

The essential scheme of the book is to develop understanding of the method and familiarity with its techniques by analyzing successively more complex and sophisticated problems. At the same time, the generality is emphasized by the range of different problems considered. Mathematical formality and rigor are kept to a minimum; in particular the calculus of variations is avoided, in keeping with the assumed mathematical background of the reader. Attention is directed almost exclusively to the generation and solution of the governing equations of the discrete systems. The reader is presumed to possess at least a basic understanding of the physical subjects treated.

While the author rightly emphasizes the value of hand calculation with small examples as a way of acquiring insight, he also emphasizes that it is only through the computer that the finite element method finds its real purpose and application. He is therefore at pains to provide in the book a detailed and well-documented computer program for one-dimensional problems. Information on other programs, applicable to many of the other topics covered in the book and available from the author, is also given.

The book really begins in Chapter 2, after a brief and rather mysterious philosophical disquisition on the idea of discretization, with an introduction of the basic concepts and a skeleton of theory. Not surprisingly, the first specific problem to be treated, in Chapter 3, is the axially loaded, elastic bar. The classic displacement analysis is derived both from the principle of minimum potential energy and from Galerkin's method. In addition, a mixed method, based on the Hellinger-Reissner principle, is presented. One-dimensional steady flow problems are next discussed and the close analogy with the elastic bar emphasized in Chapter 4. The flow problem also provides a vehicle for enlargement of the analysis into time-varying behavior in Chapter 5.

Chapter 6 contains the one-dimensional computer program, applicable to the foregoing models, and in Chapter 7 the bending behavior of beams serves as an example to introduce higher-order continuity conditions (slope as well as displacement) and, as a refinement, interpolation functions with continuity of curvature at nodes. The remaining one-dimensional problems treated are: mass transport (convection/diffusion equation), overland flow, and elastic stress-wave propagation in Chapters 8-10.

Two-dimensional problems are introduced in Chapter 11 via the St. Venant torsion problem. Four analyses are given. Besides the solutions in terms of warping and stress functions, there is a hybrid and a mixed method, all using simple triangular elements. Other two-dimensional field problems (potential flow, heat flow, and seepage) are treated in Chapter 12 to emphasize unity and generality, but the analysis uses simple (quadrilateral) isoparametric elements. Chapter 13 covers plane elasticity, using the displacement method.

In the final chapter (14), the scope of the finite element method and its applicability to heterogeneous systems is illustrated by a discussion of the modeling necessary for a building frame with slab floors and elastic foundations.

The range of technical subjects is very impressive and the versatility of the finite element method is clearly demonstrated. A student working through the book, with its constant reinforcement of ideas and tactics and its widening horizons, and making use of the computer programs available, must very quickly acquire familiarity and competence in basic finite element technology.

But there are some flaws, and these are mainly in basic,

theoretical aspects. Thus, the idea of isoparametric elements is presented rather casually, and the "natural" or "intrinsic" coordinates, fundamental to the idea, are merely and misleadingly described as "local" coordinates, with no explanation. It is also difficult to see what the complementary energy method has to offer in the analysis of the axially loaded bar.

More serious, however, is the fundamental question of the variational basis of the finite element method. The author uses a mixture of approaches; sometimes a variational principle is invoked, sometimes the Galerkin method is applied, once the principle of virtual work. Discussion of these and their relationships can only be loose and qualitative without the help of variational calculus. Although the principle of minimum total potential energy, as a condition for equilibrium, should be familiar to an undergraduate from his or her physics training, the other principles used, like the Hellinger-Reissner, must be quite mysterious. And even if these are acceptable for defining element behavior, the question of natural boundary conditions is extremely obscure without the variational argument. A brief concise exposition of the essential variational approach would have added greatly to the strength of the book, by providing a clearer theoretical base to complement the development.

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**Continuum Mechanics.** By A. J. M. Spencer. Longman Mathematical Texts, 1980. 183 Pages. Price \$13.50.

#### REVIEWED BY R. L. FOSDICK<sup>6</sup>

The author has intended this book to be an introduction to the theory of continuum mechanics in a form that is suitable for undergraduate students. I think that his aim has been achieved and that the approach and scope of material covered is reasonable for the engineering undergraduate student at the junior or senior level. In 183 pages and 11 chapters it is not possible to stray too far from the beaten track and by design the author does not do so. The usual elements of the kinematics of motion and deformation, stress, and balance laws are to be found in a general and understandable setting which is not compromised by ad hoc linearity assumptions. There is one chapter that is devoted to linear constitutive relations for elasticity, classical viscous fluids, and viscoelasticity. In addition, there is one chapter that gives a brief, but instructive description of the constitutive theory for nonlinear hyperelasticity, and of the invariance restrictions that lead to the Reiner-Rivlin fluid. Simple materials with general history dependence are merely mentioned, and the basic elements of classical plasticity theory are discussed and formulated in four pages. There are two supporting chapters at the beginning of this book on matrix algebra, vectors, and cartesian tensors and one at the end of the book on cylindrical and spherical coordinates. The notion used throughout the book is a blend of direct or matrix (i.e., index-free) and the indicial for rectangular cartesian coordinates. There is no thermomechanics covered in this book. While there is a reasonable number of exercises on the mechanics topics with an appendix of answers, there are none that deal explicitly with vectors and tensors.

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