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Discussion

Discussion: "On Global Energy Release Rate of a Permeable Crack in Piezoelectric Ceramic" (Li, S., 2003 ASME J. Appl. Mech., 70, pp. 246–252)

X.-F. Li
College of Mathematics and Computer Science, Hunan Normal University, Changsha 410081, People's Republic of China.
e-mail: xfli@post.com

K. Y. Lee
School of Mechanical Engineering, Yonsei University, Seoul 120-749, South Korea.
e-mail: KYL2813@yahoo.co.kr

In studying crack problems in a piezoelectric material, many crack models have been proposed and some fracture criteria have been established. The author of [1] presented a new permeable crack model. That is, a permeable crack is modeled as a rectangular hole having height h_0. The first-order perturbation solution in terms of small parameter h_0 is derived, and asymptotic electroelastic field, together with field intensity factors, local and global energy release rates are further determined. The obtained theoretical prediction agrees basically with experimental observation. Here, we would like to make some discussions on [1].

In deriving the results in [1], Eq. (45) is crucial. However, based on (44), (45) does not hold unless $\tilde{E}_X(X,Y)$ and $\tilde{E}_X^a(X,Y)$ are linear functions with respect to variable Y and independent of variable X. The reason is that, if denoting

$$f(X,\pm h(X))=\tilde{E}_X(X,\pm h(X))-\tilde{E}_X^a(X,\pm h(X)), \tag{1}$$

the Fourier cosine transform of $f(X,\pm h(X))$ is

$$\int_0^\infty f(X,\pm h(X))\cos(\zeta X)dX=\int_0^a f(X,\pm h_0)\cos(\zeta X)dX + \int_a^\infty f(X,0)\cos(\zeta X)dX, \tag{2}$$

rather than

$$f^*\left(\zeta,\pm h_0\frac{\sin(a\zeta)}{\zeta}\right), \tag{3}$$

where

$$f^*(\zeta,Y)=\int_0^\infty f(X,Y)\cos(\zeta X)dX. \tag{4}$$

Consequently, (45), i.e.,

$$f^*\left(\zeta,\pm h_0\frac{\sin(a\zeta)}{\zeta}\right)=0,\quad 0<\zeta<\infty, \tag{5}$$

cannot follow from (44), i.e.,

$$f(X,\pm h(X))=0,\quad -\infty<X<\infty, \tag{6}$$

where $h(x)$ is given by (12) in [1].

In addition it is seen from (81)–(83) that the height of a rectangular crack has been taken into account. However, for such a rectangular crack, (or strictly speaking a rectangular hole), $\sigma_{YZ}(X,0)$, $\epsilon_{YZ}(X,0)$, $D_Y(X,0)$, and $E_Y(X,0)$ should have no singularity near the points $(\pm a,0)$ since the points $(\pm a,0)$ are not the crack tips $(h_0>0)$. Instead, the electromechanical field near the apexes of the rectangle $(\pm a,\pm h_0)$ exhibits a singularity. Moreover, the singularity is no longer an inverse square-root singularity. The classical definition of field intensity factors is therefore employed directly except for the case of $h_0=0$.

Reference

[1] Li, S., 2003, "On Global Energy Release Rate of a Permeable Crack in a Piezoelectric Ceramic," Transactions of the ASME, J. Appl. Mech., **70**, pp. 246–252.

Closure to "On Global Energy Release Rate of a Permeable Crack in Piezoelectric Ceramic" (2003, ASME J. Appl. Mech., 70, p. 930)

S. Li
Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720

(1) For a finite height $h_0>C$, $C>0$, we define real functions

$$h(X):=\begin{cases} h_0, & |X|<a \\ 0, & |X|>a \end{cases}, \tag{1}$$

and

$$F(X,h(X)):=\tilde{E}_X(X,h(X))-\tilde{E}_X^a(X,h(X)). \tag{2}$$

It is true that in general the Fourier transform of $F(X,h(X))$,

$$F^*(\zeta):=\int_0^\infty F(X,h(X))\cos(\zeta X)dX\neq f^*(\zeta,h^*(\zeta)), \tag{3}$$

where

$$f^*(\zeta,Y):=\int_0^\infty F(X,Y)\cos(\zeta X)dX, \tag{4}$$

$$h^*(\zeta) := \int_0^\infty h(X)\cos(\zeta X)\,dX. \quad (5)$$

However, when $h_0 \to 0$,

$$\int_0^\infty F(X,0)\cos(\zeta X)\,dX = f^*(\zeta,0). \quad (6)$$

In other words, we expect

$$\lim_{h_0 \to 0} F^*(\zeta) \to f^*(\zeta,0). \quad (7)$$

During this limiting process, the four corners of the slit will merge and become the two crack tips at $(X = \pm a, 0)$. One of the main technical difficulties of fracture mechanics of piezoelectric materials is how to correctly describe this limiting process.

Ref. [1] suggests that in the Fourier transform domain the limiting process may be approximated as

$$\lim_{h_0 \to 0} F^*(\zeta) \to \lim_{h_0 \to 0} f^*(\zeta, h^*(\zeta)) = f^*\left(\zeta, h_0 \frac{\sin(a\zeta)}{\zeta}\right) \to f^*(\zeta,0) \quad (8)$$

which, the author believed, is plausible in an asymptotic sense.

Moreover, the approximation (8) becomes exact when $F(X,Y)$ is a linear function with respect variable Y, which is the difference of $\tilde{E}_X(X,Y)$ and $\tilde{E}_X^a(X,Y)$. To require the same restriction on $\tilde{E}_X(X,Y)$ and $\tilde{E}_X^a(X,Y)$ may be too strong.

(2) From the perspective of classical fracture mechanics, it is also true that for a finite rectangular slit, there is no singularity for the electrical/mechanical fields at $X = \pm a$ an $Y = 0$. The singularities will appear at the four corners of the rectangular slit, $(\pm a, \pm h_0)$, with a singularity power index different from $-1/2$.

Nonetheless, it has become a consensus now that the fracture process of a piezo-electric ceramic is in fact a coupled multiscale phenomenon. This can be argued based on both its physical nature and its mathematical structure.

Ref. [1] tried to explore the asymptotic multiscale structure of the problem. Intuitively, the crack-tip field was viewed as the outer problem, and it was assumed that it has the form of the classical solution with respect to the ''slow'' coordinate variables (therefore there is basically no slit there). On the other hand, the electrostatic problem inside the crack was viewed as an inner problem that is controlled by the slit height, h_0, which is the length scale of the problem and it is associated with the ''fast'' coordinate variable.

The essential idea of this approach is using Eq. (8) to match the outer (macro) solution with the inner (micro) solution. Of course, the asymptotic multiscale analysis could be done differently.

Reference

[1] Li, S., 2003, ''On Global Energy Release Rate of a Permeable Crack in a Piezoelectric Ceramic,'' ASME Journal of Applied Mechanics, **70**, pp. 246–252.