finite element approach to the analysis of edge and interlaminar effects in laminated composites. Their results demonstrate that numerical methods involving self-equilibrated eigenfunctions for such problems are most efficiently carried out using finite elements.

However, it should be noted that a purely numerical scheme such as that given by the authors does not reveal the explicit dependence of the *rate of decay* of edge effects on material and geometric parameters. Such results can be obtained from the qualitative analyses of Saint-Venant's principle [1–6] cited by the authors. (See also [1, 2] of this discussion for a summary of results in the anisotropic case.) For example, for plane strain of an homogeneous rectangular strip, transversely isotropic about the axial direction, it is shown by I. Choi and the discusser in [5] that the exponential stress decay rate has the asymptotic characterization

$$\gamma \sim 2\pi (G_{LT}/E_L)^{1/2}/b \quad \text{as} \quad G_{LT}/E_L \to 0, \tag{1}$$

for highly anisotropic materials. Here b is the strip width and the remaining notation is that of the paper under discussion. For the graphite/epoxy composite considered by the authors and in [5] (for which $G_{LT}/E_L = 0.03$), equation (1) yields the result $\gamma \sim 1.088/b$, which is in excellent agreement with the exact value $\gamma = 1.128/b$. We advocate use of the formula (1) for design purposes for fiber-reinforced composites (in both plane strain and generalized plane stress). Another situation considered in [6] is concerned with plane strain of a sandwich strip, composed of identical homogeneous *isotropic* face materials occupying two layers of equal thickness enclosing a dissimilar homogeneous *isotropic* core. The result analogous to (1) has the form (see equation (4.1) of [6])

$$\gamma\,(\text{total strip width}) \sim 2\left[\frac{2(f^2-3f+3)(1-\nu_f^2)E_c}{f^3(1-f)(1+\nu_c)E_f}\right]^{1/2}, \quad \text{as} \quad E_c/E_f \to 0. \tag{2}$$

Here f denotes the volume fraction of face material and E, and ν are the respective Young's moduli and Poisson ratios. It would be of interest to extend (2) to the case of laminates with anisotropic layers, as treated by the authors.

Finally, we concur with the authors' concluding remarks regarding the need for consideration of stress singularities in any discussion of edge effects in laminated plates.

References

1 Horgan, C. O., "Saint-Venant's Principle in Anisotropic Elasticity Theory," in: *Mechanical Behavior of Anisotropic Solids*, J. P. Boehler, ed., Proc. of Euromech Colloquium 115, Villard de Lans, 1979; Editions Scientifiques du CNRS, Paris, France, 1982, pp. 855–870.

2 Horgan, C. O., "Saint-Venant End Effects in Composites," *Journal of Composite Materials*, Vol. 12, 1982, in press.

Authors' Closure

There is no question regarding the usefulness and desirability of formulas describing the fundamental exponential decay rate of self-equilibrated edge effects in terms of material and geometrical parameters. However, such formulas, in the authors' opinion, are possible for a limited number of composite plates only, such as those cited by the discusser and perhaps for cross-ply and angle-ply plates with equal thickness laminates. Beyond these, the large number of parameters that must be reflected in meaningful formulas will preclude their derivations, simply due to algebraic difficulties.

The numerical analysis outlined in our paper enables an intuitive understanding of a large class of composite plate to be easily gained. The relatively low computational effort must be emphasized. Also, a great deal of information is revealed in a straightforward manner, e.g., the fundamental decay rate as well as the higher order ones and their displacement and stress distributions over the plate's cross section. Although not as elegant as a formula, the numerical technique is equally effective in sorting out the governing parameters on decay rates.

It would be a nice outcome if the numerical studies could lead to relatively simple formulas containing the essential nature of the rate of exponential decay.

A Simple and Effective Pipe Elbow Element – Interaction Effects[1]

J. F. Whatham.[2] I would like to point out that while the experimental points in Figs. 9–13 were taken from Whatham [1] and suitably acknowledged, no mention was made of the analytic solution in that paper. The results from that solution were closer to the experimental points than were the curves presented, particularly for circumferential stress; references [2–5] describe the method more fully.

The Bathe-Almeida model with only two boundary conditions cannot adequately describe the true situation in a curved pipe at a rigid flange where four strain components must vanish.

A computer program package BENDPAC written in FORTRAN IV and ASSEMBLER for an IBM 3031 computer is available from the Australian Atomic Energy Commission,

[1] By S. B. Dong and D. B. Goetschel, and published in the March, 1982, issue of the ASME Journal of Applied Mechanics, Vol. 49, pp. 129–135.

[2] Professor of Mechanics, Department of Metallurgy, Mechanics and Materials Science, Michigan State University, East Lansing, Mich. 48824.

[1] By K. J. Bathe and C. A. Almeida, and published in the March, 1982 issue of the ASME Journal of Applied Mechanics, Vol. 49, 1982, pp. 165–171.

[2] Australian Atomic Energy Commission, Lucas Heights Research Laboratories, New Illawarra Road, Lucas Heights, NSW Australia.

and the programs contained therein are designed to give an analytic solution for the following:

(*a*) Stresses and deflections in pipe bends terminated by flanges, infinitely long tangent pipes or short equal length flange-ended tangents when under pressure (PRESEF), or pure in-plane bending (BENDEF).

(*b*) Stresses in flange-ended pipe bends from in-plane end loading other than pure bending (SHEFEF), coil spring type out-of-plane loading (COILEF), or any other type of out-of-plane loading (TURNEF).

(*c*) Flexibility matrices for flange-ended pipe bends under any in-plane end loading (FLEXIN) or out-of-plane end loading (FLEXOT).

Individual programs PRESEF and BENDEF are available from the National Energy Software Center, Argonne National Laboratory, while the total BENDPAC package has been recently supplied to the Center.

References

1 Whatham, J. F., "In-Plane Bending of Flanged Pipe Elbows," *Proceedings, Metal Structures Conference*, the Institution of Engineers, Perth, Australia, Nov. 30-Dec. 1, 1978, also: *Transactions, Institution of Engineers, Australia*, Vol. CE21, No. 2, 1979, pp. 80-85.

2 Whatham, J. F., and Thompson, J. J., "The Bending and Pressurizing of Pipe Bends With Flanged Tangents," *Journal of Nuclear Engineering and Design*, Vol. 54, No. 1, 1979, pp. 17-28.

3 Whatham, J. F., "Thin Shell Equations for Circular Pipe Bends," *Journal of Nuclear Engineering and Design*, Vol. 65, No. 1, 1981, pp. 77-89.

4 Whatham, J. F., "Thin Shell Analysis of Circular Pipe Bends," *Transactions, Institution of Engineers, Australia*, Vol. CE23, No. 4, 1981, pp. 234-245.

5 Whatham, J. F., "Thin Shell Analysis of Non-Circular Pipe Bends," *Journal of Nuclear Engineering and Design*, Vol. 67, No. 2, 1981, pp. 287-296.

Authors' Closure

As was stated, the objective in our research is to obtain a *simple* pipe element that can be used in practical and general analyses of large piping systems – static, dynamic, materially nonlinear, and large displacement analyses of assemblages of many bends and straight sections [1-4]. Clearly, in this development we must balance the simplicity of formulation and solution cost versus the predictive capability of the element, and our primary objective is to predict the flexibility of the structure accurately.

There is no doubt that for single pipe bends with or without straight sections or flanges, more accurate solutions on detailed stress distributions, than obtained with our simple pipe model, can be calculated. For example, finite element idealizations using isoparametric shell elements can be used to obtain very high accuracy solutions for linear and nonlinear conditions [5], or for certain solutions Whatham's programs can be employed.

References

1 Bathe, K. J., and Almeida, C. A., "A Simple and Effective Pipe Elbow Element — Linear Analysis," ASME Journal of Applied Mechanics, Vol. 47, 1980, pp. 93-100.

2 Bathe, K. J., and Almeida, C. A., "A Simple and Effective Pipe Elbow Element — Interaction Effects," ASME Journal of Applied Mechanics, Vol. 49, 1982, pp. 165-171.

3 Bathe, K. J., and Almeida, C. A., "A Simple and Effective Pipe Elbow Element — Pressure Stiffening Effects," ASME Journal of Applied Mechanics, Vol. 49, 1982, pp. 914-916.

4 Bathe, K. J., Almeida, C. A., and Ho, L. W., "A Simple and Effective Pipe Elbow Element — Nonlinear Analysis," *J. Computers and Structures*, in press.

5 Bathe, K. J., *Finite Element Procedures in Engineering Analysis*, Prentice-Hall, Englewood Cliffs, N.J., 1982.

Characteristic Forms of Differential Equations for Wave Propagation in Nonlinear Media[1]

M. Ziv[2]. I was most surprised to see this paper whose objectives and outline are systematically the same as those in my paper [1]. These two papers seem too similar to be a coincidence. My surprise stems from the fact that my manuscript was first submitted on June 17, 1980 for publication to Professor Ting, acting as an Associate Editor of the Journal of Applied Mechanics. Based on his own opinion that my analysis is wrong, Ting rejected my manuscript from being published in the Journal of Applied Mechanics. Yet, a few months later, Ting submitted his paper, without referring to my work, presenting results which are obtainable from my manuscript.

In his paper, Professor Ting claims that his results are equations (21) and (64). But it is immediately seen that his equation (21) is a special case of my result, equation (35) of [1]. Upon a straightforward reduction, my result, equation (17) of [1], also agrees with his equation (64). Obviously, while rejecting my work, Professor Ting has adapted my objectives only to arrive at results already available to him from my manuscript. Yet, no mention of my manuscript is found in his paper.

In order to show that equation (35) of [1] becomes Ting's equation (21) upon reduction, the following changes in representations are made in equation (35) of [1]: the body force is deleted, the distinction between upper case letters and lower case letters is neglected, the symbols become those of Ting's, and finally rectangular Cartesian coordinates are used taking, for example, the x-direction. With these simplifications, equation (25) of [1] now reads

$$\frac{d\sigma_{xx}}{dt} - \rho_0 c \frac{dv_x}{dt} = M_{xxqp} v_{q,p} - \rho_0 c^2 v_{x,x} - c\sigma_{xp,p} + c\sigma_{xx,x} \quad \text{along} \quad dx/dt = c$$

where M_{kqdl} is the term $\dfrac{\partial^2 w}{\partial E_K^N \partial E_D^Q} x^d_{,D} x^N_{l,} + T^K_Q \delta^d_l$

in equation (35) of [1].

When Ting's equation (21) is now written in terms of his equation (15), i.e., $b_i = cn_i$ and the x-direction, for example, is taken, Ting's equation (21) becomes the one written in the foregoing. It was thus shown that equation (21) of Ting's paper is a special case obtained directly from equation (35) of [1]. In the same manner, it can be easily shown that equation (64) of Ting's paper is a special case of equation (17) of [1].

Also, it should be noted that equation (43) of Ting's paper is equation (38) of [2] when taking the direction cosines to be $n_r = 1$ and $n_\theta = 0$. Reference [2] was listed in my manuscript. No reference was made, however, to these works and no new results are given in Ting's paper.

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2 Ziv, M., "Two-Spatial Dimensional Elastic Wave Propagation by the Theory of Characteristics," *Int. J. Solids and Structures*, Vol. 5, 1969, pp. 1135-1151.

[1] By T. C. T. Ting and published in the December, 1981, issue of the ASME Journal of Applied Mechanics, Vol. 48, pp. 743-748.

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