The Modified $\sigma$-$\omega$ Model for Relativistic Nuclear Matter

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We have developed the modified $\sigma$-$\omega$ model. It includes the modified scalar and vector vertices expressed by the operators like the positive- and negative-energy projections and by two additional parameters. These vertices finally result in the effective density-dependent $NN\sigma$ and $NN\omega$ coupling constants. Comparing our results with the Dirac-Brueckner-Hartree-Fock (DBHF) calculations, the one-boson-exchange-model of $NN$ force and the empirical value of the incompressibility, we can see that the two parameters should be nearly equal and that their values should be between 0.4 and 0.6. This favorable result is due to opposite effects by the modified scalar and vector vertices. At the saturation density, provided that values of the two parameters are 0.6, our result is almost equivalent to the DBHF result. The strengths of the potentials at all the densities and the energy per particle at the densities higher than the saturation point decrease, as the density-dependencies of the coupling constants become stronger. Consequently, it is found that the modified $\sigma$-$\omega$ model is able to reproduce the reasonable values of the coupling constants, potentials and incompressibility, and so may overcome the shortcomings of the original Walecka $\sigma$-$\omega$ model.

§ 1. Introduction

Recently, the relativistic density-dependent Hartree-Fock (RDHF) theory draws much attention. It is an effective model for the Dirac-Brueckner-Hartree-Fock (DBHF) theory, which is able to describe the low-energy $NN$ scattering data, deuteron properties and nuclear matter saturation simultaneously, and so is the most reliable model at present. The RDHF includes the effective density-dependent meson-nucleon coupling constants, which reflect the information of the relativistic $NN$ $G$-matrix in a nuclear medium. Another effective model of the DBHF theory is proposed by Gmuca. He used the extended $\sigma$-$\omega$ model including nonlinear $\sigma$ and $\omega$ self-coupling terms, and determined the model parameters to reproduce the results of the DBHF calculation. The RDHF and Gmuca’s approach are essentially different from each other. The key ingredient in the RDHF model is the density-dependent couplings, while in Gmuca’s model it is the dressed meson masses. In the present study, we are also interested in the approach using the effective coupling constants.

Both the RDHF and Gmuca’s model fully depend on the results of the DBHF calculation. On the other hand, Zimanyi and Moszkowski (ZM) developed the derivative scalar coupling (DSC) model, in which the usual $NN\sigma$ coupling $\bar{\psi}\psi\sigma$ is replaced by $\bar{\psi}(\not{p}/M)^{\lambda}\psi\sigma$. This model is equivalent to employing an effective $NN\sigma$ coupling constant $\hat{g}^{N}_{N\sigma} = (M^*/M)g^{N\sigma}$. Here, $M^*$ is the relativistic effective mass of a nucleon in the medium, and thus $\hat{g}^{N\sigma}$ is density-dependent. In this sense, the approach of the DSC model resembles the RDHF model. The DSC model has the following attractive features: it produces a reasonable incompressibility of nuclear matter, it does not depend on the DBHF results unlike the RDHF model and it includes only two fitted parameters in contrast to five parameters in Gmuca’s model. How-
ever, the DSC model is not able to reproduce the properties of finite nuclei. Koepf et al.\textsuperscript{13} pointed out that this failure is due to relatively large effective mass $M^*/M \approx 0.85$ in the DSC model, compared to $M^*/M \approx 0.55$ in the Walecka $\sigma$-$\omega$ model.\textsuperscript{14} The small effective mass or large scalar potential is necessary for the large spin-orbit splitting.\textsuperscript{15} In fact, the DBHF calculations also yield rather small effective mass $M^*/M \approx 0.6$.\textsuperscript{8,9} The DSC model only produces relatively weak strengths of the scalar and vector potentials. Furthermore, the $NN\omega$ coupling in this model is not density-dependent in contrast to the RDHF model. It is more natural that both the effective $NN\sigma$ and $NN\omega$ coupling constants are density-dependent. In spite of these shortcomings, the idea of the DSC model is attractive because of its simplicity and independence on the DBHF result. Therefore, this model is extensively applied to hypernuclear matter, neutron star and nuclear matter at finite temperature, etc.\textsuperscript{16}

In order to overcome the DSC model and to develop more useful phenomenological model to be compared with the DBHF calculation, we note that the success of the relativistic models is mainly due to the couplings between positive and negative energy states of the nucleons (the $+-$ couplings). In the relativistic impulse approximation (RIA) for nucleon-nucleus ($NA$) elastic scattering at intermediate energies it has been found that the $+-$ couplings play an important role in the differences between the relativistic and nonrelativistic descriptions.\textsuperscript{17} However, the effect of the $+-$ couplings in the RIA is too strong to reproduce the $NA$ scattering observables at the incident energies below 300 MeV.\textsuperscript{18} Tjon and Wallace remedied this problem by developing the generalized RIA (GRIA),\textsuperscript{19} which produces the different $+-$ coupling strengths from the usual RIA. Furthermore, the $G$-matrix elements between positive and negative energy states of the nucleons are different from the corresponding matrix elements of one-boson-exchange-potentials.\textsuperscript{20} Thus, it is also interesting and worthwhile to investigate the effect of the $+-$ couplings in the relativistic mean-field calculation of nuclear matter.

In this work, we construct a modified $\sigma$-$\omega$ model, in which $NN\sigma$ and $NN\omega$ vertices are artificially modified so as to correct the couplings between positive and negative energy states of the nucleons. This correction is performed by introducing the projection operators to positive and negative energy. Because these operators contain derivative couplings, our model is a kind of the generalization of the DSC model. In addition, the modified vertices effectively produce density-dependent $NN\sigma$ and $NN\omega$ coupling constants. Thus, the modified $\sigma$-$\omega$ model can be compared with the DBHF or RDHF model. In the next section, the modified $\sigma$-$\omega$ model is developed. In § 3 we show the numerical results for symmetric nuclear matter and analyze the properties of our model. Finally, we summarize this work in § 4.

\section{The modified $\sigma$-$\omega$ model}

\subsection{The modified vertices and effective coupling constants}

In this section, according to the motivation mentioned in § 1, we will formulate the modified $\sigma$-$\omega$ model. For this purpose, the operators $A^\pm(p)$ like the positive- and negative-energy projections are introduced:
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\[ \Lambda^\pm(p) = \pm \frac{p + M}{2M}, \quad (1) \]

where $p$ is the four-momentum of a nucleon and $M$ is the nucleon mass. It is noted that for the vertex $\Gamma = I$ or $\gamma^\mu$, an identity
\[ \Gamma = \Lambda^+(p') \Gamma \Lambda^+(p) + \Lambda^-(p') \Gamma \Lambda^-(p) + \Lambda^+(p') \Gamma \Lambda^-(p) + \Lambda^-(p') \Gamma \Lambda^+(p), \quad (2) \]
is satisfied. Then, introducing two additional parameters $0 \leq \lambda_\sigma, \lambda_\omega \leq 1$, we modify the scalar $(NN\sigma)$ and vector $(NN\omega)$ vertices as follows.

\[ I \rightarrow \Lambda^+(p') \Lambda^+(p) + \Lambda^-(p') \Lambda^-(p) + \lambda_\omega (\Lambda^+(p') \Lambda^-(p) + \Lambda^-(p') \Lambda^+(p)) \quad (3) \]
\[ (1 - \lambda_\sigma) \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \phi \left( \frac{1}{2M^2} \right) + (1 + \lambda_\sigma) M^2 \phi \quad (4) \]
\[ \gamma^\mu \rightarrow \lambda_\omega (\Lambda^+(p') \gamma^\mu \Lambda^+(p) + \Lambda^-(p') \gamma^\mu \Lambda^-(p)) + \Lambda^+(p') \gamma^\mu \Lambda^+(p) + \Lambda^-(p') \gamma^\mu \Lambda^-(p) \quad (5) \]
\[ = \frac{(\lambda_\omega - 1) \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \phi + (\lambda_\omega + 1) M^2 \gamma^\rho \phi}{2M^2} \quad (6) \]

If $\lambda_\sigma = \lambda_\omega = 1$, the Walecka $\sigma$-$\omega$ model is recovered. These modified vertices are the essential ingredients of our model. They are not the physical meson-nucleon vertices but the purely phenomenological ones. The parameters artificially correct the couplings between positive and negative energy states of the nucleons. (The coupling strengths of the third and fourth terms of Eqs. (3) and (5) should be the same owing to the time reversal invariance.) It is noted that the modifications of the scalar and vector vertices are different from each other. This seems to be inconsistent but is really necessary to produce reasonable effective $NN\omega$ coupling constant, as will be seen later. Employing the vertices (4) and (6) and the mean-field approximation for the $\sigma$- and $\omega$-meson fields, our model Lagrangian is

\[ \mathcal{L} = \bar{\phi} (\not{\sigma} - M) \phi - (1/2) m_\sigma^2 \langle \sigma \rangle^2 + (1/2) m_\omega^2 \langle \omega \rangle^2 + \frac{1}{2M^2} g_{NN\sigma} \langle \sigma \rangle \left( (1 - \lambda_\sigma) \bar{\phi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \phi + (1 + \lambda_\sigma) M^2 \phi \right) \]
\[ - \frac{1}{2M^2} g_{NN\omega} \langle \omega \rangle \left( (\lambda_\omega - 1) \bar{\phi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \phi + (\lambda_\omega + 1) M^2 \gamma^\rho \phi \right), \quad (7) \]

where $m_{\sigma(\omega)}$ is the $\sigma(\omega)$-meson mass and $g_{NN\sigma(\omega)}$ is the $NN\sigma(\omega)$ coupling constant.

The Euler-Lagrange equation for the nucleon field $\phi$ becomes

\[ (\not{\sigma} - M) \phi + \frac{1}{2M^2} g_{NN\sigma} \langle \sigma \rangle \left( (1 - \lambda_\sigma) \not{\phi} + (1 + \lambda_\sigma) M^2 \phi \right) \phi \]
\[ - \frac{1}{2M^2} g_{NN\omega} \langle \omega \rangle \left( (\lambda_\omega - 1) \not{\phi} + (\lambda_\omega + 1) M^2 \gamma^\rho \phi \right) \phi = 0. \quad (8) \]

This equation is not linear in $\not{\phi}$. Thus, we suppose that $\phi$ satisfies the following Dirac equation including the scalar and vector potentials,

\[ (\not{\sigma} - M^* \gamma^\rho \gamma^\sigma \phi = 0. \quad (9) \]
Then, Eq. (8) is expressed by

\[
(a_1\beta/M + b_1\gamma^0 + c_1)(b_2\gamma^0 + c_2)(\beta - \gamma^0 V - M^*)\psi = 0.
\]

Comparing the coefficients of Eq. (8) with those of (10), we obtain

\[
a_1b_2 = \frac{1}{2} \left(1 - \lambda_v\right)g_{NNω} \frac{\langle \omega_0 \rangle}{M},
\]

\[
a_1c_2 = \frac{1}{2} \left(1 - \lambda_s\right)g_{NNσ} \frac{\langle \sigma \rangle}{M},
\]

\[
a_1c_2v + a_1b_2m^* = 0,
\]

\[
b_1c_2 + b_2c_1 = 0,
\]

\[
b_1b_2 + c_1c_2 - (a_1b_2v + a_1c_2m^*) = 1,
\]

\[
(b_1b_2 + c_1c_2)v + (b_1c_2 + b_2c_1)m^* = \frac{1}{2} \left(1 + \lambda_v\right)g_{NNω} \frac{\langle \omega_0 \rangle}{M},
\]

\[
(b_1b_2 + c_1c_2)m^* + (b_1c_2 + b_2c_1)v = 1 - \frac{1}{2} \left(1 + \lambda_s\right)g_{NNσ} \frac{\langle \sigma \rangle}{M},
\]

where \(m^* = M^*/M\) and \(v = V/M\). Replacing \(g_{NNσ}\langle \sigma \rangle/M\) and \(g_{NNω}\langle ω_0 \rangle/M\) on the r.h.s. of Eqs. (16) and (17) with \(a_1b_2\) and \(a_1c_2\) by means of Eqs. (11) and (12), and then eliminating \(b_1b_2 + c_1c_2\) from Eqs. (16) and (17), we have

\[
(b_1c_2 + b_2c_1)(v - m^2) = \left(1 - \frac{1 + \lambda_s}{1 - \lambda_s} a_1c_2\right)v - \frac{1 + \lambda_v}{1 - \lambda_v} a_1b_2m^* = 0.
\]

The second equality holds because of Eq. (14). Similarly, eliminating \(b_1b_2 + b_2c_1\) and utilizing Eq. (15),

\[
(b_1b_2 + c_1c_2)(v + m^*) = 1 - \frac{1 + \lambda_s}{1 - \lambda_s} a_1c_2 + \frac{1 + \lambda_v}{1 - \lambda_v} a_1b_2
\]

\[
= (v + m^*)(1 + a_1b_2v + a_1c_2m^*).
\]

Furthermore, eliminating \((1 + \lambda_v)/(1 - \lambda_v))a_1b_2\) and \((1 - (1 + \lambda_s)/(1 - \lambda_s))a_1c_2\) from Eqs. (18) and (19),

\[
1 - \frac{1 + \lambda_s}{1 - \lambda_s} a_1c_2 = m^*(1 + a_1b_2v + a_1c_2m^*),
\]

\[
\frac{1 + \lambda_v}{1 - \lambda_v} a_1b_2 = v(1 + a_1b_2v + a_1c_2m^*).
\]

Then, substituting Eq. (13) into (20) and (21), these become

\[
\left(\frac{1 + \lambda_s}{1 - \lambda_s} + (m^*)^2 - v^2\right)a_1c_2 = 1 - m^*,
\]

\[
\left(\frac{1 + \lambda_v}{1 - \lambda_v} + (m^*)^2 - v^2\right)a_1b_2 = v.
\]
Finally, utilizing Eqs. (11) and (12) again, we obtain

\[ g_{NNa}^{\ast} \langle \sigma \rangle = M - M^* , \]  
\[ g_{NNw}^{\ast} \langle \omega_0 \rangle = V , \]  
where the effective \( NNa \) and \( NNw \) coupling constants, \( g_{NNa}^{\ast} \) and \( g_{NNw}^{\ast} \), are

\[ g_{NNa}^{\ast} = \frac{1}{2} \left[ (1+\lambda_s) + (1-\lambda_w)\left((m^*)^2-v^2\right) \right] g_{NNa} , \]  
\[ g_{NNw}^{\ast} = \frac{1}{2} \left[ (1+\lambda_w) + (1-\lambda_w)\left((m^*)^2-v^2\right) \right] g_{NNw} . \]  

If \( \lambda_s = \lambda_w = 1 \), \( g_{NNa}^{\ast} = g_{NNa} \) and \( g_{NNw}^{\ast} = g_{NNw} \) and so the Walecka \( \sigma-\omega \) model is recovered. At this stage, our modified \( NNa \) and \( NNw \) vertices (3) and (5) have been reduced to the effective density-dependent \( NNa \) and \( NNw \) coupling constants (26) and (27). The DBHF theory also produces effective density-dependent coupling constants by the reduction of it to the RDHF model. Therefore, the comparison between our calculation and the DBHF result is interesting and worthwhile.

At zero density limit, \( g_{NNa}^{\ast} \) returns to the free coupling constant

\[ \lim_{\rho_v \to 0} g_{NNa}^{\ast} = g_{NNa} , \]  

since \( \lim_{\rho_v \to 0} m^* = 1 \) and \( \lim_{\rho_v \to 0} v = 0 \). Although this condition seems to be a matter of course, it is just the result of the modified vertices (3) and (5). If the vector vertex is modified as

\[ \gamma^\mu \rightarrow \Lambda^\mu(p') \gamma^\mu \Lambda^\mu(p) + \Lambda^{-\mu}(p') \gamma^\mu \Lambda^{-\mu}(p) + \lambda_s(\Lambda^\mu(p') \gamma^\mu \Lambda^{-\mu}(p) + \Lambda^{-\mu}(p') \gamma^\mu \Lambda^\mu(p)) , \]  

the effective \( NNw \) coupling constant becomes

\[ g_{NNw}^{\ast} = \frac{1}{2} \left[ (1+\lambda_w) - (1-\lambda_w)\left((m^*)^2-v^2\right) \right] g_{NNw} . \]  

In this case, \( g_{NNw}^{\ast} \) cannot satisfy the proper condition (28). In other words, Eq. (28) prescribes for us to select the modified vertices (3) and (5) rather than (3) and (29).

2.2. Self-consistency equations

Based on the requirement that the nucleon field \( \phi \) obeys the Dirac equation (9), we remake the effective model Lagrangian \( \mathcal{L}_{\text{eff}} \):

\[ \mathcal{L}_{\text{eff}} = \bar{\phi}(\not p - \gamma^\mu V - M^*) \phi - (1/2) m_s^2 \langle \sigma \rangle^2 + (1/2) m_w^2 \langle \omega_0 \rangle^2 . \]  

The \( \sigma \) and \( \omega \) mean-fields, \( \langle \sigma \rangle \) and \( \langle \omega_0 \rangle \), are expressed in terms of \( m^* \) and \( v \) through Eqs. (24) and (25). The energy per particle \( W \) for symmetric nuclear matter is written in units of \( M \) as

\[ \frac{W}{M} = \frac{\langle E_\ast \rangle}{M} + v - 1 + \frac{2}{C_{\text{f},\not p}} \left( \frac{1-m^*}{(1+\lambda_s) + (1-\lambda_w)\left((m^*)^2-v^2\right)} \right)^2 \]  
\[ - \frac{2}{C_{\text{f},\not p}} \left( \frac{v}{(1+\lambda_w) + (1-\lambda_w)\left((m^*)^2-v^2\right)} \right)^2 , \]  

\[ (32) \]
where $\langle E^* \rangle$ is the average kinetic energy, $\bar{\rho} = \rho_v / \rho_0$ with the vector density $\rho_v$ and the nuclear matter saturation density $\rho_0$. $C_{s(v)}$ are defined by

$$C_{s(v)} = \frac{g_{NN}(\omega) \rho_0}{m^2(\omega) M}.$$  \hfill (33)

The effective nucleon mass $m^*$ and vector potential $v$ are determined self-consistently by extremization of $W$,

$$\frac{\partial W}{\partial m^*} = 0 , \quad \frac{\partial W}{\partial v} = 0 . \hfill (34)$$

These equations are rewritten as

$$C_s = - \frac{4c}{a_s b_v \bar{\rho}} (1 - m^*) , \hfill (36)$$

$$C_v = - \frac{4c}{a_v b_v \bar{\rho}} v , \hfill (37)$$

where

$$a_s = (1 + \lambda_s) + (1 - \lambda_s)((m^*)^2 - v^2) , \hfill (38)$$

$$a_v = (1 + \lambda_v) + (1 - \lambda_v)((m^*)^2 - v^2) , \hfill (39)$$

$$b_s = [(1 + \lambda_v) + (1 - \lambda_v)((m^*)^2 + v^2)](\rho_s / \rho_v) + 2(1 - \lambda_v)m^* v , \hfill (40)$$

$$b_v = [(\lambda_v + 1) + (1 - \lambda_v)((m^*)^2 + v^2)](\rho_s / \rho_v) + 2(1 - \lambda_v)(1 - m^*) v(\rho_s / \rho_v) , \hfill (41)$$

$$c = [(\lambda_v + 1) + (1 - \lambda_v)((m^*)^2 + v^2)] \{1 + \lambda_v + (1 - \lambda_v)((m^*)^2 + v^2)\}$$

$$- 4(1 - \lambda_v)(1 - \lambda_v)(1 - m^*) m^* v^2 (\rho_s / \rho_v) . \hfill (42)$$

The coupling constants, $C_s$ and $C_v$, are determined to fulfill the saturation condition

$$\frac{\partial W}{\partial \bar{\rho}}|_{\bar{\rho}=1} = 0 , \hfill (43)$$

$$W_0 = W(\bar{\rho} = 1) = - 15.75 \text{ MeV} . \hfill (44)$$

Substituting Eqs. (36) and (37) into (43) and (44), we have the self-consistency equation for $m^*$ at the nuclear matter saturation density:

$$\frac{1}{4} \left( \frac{E^*_s}{M} - m^* \frac{\rho_s}{\rho_v} \right) - \frac{a_s b_s}{2c} (1 - m^*) + \frac{a_v b_v}{2c} v = 0 , \hfill (45)$$

where $E^*_s = (k_F^2 + (M^*)^2)^{1/2}$, $\rho_s$ is the scalar density and $v$ is expressed in terms of $m^*$ through

$$v = 1 + \frac{W_0}{M - E^*_s / M} . \hfill (46)$$

Solving the nonlinear equation (45) for given $\lambda_s$ and $\lambda_v$, we obtain the value of $m^*$ at saturation. It is used to calculate the value of $v$ at saturation through Eq. (46). Then, substituting these values into Eqs. (36)~(42), we can determine the coupling constants $C_s$ and $C_v$. Now, Eqs. (36) and (37) turn to the nonlinear coupled self-
consistency equations for calculating the values of $m^*$ and $v$ at any nuclear matter density. Their solutions are used to calculate the density-dependencies of the scalar and vector potentials, energy per particle and effective coupling constants. In those calculations, $\lambda_s$ and $\lambda_v$, which define the modified vertices, are treated as free parameters. Their appropriate values will be determined by the comparison between the results of our and the DBHF calculations.

§ 3. The analyses and results

According to the procedure described in § 2.2, we can calculate the scalar and vector potentials, energy per particle and incompressibility of symmetric nuclear matter. For these calculations, values of the parameters $\lambda_s$ and $\lambda_v$ must be given. Equations (3) and (5) suggest that the modified scalar and vector vertices yield opposite effects. Therefore, we first investigate the effect of the modified vertices and determine reasonable values of the parameters. In the following calculations, we employ $k_F=1.30\text{ fm}^{-1}$.

Figures 1(a) and 2(a) display the scalar ($U_s$) and vector ($U_v$) potentials at the saturation density for several values of $\lambda_v$ as functions of $\lambda_s$, while Figs. 1(b) and 2(b) display them for several values of $\lambda_s$ as functions of $\lambda_v$. It is seen that the strengths of the potentials in the part (a) decrease, as $\lambda_s$ goes down to 0. On the other hand, the strengths in the part (b) increase, as $\lambda_v$ goes down to 0. Thus, it has been confirmed that the modified scalar and vector vertices have opposite effects on the potentials. In order to search for available values of the parameters, it is worthwhile to compare our results with the DBHF calculations. This comparison is reasonable because the DBHF theory is the established and the most reliable model at present, and because both our model and the DBHF theory produce the effective density-dependent $NN\sigma$ and $NN\omega$ coupling constants. The scalar and vector potentials at saturation by the
DBHF calculation are $U_s \approx -375$ MeV and $U_v \approx 290$ MeV, respectively. In Figs. 1 and 2, these values are depicted by dotted lines. (It is noted that the saturation point derived from the DBHF calculation of Ref. 8) is not the same as that adopted in our calculation. We can however expect that reasonable values of the potentials exist around the dotted lines.) The DBHF results require that the parameters should satisfy $|\lambda_s - \lambda_v| < 0.1$. This restriction is due to our results that the slopes of the curves in Figs. 1 and 2 rapidly change around $U_s \approx -350$ MeV and $U_v \approx 250$ MeV. In fact, the scalar potentials steeply decrease in the region $-700$ MeV $\leq U_s \leq -400$ MeV and the vector potentials steeply increase in the region $300$ MeV $\leq U_v \leq 550$ MeV.

Next, we investigate the scalar ($g_{NNs}$) and vector ($g_{NNv}$) coupling constants. Figures 3(a) and 4(a) display $(g_{NNs})^2/(4\pi)$ and $(g_{NNv})^2/(4\pi)$ for several values of $\lambda_s$ as functions of $\lambda_v$, while Figs. 3(b) and 4(b) display them for several values of $\lambda_v$ as functions of $\lambda_s$. In these calculations we assume $m_\sigma = 550$ MeV and $m_\omega = 783$ MeV. In the comparison between Figs. 4(a) and (b), we can also see that the modified scalar and vector vertices have opposite effects on the vector coupling constant. It decreases as $\lambda_s$ goes down to 0, while it increases as $\lambda_v$ goes down to 0. The scalar coupling constant is not a monotonic function of the parameters. (See Fig. 3(a).) Reasonable values of the coupling constants are able to be found in the one-boson-exchange-model (OBEM) of $NN$ potential, which suggests $(g_{NNs})^2/(4\pi) < 10$ and $(g_{NNv})^2/(4\pi) < 20$. These boundaries are depicted by dotted lines in Figs. 3 and 4. The dotted lines also indicate the border on which the slopes of the curves rapidly change. In fact, the scalar and vector coupling constants steeply increase in the region $(g_{NNs})^2/(4\pi) > 10$ and $(g_{NNv})^2/(4\pi) > 15$, respectively. It is noted that our model includes only $\sigma$- and $\omega$-mesons while the OBEM includes the other mesons. Therefore, we do not have to impose severely the conditions by the OBEM on our coupling constants. In this work we will allow that $(g_{NNs})^2/(4\pi)$ is somewhat larger than 10. $(g_{NNv})^2/(4\pi) \approx 10.$ From all these considerations, it is seen that available values of the parameters are restricted by $\lambda_s \leq \lambda_v$.

Furthermore, we investigate the incompressibility $K$. Figure 5(a) ((b)) shows it
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Fig. 3. (a) The $NN\sigma$ coupling constant $(g_{NN\sigma})^2/(4\pi)$ for various values of $\lambda_s$ as a function of $\lambda_s$. (b) That for various values of $\lambda_s$ as a function of $\lambda_s$. The $\sigma$-meson mass is assumed to be $m_{\sigma} = 550$ MeV. The dotted line is the upper limit of the coupling constant by the OBEM of $NN$ potential.

Fig. 4. The same as Fig. 3 but for the $NN\omega$ coupling constant $(g_{NN\omega})^2/(4\pi)$. The $\omega$-meson mass is assumed to be $m_{\omega} = 783$ MeV. The dotted lines are the upper and lower limits of the coupling constant by the OBEM of $NN$ potential.

for several values of $\lambda_v$ ($\lambda_s$) as a function of $\lambda_s$ ($\lambda_v$). The empirical value of $K$ is $200$ MeV $\leq K \leq 300$ MeV. These boundaries are depicted by dotted lines. Figure 5(a) exhibits a characteristic feature. All the curves approach $K \approx 200$ MeV as $\lambda_s$ goes down to 0, and steeply increase in the region $\lambda_s > \lambda_v$. As a result of this feature, available values of the two parameters are restricted by $\lambda_s < 0.7$ and $\lambda_v > \lambda_s - 0.1$. The same restrictions are also derived from Fig. 5(b), where the incompressibility steeply increases in the region $\lambda_v < \lambda_s$.

From all the above analyses, it is concluded that the two parameters $\lambda_s$ and $\lambda_v$
should be nearly equal, $\lambda_s \approx \lambda_v$. In order to confirm this further, we show the scalar and vector potentials for $\lambda_v = \lambda_s$, $\lambda_v = \lambda_s + 0.1$ and $\lambda_v = \lambda_s + 0.2$ in Figs. 6 and 7. The strengths of the DBHF calculation are depicted by dotted lines. They agree with our results using $\lambda_s \approx \lambda_v \approx 0.6$. The curves for $\lambda_v = \lambda_s + 0.1$ and $\lambda_v = \lambda_s + 0.2$ cannot cross the dotted lines. We also show the incompressibility in Fig. 8. The empirical values and the condition $\lambda_v = \lambda_s$ restrict the values of the parameters to $0.3 \leq \lambda_{\text{emp}} \leq 0.6$. The condition $\lambda_v = \lambda_s + 0.1$ or $\lambda_v = \lambda_s + 0.2$ relaxes this restriction. The DBHF result $K = 290$ MeV agrees with our calculation using $\lambda_s \approx \lambda_v \approx 0.6$, again. Consequently, the DBHF model corresponds to the upper allowed limit of the modified vertices.

For completeness, the scalar and vector coupling constants, the effective mass of

Fig. 5. (a) Incompressibility of nuclear matter for various values of $\lambda_v$ as a function of $\lambda_s$. (b) That for various values of $\lambda_v$ as a function of $\lambda_s$. The dotted lines indicate the empirical upper and lower limits.

Fig. 6. The scalar potential at saturation for $\lambda_v = \lambda_s$, $\lambda_v = \lambda_s + 0.1$ and $\lambda_v = \lambda_s + 0.2$ as a function of $\lambda_s$. The dotted line is the result by the DBHF calculation of Ref. 8).

Fig. 7. The same as Fig. 6 but for the vector potential.
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Table I. Values of the scalar and vector coupling constants, $(g_{\text{NN}a}^2)/(4\pi)$ and $(g_{\text{NN}w}^2)/(4\pi)$, the effective mass $m^*$ at the saturation density, the scalar $U_s$ and vector $U_v$ potentials at the saturation density and the incompressibility $K$ for $\lambda_s = \lambda_v = 1.0, 0.8, 0.6, 0.4, 0.2, 0.0$, respectively.

<table>
<thead>
<tr>
<th>$\lambda_{s(v)}$</th>
<th>$(g_{\text{NN}a}^2)/(4\pi)$</th>
<th>$(g_{\text{NN}w}^2)/(4\pi)$</th>
<th>$m^*$ (MeV)</th>
<th>$U_s$ (MeV)</th>
<th>$U_v$ (MeV)</th>
<th>$K$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<td>15.14</td>
<td>0.541</td>
<td>-431</td>
<td>354</td>
<td>547</td>
</tr>
<tr>
<td>0.8</td>
<td>10.77</td>
<td>16.55</td>
<td>0.571</td>
<td>-403</td>
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<td>390</td>
</tr>
<tr>
<td>0.6</td>
<td>11.62</td>
<td>17.62</td>
<td>0.603</td>
<td>-372</td>
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</tr>
<tr>
<td>0.4</td>
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<td>18.20</td>
<td>0.637</td>
<td>-341</td>
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</tr>
<tr>
<td>0.2</td>
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<td>18.26</td>
<td>0.669</td>
<td>-310</td>
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</tr>
<tr>
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<td>17.86</td>
<td>0.699</td>
<td>-282</td>
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<td>163</td>
</tr>
</tbody>
</table>

Fig. 8. The same as Fig. 6 but for incompressibility. The dotted lines indicate the empirical upper and lower limits.

Finally, under the condition $\lambda_s = \lambda_v$, we investigate the behavior of the energy per particle, the scalar and vector potentials, and the effective coupling constants at low and high densities in Figs. 9~12. The dotted points in Figs. 9~11 are the results of the DBHF calculations. As may be conjectured from Eqs. (26) and (27), the density-dependence of the effective coupling constant in Fig. 12 becomes stronger, as $\lambda_{s(v)}$ goes down to 0. The strengths of the potentials in the whole range of $\bar{\rho}$ and the energy per particle in the range $\bar{\rho} > 1$ decrease, as the effective coupling constant depends on the density more strongly. The density-dependencies of the potentials and energy per particle in the modified $\sigma$-$\omega$ model are different from those in the DBHF theory. As mentioned above, the DBHF results at $\bar{\rho} = 1$ coincide with our results using $\lambda_s = \lambda_v \approx 0.6$. At $\bar{\rho} = 2$, the potentials by the DBHF calculation nearly equal our calculations using $\lambda_s = \lambda_v = 0.4$. For the scalar potentials in the range $\bar{\rho} > 2$, the DBHF results coincide with our results using $\lambda_{s(v)} < 0.4$, while the vector potentials by the DBHF calculation agree with ours using $\lambda_{s(v)} > 0.4$. For the energy per particle in the range $\bar{\rho} > 2$, the DBHF calculations correspond to ours using $\lambda_{s(v)} > 0.6$. Because the allowed values of the parameters have already been restricted to $0.3 \leq \lambda_{s(v)} \leq 0.6$, this result indicates that the DBHF model and the modified $\sigma$-$\omega$ model are essentially different from each other at high densities.
§ 4. Summary

In order to overcome the shortcomings of the Walecka $\sigma$-$\omega$ model and the DSC model, we introduced the modified scalar and vector vertices. These are expressed by the scalar and vector Dirac gamma matrices times the operators like the positive- and negative-energy projections, and include two additional parameters whose values lie between 0 and 1. If both the parameters are taken to be 1, our model returns to
The Modified $\sigma$-$\omega$ Model for Relativistic Nuclear Matter

The values of the parameters less than 1 correct the couplings between positive and negative energy states of the nucleons in the Walecka model. The appropriate values of the parameters should be determined to reproduce the properties of nuclear matter.

The Euler-Lagrange equation for the nucleon field derived from the Lagrangian including the modified vertices has a complicated form unlike the Dirac equation. We therefore required that the nucleon field obeys the Dirac equation including the scalar and vector potentials. This requirement leads to the expressions of the mean-$\sigma$ and $-\omega$ fields in terms of the effective nucleon mass and vector potential. If the effective $NN\sigma$ and $NN\omega$ coupling constants are introduced, the formal expression of the energy density is the same as the Walecka $\sigma$-$\omega$ model. The coupled self-consistency equations for the effective nucleon mass and vector potential are determined by extremization of the energy density. These equations are rewritten to the expressions of the $NN\sigma$ and $NN\omega$ coupling constants. Substituting them to the equation for the saturation condition of nuclear matter, we finally obtained the self-consistency equation for the effective nucleon mass.

We first calculated the scalar and vector potentials at the saturation density, the $NN\sigma$ and $NN\omega$ coupling constants, and the incompressibility as functions of the two additional parameters characterizing the modified vertices. The results were compared with those by the DBHF calculations, the OBEM of $NN$ force and the empirical value, respectively. By these comparisons we can see that the two parameters should be nearly equal and that their values should be between 0.4 and 0.6. Consequently, we have only one additional parameter in fact and the available value of the parameter is severely restricted. It has to be emphasized that this favorable result is due to opposite effects by the modified scalar and vector vertices. Then, the energy per particle, the scalar and vector potentials and the effective coupling constant were calculated as functions of density, and compared with the DBHF results. At the saturation density, our results are almost the same as the DBHF ones, provided that the values of the two parameters are 0.6. The strengths of the potentials at all the densities and the energy per particle at the densities higher than the saturation point decrease, as the density-dependence of the coupling constants become stronger. The density-dependencies of the potentials and energy per particle in our model are somewhat different from those in the DBHF theory. These differences are prominent at low and high densities.

It has been found that the modified $\sigma$-$\omega$ model is able to reproduce the reasonable values of the coupling constants, potentials and incompressibility, and so may overcome the shortcomings of the Walecka $\sigma$-$\omega$ model and the DSC model. In order to certify the validity of the present model, it is necessary to apply the model to the calculations of finite-nuclear structure and elastic nucleon-nucleus scattering at intermediate energies. Furthermore, the extensions of the model to asymmetric- and hyper-nuclear matter are interesting. We have constructed the modified $\sigma$-$\omega$ model, inspired by several suggestions that the $+ -$ couplings in the usual $\sigma$-$\omega$ (or scalar-vector) model should be corrected. However, this model has no theoretical foundations at present. Thus, it is a remaining important subject to investigate the physical origin of the modified vertices.
References