Sensitivity analysis of bridge pier scour depth predictive formulae

Roberto Gaudio, Ali Tafarojnoruz and Samuele De Bartolo

ABSTRACT

Sensitivity analysis is an approach to recognising the behaviour of models and relative importance of causative factors. In this paper, behaviours of six pier scour depth empirical formulae are evaluated on the basis of an analytical method. The sensitivity of predicted scour depth is analysed with respect to the following independent parameters: approach flow depth, riverbed slope and median sediment size. Also their combined influence is studied examining the relative importance of each parameter with respect to the total variation of the maximum scour depth. Results show that: (1) sensitivity significantly depends on flow intensity for most of the selected formulae, whereas for the others it is a constant value or depends on other influencing parameters; (2) different formulae demonstrate various level of sensitivity to the input variables, so that, for a certain error in the input variables, the error in the results may vary consistently; (3) some formulae are very sensitive to the input parameters under some conditions, hence an error in an input variable may be amplified in the output results; and (4) most of the formulae are more sensitive to the variations of the influencing parameters in clear-water than in live-bed conditions.

Key words | bridge pier, local scour, prediction and variation, sensitivity analysis

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NOTATION

b	pier width [L]
b_e	pier width projected orthogonally to the approach flow [L]
c	relative variation of F_i or relative variation ($c\Delta F_i/F_i$)
D^*	effective pier diameter [L]
d_{50}	median grain size [L]
d_{se}	maximum scour depth at equilibrium [L]
Fr	Froude number, $U/(gh)^{0.5}$
Fr_c	critical Froude number, $U_c/(gh)^{0.5}$
F_i	factor that influence O
f	unknown function
g	acceleration due to gravity [LT^{-2}]
h	approach flow depth [L]
K_1	correction factor for pier nose shape
K_2	correction factor for angle of attack
K_3	correction factor for bed condition
K_4	correction factor for bed armouring
K_d	sediment size factor
K_G	channel geometry factor
K_{hb}	flow depth-foundation size factor [L]
K_I	flow intensity factor
K_S	pier shape factor
K_w	correction factor for wide piers in shallow water
K_θ	pier alignment factor
n	Manning's roughness coefficient [L$^{-1/3}$T]
O	model output
O_0	value of O at some specified level of each F_i
P_{F_i}	percentage variation aliquot related to variation of influencing factor F_i
P_h	percentage variation aliquot related to variation of h
P_S	percentage variation aliquot related to variation of S
$P_{d_{50}}$	percentage variation aliquot related to variation of d_{50}
R_h	hydraulic radius [L]
S	riverbed slope
S_{F_i}	relative variation of O_0

S_f	energy gradient
U	mean approach flow velocity $[LT^{-1}]$
U_c	critical flow velocity for sediment motion $[LT^{-1}]$
Δ	relative submerged grain density
$\Delta d_{se,\Delta h}$	variation of the equilibrium scour depth due to Δh $[L]$
$\Delta d_{se,\Delta S}$	variation of the equilibrium scour depth due to ΔS $[L]$
$\Delta d_{se,\Delta d50}$	variation of the equilibrium scour depth due to Δd_{50} $[L]$
ΔF_i	variation of factor F_i
Δh	variation of h $[L]$
ΔS	variation of S
Δd_{50}	variation of d_{50} $[L]$
$\varepsilon_{d_{50}}$	specific sensitivity to d_{50}
ε_{F_i}	specific sensitivity to F_i
ε_h	specific sensitivity to h
ε_S	specific sensitivity to S
ϕ	pier nose shape coefficient

INTRODUCTION

Sensitivity analysis is an essential modelling tool for the proper application of a model (or formula), since it enables the model user to understand the relative importance of variables and the effects of input errors on computed outputs (McCuen 2003). In the fields of hydraulics and water resources, sensitivity analysis was widely studied using several methods. For instance, Schulz & Huwe (1999) used fuzzy set theory to perform sensitivity analysis of water transport modelling in a layered soil profile, Radwan *et al.* (2004) carried out sensitivity analysis for river water quality modelling, Hall *et al.* (2005) applied variance-based global sensitivity analysis for simulation of a flood on a river reach, Mishra (2009) showed that the uncertainty and sensitivity analysis techniques can be applied systematically to field-scale hydrologic models. In the field of sediment transport an interesting study was conducted by Sirangelo & Versace (1983) and more recently by Pinto *et al.* (2006), who used the Monte Carlo method to consider one bed-load and four total-load formulae. Their results show that the accuracy in total sediment transport evaluations is mainly determined by uncertainty in flow velocity and median sediment size.

Regarding bridge pier scour equations, once a formula is employed for the prediction of the maximum scour depth, inaccurate output may be due to the selected empirical formula and/or uncertainty in the input variables. According to Samadi *et al.* (2009), parameter uncertainty can be due to: (1) measurement error (e.g., personal bias in reading the flow depth measuring scale, finite instrument resolution for measuring the bed material size or estimation of river bed slope through small scale maps); or (2) the inherent natural variability of the parameter itself (e.g., the river bed material characteristics, as the median grain diameter, may vary spatially around the bridge pier; such properties may also change owing to the bed form propagations).

The prediction can therefore be significantly different from the real scour depth, owing to the error aliquot deriving from the selection of an inappropriate formula (e.g., an envelope formula which overestimates too much, an interpolation formula with low coefficient of determination, a formula which is used out of its range of application, etc.). Furthermore, parameter uncertainty may also lead to unreliable predictions. In this case, sensitivity analysis may link the uncertainty of the scour influencing parameters to the reliability of predicted scour depth. A significant overestimation in prediction of scour depth results in uneconomic construction of bridge and countermeasures, whereas an underestimation may reduce their safety. In particular, a more accurate estimation of pier scour depth plays an important role in the design of some types of scour countermeasures, since a high efficiency in application of some of them (e.g., a slot through pier) is obtained when they are extended also inside the scour (Tafarojnoruz *et al.* 2010a, 2012; Gaudio *et al.* 2012).

Maximum pier scour depth is generally estimated by means of empirical formulae as a function of several influencing parameters (Tafarojnoruz *et al.* 2010b). Since each effective parameter has a particular influence on scour depth, different sensitivities in the prediction of scour depth are expected with respect to each influencing parameter. In this field, Yucel (1992) and Glenn (1994) carried out a preliminary study of sensitivity analysis for bridge scour equations proposed by the Federal Highway Administration (FHWA). Dunn & Smith (1993) performed a

sensitivity analysis case study based on cross-section data from 15 bridge sites. The cross-sections were used to compute the water surface profile through each bridge for several recurrence-interval design discharges. The results were presented in the form of scour depth as a function of exceeding-probability. Yanmaz (2001) carried out a reliability-based assessment of bridge pier scour depth by analysing two equations derived from an experimental dataset. He performed the uncertainty analysis of the two equations on the basis of a first-order Taylor expansion as well as a probability distribution of influencing pier scour parameters.

In this work six well-known formulae for the assessment of the equilibrium maximum scour depth at a bridge pier were selected in order to perform the sensitivity analysis with respect to the most important independent variables. They were respectively proposed by Breusers *et al.* (1977) (hereinafter BR), Jain & Fischer (1979) (JF), Froehlich (1988) (FL), Melville (1997) (ML), FHWA (HEC-18) (HC; Richardson & Davis 2001) and Sheppard *et al.* (2004) (SH). Table 1 furnishes the selected pier scour formulae, d_{se} being the maximum scour depth at equilibrium, b the pier width, h the approach flow depth, d_{50} the median sediment size, U the mean approach flow velocity and U_c the critical approach flow velocity for the inception of sediment motion. Figure 1 shows flow and local scour around a bridge pier.

Although a large number of empirical formulae was proposed to estimate pier scour depth, the above six formulae have some peculiarities which induced us to select them. The formulae were selected based on the following categories: (a) formulae which were originally developed on the basis of a large number of laboratory datasets or after comparison and analysis of several formulae. The ML, BR and JF formulae fall in this category (see Breusers *et al.* 1977; Jain & Fischer 1979; Melville 1997); and (b) formulae which exhibit the highest accuracy in the prediction of large scale/field scour depth. The HC, FL and SH formulae belong to this category (see, e.g., Landers & Mueller 1996; Sheppard *et al.* 2004; Mueller & Wagner 2005; Mohamed *et al.* 2006; Tafarojnoruz 2012).

Up to now, several studies have been carried out to compare the accuracy of pier scour formulae using field data (e.g., Jones 1984; Johnson 1995). In a more recent study,

Gaudio *et al.* (2010) showed that different formulae produce significantly different predictions in both laboratory and field studies. Such researches help the designer to select the best formulae for the specific study, reducing the error aliquot due to selection of inappropriate formulae, but do not guarantee to reduce the unreliability of calculated scour depth due to uncertainty of the effective parameters. In fact, in field applications, some parameters may be assessed with different level of accuracy: e.g., the channel slope may be estimated on the basis of small-scale maps or by costly topographic measurements; the median grain size by available reports on the watercourse or by geotechnical analysis of one or more sediment samples collected at the bridge site; the water depth by hydrological studies and with the application of rainfall–runoff models or by direct measurements. In such cases, the knowledge of the sensitivity of a predictive formula to the effective parameters can be of help to improve the scour depth prediction, through the acquisition of more accurate (and, consequently, more expensive) data. The aim of the present study is to show the importance of sensitivity analysis in pier scour estimation, once the input parameters are measured independently, with an independent level of uncertainty. The uncertainty is also assumed to depend only on the measurement error; hence, other types of uncertainty, e.g. possible uncertainty due to inherent natural variability of the parameters, were neglected. In fact, for most practical cases, in addition to uncertainty due to the measurement error, the uncertainty due to the temporal or spatial variation of the parameters is also considerable. The importance of such uncertainty is notable, if the uncertainty of a parameter amplifies that one of the other parameters. For instance, variation of riverbed slope due to bed degradation may lead to an increase of flow velocity variations. In similar cases, the sensitivity effects on an output parameter may be evaluated through a calibrated bed-morphodynamic model.

METHODOLOGY

Let us consider a uniform circular pier under the following conditions: the bed material is made of uniform sediments with relative submerged grain density $\Delta = 1.65$, no bed

Table 1 | Selected pier scour formulae

| Abbreviation	Equation	Notes	Reference
BR	$\dfrac{d_{se}}{b} = 2 \cdot f\left(\dfrac{U}{U_c}\right) \tanh\left(\dfrac{h}{b}\right) \cdot K_s \cdot K_\theta$	$f\left(\dfrac{U}{U_c}\right) = \begin{cases} 0 & \text{for } \dfrac{U}{U_c} \le 0.5 \\ 2\dfrac{U}{U_c} - 1 & \text{for } 0.5 \le \dfrac{U}{U_c} < 1 \\ 1 & \text{for } \dfrac{U}{U_c} \ge 1 \end{cases}$ K_S=pier shape factor; K_θ = pier alignment factor.	Breusers et al. (1977)
JF	$d_{se} = 1.84 b\left(\dfrac{h}{b}\right)^{0.3} \mathrm{Fr}_c^{0.25}$, valid for maximum clear-water scour; $d_{se} = 2.0 b\left(\dfrac{h}{b}\right)^{0.5}(\mathrm{Fr} - \mathrm{Fr}_c)^{0.25}$ for Fr–$\mathrm{Fr}_c \ge 0.2$	$\mathrm{Fr} = U/(gh)^{0.5}$ = Froude number; $\mathrm{Fr}_c = U_c/(gh)^{0.5}$ = critical Froude number; for $0 < \mathrm{Fr}$–$\mathrm{Fr}_c < 0.2$, the largest value obtained from the two equations is to be taken.	Jain & Fischer (1979)
FL	$d_{se} = 0.32 b\phi \mathrm{Fr}^{0.2}\left(\dfrac{b_e}{b}\right)^{0.62}\left(\dfrac{h}{b}\right)^{0.46}\left(\dfrac{b}{d_{50}}\right)^{0.08} + b$	b_e = width of the bridge pier projected orthogonally to the approach flow direction; ϕ = coefficient based on the pier nose shape.	Froehlich (1988)
ML	$d_{se} = K_{hb} K_I K_d K_S K_\theta K_G$	$K_{hb} = \begin{cases} 2.4b & \text{for } b/h < 0.7 \\ 2\sqrt{hb} & \text{for } 0.7 < b/h < 5.0 \\ 4.5\,h & \text{for } b/h > 5.0 \end{cases}$ $K_I = \begin{cases} \dfrac{U}{U_c} & \text{for } \dfrac{U}{U_c} < 1 \\ 1 & \text{for } \dfrac{U}{U_c} \ge 1 \end{cases}$ $K_d = \begin{cases} 0.57\log\left(2.24\dfrac{b}{d_{50}}\right) & \text{for } \dfrac{b}{d_{50}} \le 25 \\ 1 & \text{for } \dfrac{b}{d_{50}} > 25 \end{cases}$ K_G = channel geometry factor.	Melville (1997)
HC	$\dfrac{d_{se}}{b} = 2 K_1 K_2 K_3 K_4 K_w \left(\dfrac{h}{b}\right)^{0.35} \mathrm{Fr}^{0.43}$	K_1, K_2, K_3, and K_4 are correction factors accounting for the pier nose shape, flow angle of attack, presence of bed forms, and bed armouring. K_w was suggested by Johnson & Torrico (1994) for wide piers in shallow water when $h/b < 0.8$, $b/d_{50} > 50$ and $\mathrm{Fr} < 1$: $K_w = 2.58\left(\dfrac{h}{b}\right)^{0.34}\mathrm{Fr}^{0.65} \quad \text{for } \dfrac{U}{U_c} < 1$ $K_w = 1.0\left(\dfrac{h}{b}\right)^{0.13}\mathrm{Fr}^{0.25} \quad \text{for } \dfrac{U}{U_c} \ge 1$ Note that in the last equation the exponent of Fr is 0.25 (and not 0.20 as in the original paper by Johnson & Torrico (1994)).	Richardson & Davis (2001)
SH	$\dfrac{d_{se}}{D^*} = 2.5 \cdot f_1\left(\dfrac{h}{D^*}\right) \cdot f_2\left(\dfrac{U}{U_c}\right) \cdot f_3\left(\dfrac{D^*}{d_{50}}\right)$	D^* = effective diameter of the pier; $f_1\left(\dfrac{h}{D^*}\right) = \tanh\left[\left(\dfrac{h}{D^*}\right)^{0.4}\right]$ $f_2\left(\dfrac{U}{U_c}\right) = 1 - 1.75\left[\ln\left(\dfrac{U}{U_c}\right)\right]^2$ $f_3\left(\dfrac{D^*}{d_{50}}\right) = \dfrac{D^*/d_{50}}{0.4\left(\dfrac{D^*}{d_{50}}\right)^{1.2} + 10.6\left(\dfrac{D^*}{d_{50}}\right)^{-0.13}}$	Sheppard et al. (2004)
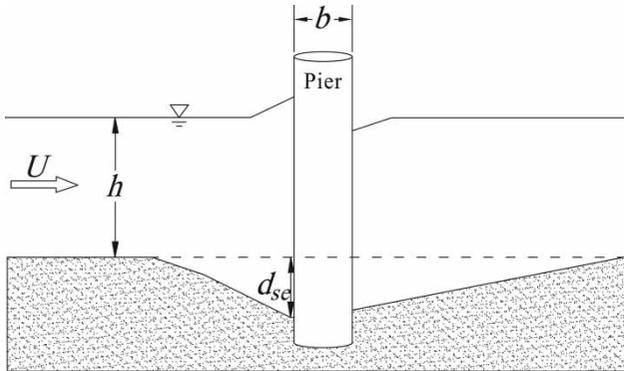
Figure 1 | Schema of flow and local scour around a bridge pier.

armouring is then considered; and the flow regime upstream to the pier is assumed to be uniform ($S = S_f$) and fully turbulent in a wide channel ($R_h \approx h$) with S, S_f and R_h being the riverbed slope, the energy gradient and the hydraulic radius, respectively.

In this research, the well-known Manning and Strickler equations,

$$U = R_h^{2/3} S^{0.5}/n \quad \text{and} \quad n = 0.041 d_{50}^{1/6} \tag{1}$$

were used to compute the mean approach flow velocity, U (n is the Manning roughness coefficient). U_c was also assessed by the Neill (1968) equation,

$$U_c = 1.41 (\Delta g d_{50})^{0.5} (h/d_{50})^{1/6} \tag{2}$$

g being acceleration due to gravity. Therefore, the flow intensity, U/U_c, can be calculated as follows:

$$\frac{U}{U_c} = 4.3 \sqrt{\frac{S \cdot h}{d_{50}}} \tag{3}$$

Regarding the above assumptions, the equilibrium maximum scour depth, d_{se}, in the selected formulae is expressed as a function of b, d_{50}, h, U, and U_c. Since U and U_c can be defined as functions of h, d_{50} and S, the influencing parameters in estimating the maximum scour depth can be reduced to b, h, d_{50}, and S. Although pier width is an important parameter in scour depth calculations, sensitivity analysis of scour depth with respect to pier width was not considered in this study. In fact, for a certain uniform circular pier, the pier width is a deterministic factor; it does not vary during the bridge life span and no uncertainty is assumed to affect it.

In this study, the bed form factor in the HC formula, K_3 (see Table 1), was assumed to be constant. In fact, sensitivity of the HC formula to this parameter can be calculated easily based on dune height (see Table 6.3 in Richardson & Davis 2001).

The general mathematical definition of sensitivity can be expressed using the Taylor series expansion of the explicit function (McCuen 2003):

$$O = f(F_1, F_2, \ldots, F_n) \tag{4}$$

where O is the model output and the F_i are factors which influence O. The change in O resulting from change in F_i can be obtained by using the Taylor series as follows:

$$f(F_i + \Delta F_i, F_{j|j \neq i}) = O_0 + \frac{\partial O_0}{\partial F_i} \Delta F_i + \frac{1}{2!} \frac{\partial^2 O_0}{\partial F_i^2} (\Delta F_i)^2 + \ldots \tag{5}$$

where O_0 is the value of O at some specified level of each F_i. In general, the nonlinear terms of Equation (5) are small in comparison with the linear terms; hence, Equation (5) reduces to:

$$f(F_i + \Delta F_i, F_{j|j \neq i}) = O_0 + \frac{\partial O_0}{\partial F_i} \Delta F_i \tag{6}$$

Thus, the approximate incremental change in O can be obtained as follows:

$$\Delta O_0 = \left(\frac{\partial O_0}{\partial F_i} \right) \Delta F = f(F_i + \Delta F_i, F_{j|j \neq i}) - O_0 \tag{7}$$

Since the variation ΔF_i can be defined as proportional to F_i (i.e., $\Delta F_i = c \cdot F_i$ c being a coefficient), then the relative incremental change in O can be written as follows and assumed as a measure of sensitivity:

$$S_{F_i} = \frac{\Delta O_0}{O_0} = \frac{1}{O_0} \cdot \left(\frac{\partial O_0}{\partial F_i} \right) \cdot c \cdot F_i = c \cdot \varepsilon_{F_i} \tag{8}$$

where S_{F_i} and ε_{F_i} are the relative variation of O_0 and the specific sensitivity of O_0, respectively, with respect to F_i,

the dimensionless coefficient $c = \Delta F_i / F_i$ being the relative variation of F_i. According to Equation (8), ε_{F_i} can be obtained through the following equation:

$$\varepsilon_{F_i} = \frac{1}{O_0} \cdot \left(\frac{\partial O_0}{\partial F_i} \right) \cdot F_i \qquad (9)$$

Equation (8) implies that knowledge of ε_{F_i} permits calculating S_{F_i} for various values of c. For instance, a 5% variation in F_i ($\Delta F_i = 0.05 \cdot F_i$, $c = 0.05$) results in $S_{F_i} = 0.05 \cdot \varepsilon_{F_i}$. In the following sections, the key parameter ε_{F_i} for each influencing parameter of the selected pier scour formulae is calculated. It will be demonstrated that specific sensitivity ε_{F_i} of some selected bridge scour equations is a constant or a simple function, whereas for the remaining ones it is a complex function of the influencing input parameters.

RESULTS

Specific sensitivity of d_{se} to h

Depending on the selected formula, the sensitivity of the equilibrium scour depth, d_{se}, to the approach flow depth, ε_h, is various. In clear-water conditions, ε_h in the BR formula

is a function of flow intensity and h/b, as follows (see also Figure 2(a)):

$$\varepsilon_h = 0.5 + \frac{1}{4U/U_c - 2} + \frac{2h}{b} \operatorname{csch}(2h/b) \qquad (10)$$

Equation (10) was obtained according to Equation (9) in which F_i and O_0 are h and $d_{se} = f(h,\dots)$, respectively. $\partial O_0 / \partial F_i$ also denotes the derivative of $d_{se} = f(h,\dots)$ with respect to h, i.e., $\partial d_{se} / \partial h$. Analogously, ε_S, and $\varepsilon_{d_{50}}$ were obtained for $F_i = S$ and d_{50}, respectively, as presented in the following sections.

Figure 2(a) clarifies that BR formula is considerably sensitive to h for flow intensity values less than about 0.7. For h/b values higher than about 4, the third term of Equation (10) can be ignored and ε_h can be estimated only through flow intensity. On the other hand, in live-bed conditions, ε_h is just a function of h/b (Equation (11) and Figure 2(b)). In fact, this formula is more sensitive to h in clear-water than in live-bed conditions, and in live-bed conditions the influence of h on predictions of d_{se} is negligible for $h/b > 4$:

$$\varepsilon_h = \frac{2h}{b} \operatorname{csch}(2h/b) \qquad (11)$$

Specific sensitivity of the JF formula to h in clear-water conditions is constant ($\varepsilon_h = 0.217$). However, in live-bed
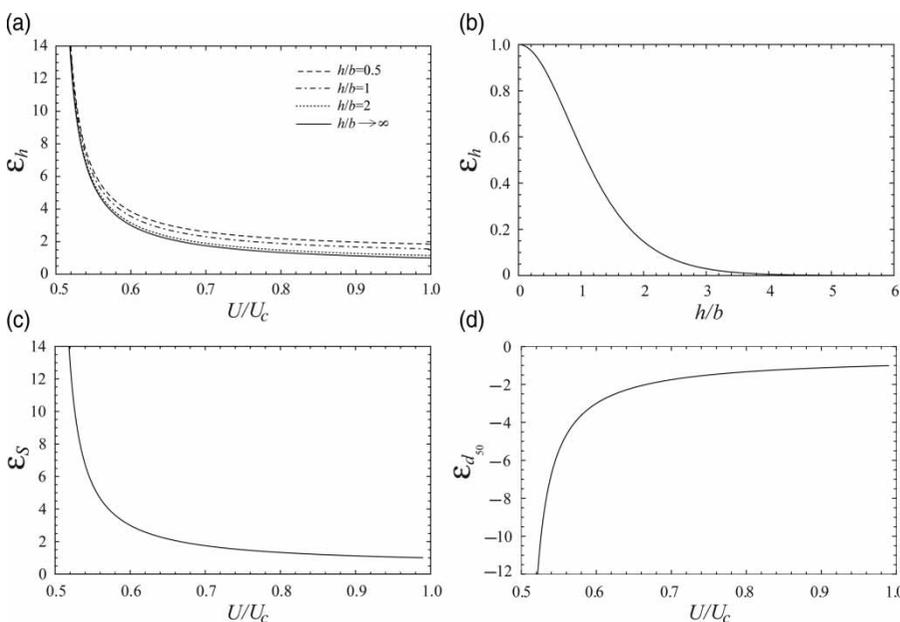

Figure 2 | Specific sensitivity of BR formula to: (a) h in clear-water conditions; (b) h in live-bed conditions; (c) S in clear-water conditions; and (d) d_{50} in clear-water conditions.
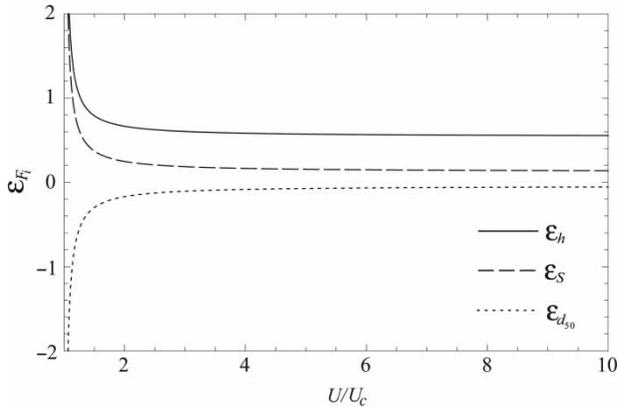
Figure 3 │ Specific sensitivity of the JF formula to h, S, and d_{50} in live-bed conditions.

conditions it is higher, especially near the threshold of sediment movement, i.e. $1 < U/U_c < 1.5$ (Equation (12) and Figure 3), whereas for $U/U_c > 2$, an almost constant value of 0.6 is obtained:

$$\varepsilon_h = 0.5 + \frac{0.083(1 + 0.5U/U_c)}{U/U_c - 1} \tag{12}$$

The FL formula was recommended for live-bed scour conditions. Its specific sensitivity to h is less than 0.495 and it is also a function of all the influencing parameters, as it depends on the first value of d_{se} (Equation (13)):

$$\varepsilon_h = 0.495 \frac{d_{se} - b}{d_{se}} \tag{13}$$

The sensitivity of d_{se} to h in the recommended SH formula for clear-water conditions decreases as flow intensity increases. Equation (14) and Figure 4(a) show that ε_h is a function of flow intensity and h/b; for $h/b > 4$ this parameter can be neglected and Equation (14) reduces to Equation (15). The range of variations of ε_h is considerably less than for the BR formula (Figures 2(a) and 4(a)):

$$\varepsilon_h = \frac{-1.75\ln(U/U_c) + (h/b)^{0.4}\{0.8 - 1.4[\ln(U/U_c)]^2\}}{\operatorname{csch}[2(h/b)^{0.4}]} \bigg/ \left(1 - 1.75[\ln(U/U_c)]^2\right) \tag{14}$$

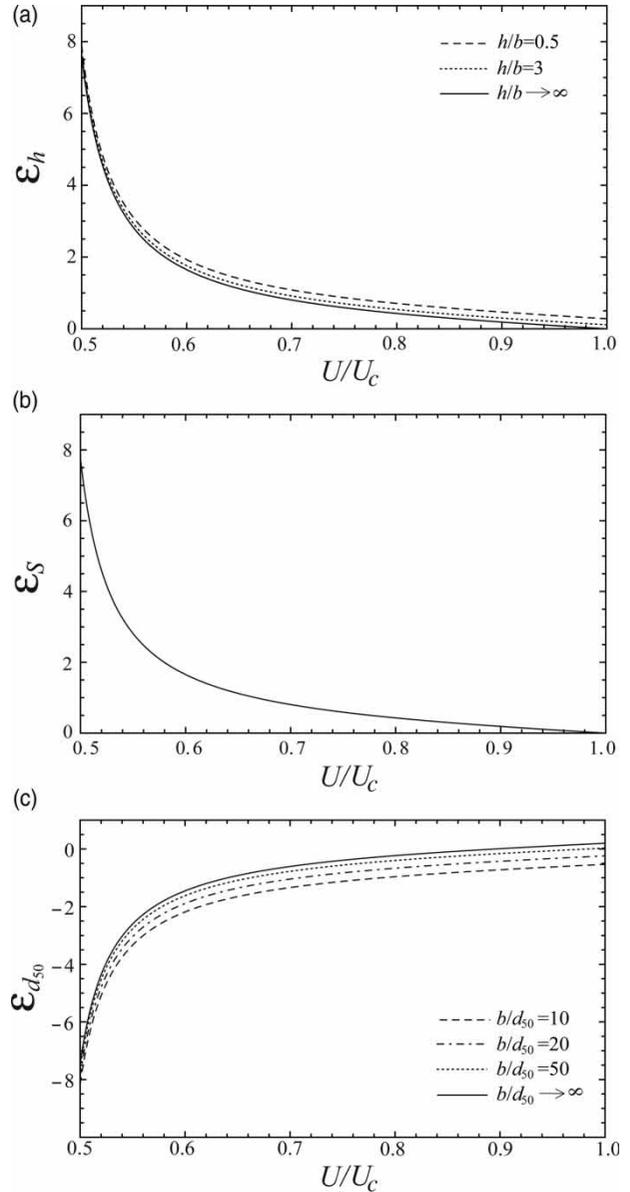$$\varepsilon_h = \frac{-1.75\ln(U/U_c)}{1 - 1.75[\ln(U/U_c)]^2} \tag{15}$$

Figure 4 │ Specific sensitivity of the SH formula to: (a) h; (b) S; and (c) d_{50}, all in clear-water conditions.

The remaining two selected formulae (i.e., ML and HC) have similar behaviours in the whole range of flow intensity values; in fact, in these two cases, ε_h does not depend on flow intensity (Tables 2 and 3). Table 2 shows that specific sensitivity of the ML formula to influencing parameters in live-bed conditions is less than in clear-water conditions for certain b/h and b/d_{50} values. In particular, Table 3 indicates that application of K_w increases the specific sensitivity to influencing parameters in the HC formula.

Table 2 | Specific sensitivity of the ML formula to the influencing parameters

Conditions	Clear-water conditions			Live-bed conditions		
	ε_h	ε_S	$\varepsilon_{d_{50}}$	ε_h	ε_S	$\varepsilon_d 50$
$\dfrac{b}{h} < 0.7, \dfrac{b}{d_{50}} \leq 25$	0.5	0.5	Equation (25)	0	0	Equation (26)
$0.7 < \dfrac{b}{h} < 5.0, \dfrac{b}{d_{50}} \leq 25$	1	0.5	Equation (25)	0.5	0	Equation (26)
$\dfrac{b}{h} > 5.0, \dfrac{b}{d_{50}} \leq 25$	1.5	0.5	Equation (25)	1	0	Equation (26)
$\dfrac{b}{h} < 0.7, \dfrac{b}{d_{50}} > 25$	0.5	0.5	−0.5	0	0	0
$0.7 < \dfrac{b}{h} < 5.0, \dfrac{b}{d_{50}} > 25$	1	0.5	−0.5	0.5	0	0
$\dfrac{b}{h} > 5.0, \dfrac{b}{d_{50}} > 25$	1.5	0.5	−0.5	1	0	0

Table 3 | Specific sensitivity of the HC formula to the influencing parameters

Scour condition	K_w	ε_h	ε_S	$\varepsilon_{d_{50}}$
Clear-water	1	0.42	0.215	−0.07
	$2.58\left(\dfrac{h}{b}\right)^{0.34}\mathrm{Fr}^{0.65}$	0.87	0.54	−0.18
Live-bed	1	0.42	0.215	−0.07
	$\left(\dfrac{h}{b}\right)^{0.13}\mathrm{Fr}^{0.25}$	0.59	0.34	−0.11

Specific sensitivity of d_{se} to S

The behaviour of specific sensitivity to riverbed slope, ε_S, was often found to be similar to the previous one about approach flow depth. In clear-water conditions, sensitivity of the BR formula follows a monotonic trend, exhibiting a relatively high sensitivity to S for $U/U_c < 0.7$ [Equation (16) and (Figure 2(c))]. In contrast, in live-bed conditions this formula is not sensitive to S, since it is not a function of velocity nor flow intensity:

$$\varepsilon_S = \frac{U/U_c}{2U/U_c - 1} \tag{16}$$

The JF formula is less sensitive to S with respect to h. In clear-water conditions, this formula is independent from S;

however, in live-bed conditions it is a function of flow intensity, as expressed by the following relationship (Figure 3):

$$\varepsilon_S = \frac{0.124}{1 - U_c/U} \tag{17}$$

The FL formula is significantly less sensitive to S with respect to h, and ε_S is less than 0.1 as follows:

$$\varepsilon_S = 0.1\frac{d_{se} - b}{d_{se}} \tag{18}$$

Specific sensitivity of the SH formula to riverbed slope is just a function of flow intensity (Figure 4(b)) and can be estimated using the following equation:

$$\varepsilon_S = \frac{\ln(U/U_c)}{[\ln(U/U_c)]^2 - 0.57} \tag{19}$$

Sensitivities of the remaining two formulae (i.e., ML and HC) to riverbed slope do not depend on flow intensity (Tables 2 and 3). In particular, the ML formula is not sensitive to S in live-bed conditions, since in this case it is not a function of flow velocity nor flow intensity.

Specific sensitivity of d_{se} to d_{50}

Sensitivity of most of selected formulae to d_{50} (i.e. $\varepsilon_{d_{50}}$) is negative in different conditions, i.e. for a positive value of ΔF_i (or c when F_i is positive) a negative value of S_{F_i} is expected. In other words, the reason for negative values of ε_{F_i} is that d_{se} decreases as d_{50} increases. In the BR formula, $\varepsilon_{d_{50}}$ is a function of flow intensity in clear-water conditions and can be calculated using Equation (20); however, the BR formula is not sensitive to sediment size in live-bed conditions. Figure 2(d) analogously shows that the BR formula is notably sensitive to d_{50} for flow intensity values less than 0.7:

$$\varepsilon_{d_{50}} = \frac{U/U_c}{1 - 2U/U_c} \tag{20}$$

Specific sensitivity of the JF formula is constant ($\varepsilon_{d_{50}} = 0.083$) in clear-water conditions. On the other hand, in live-bed conditions (Figure 3) specific sensitivity is determined based on flow intensity as follows:

$$\varepsilon_{d_{50}} = \frac{-0.088 - 0.041 U/U_c}{U/U_c - 1} \tag{21}$$

Absolute specific sensitivity of the FL formula to d_{50} is almost equal to ε_S and less than 0.113, according to the following formula:

$$\varepsilon_{d_{50}} = -0.113 \frac{d_{se} - b}{d_{se}} \tag{22}$$

Specific sensitivity of the SH formula to d_{50} is also a function of b/d_{50} and flow intensity, as shown in Equation (23):

$$\varepsilon_{d_{50}} = \frac{947.93 \left[-0.43 + \ln\left(\dfrac{U}{U_c}\right) \right] \cdot \left[1.32 + \ln\left(\dfrac{U}{U_c}\right) \right] - 6.32 \left(\dfrac{b}{d_{50}}\right)^{1.33} \cdot \left[-5.11 + \ln\left(\dfrac{U}{U_c}\right) \right] \cdot \left[0.11 + \ln\left(\dfrac{U}{U_c}\right) \right]}{\left[479.35 + 18.05 \left(\dfrac{b}{d_{50}}\right)^{1.33} \right] \cdot \left[1 - 1.75 \left(\ln\dfrac{U}{U_c}\right)^2 \right]} \tag{23}$$

For high values of b/d_{50} (greater than about 50), $\varepsilon_{d_{50}}$ can be assumed to be much less dependent on b/d_{50}

(Figure 4(c)), and Equation (23) reduces to the following one:

$$\varepsilon_{d_{50}} = \frac{-0.35[-5.11 + \ln(U/U_c)] \cdot [0.11 + \ln(U/U_c)]}{1 - 1.75[\ln(U/U_c)]^2} \tag{24}$$

For the ML equation, different sensitivity values to d_{50} are expected based on the value of b/d_{50}. If $b/d_{50} > 25$, a constant value of $\varepsilon_{d_{50}}$ (i.e., -0.5) was obtained in clear-water conditions. However, the ML formula is not sensitive to d_{50} in live-bed conditions when $b/d_{50} > 25$. In contrast, for $b/d_{50} \leq 25$, sensitivity of the ML formula is a function of b/d_{50} and can be estimated using Equations (25) and (26) in clear-water and live-bed conditions, respectively (Figure 5(a)):

$$\varepsilon_{d_{50}} = \frac{-1.4 - 0.5 \ln(b/d_{50})}{\ln(2.24 b/d_{50})} \tag{25}$$

$$\varepsilon_{d_{50}} = \frac{-1}{\ln(2.24 b/d_{50})} \tag{26}$$

The absolute value of $\varepsilon_{d_{50}}$ slightly decreases by increasing b/d_{50} and the ML formula is more sensitive to d_{50} in clear-water than in live-bed conditions for a certain b/d_{50} value.

For the HC formula, constant values for $\varepsilon_{d_{50}}$ were obtained (Table 3).

Combined specific sensitivity

In order to analyse the combined effect of variations on d_{se}, all the effective parameters (except b, as discussed before) are varied simultaneously, so that the total absolute variation can be calculated by using the following equation:

$$\text{Total absolute variation} = |\Delta d_{se,\Delta h}| + |\Delta d_{se,\Delta S}| + |\Delta d_{se,\Delta d_{50}}| \tag{27}$$

where, $\Delta d_{se,\Delta h}$, $\Delta d_{se,\Delta S}$, $\Delta d_{se,\Delta d_{50}}$ are variation of d_{se} due to variations of h, S, d_{50} (i.e., Δh, ΔS, and Δd_{50}), respectively. The percentage variation aliquot P_{F_i} (P_h, P_S, and $P_{d_{50}}$) related
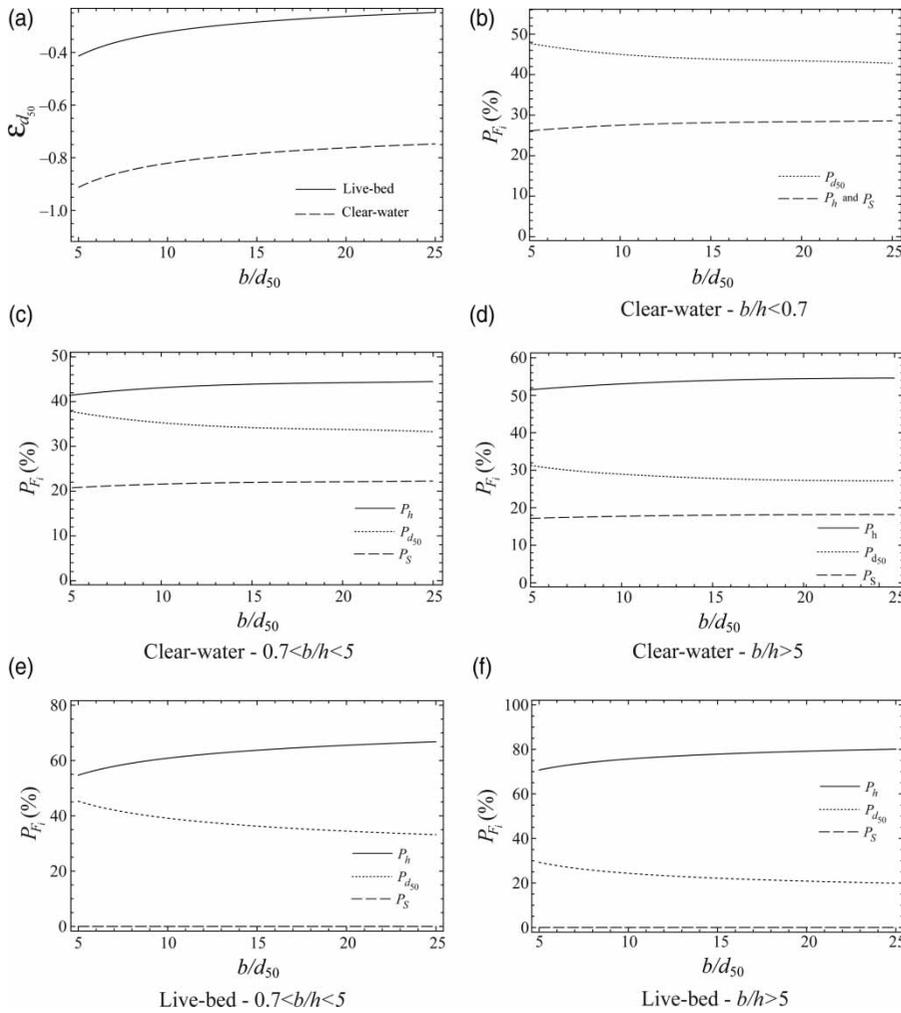
Figure 5 │ Specific sensitivity of the ML formula to (a) d_{50}, and (b)–(f) percentage of variation aliquot for $b/d_{50} \leq 25$.

to variation of each influencing parameter F_i (h, S, and d_{50}) can be defined as follows:

$$P_h = \frac{|\Delta d_{se,\Delta h}|}{|\Delta d_{se,\Delta h}| + |\Delta d_{se,\Delta S}| + |\Delta d_{se,\Delta d_{50}}|} \cdot 100 \quad (28a)$$

$$P_S = \frac{|\Delta d_{se,\Delta S}|}{|\Delta d_{se,\Delta h}| + |\Delta d_{se,\Delta S}| + |\Delta d_{se,\Delta d_{50}}|} \cdot 100 \quad (28b)$$

$$P_{d_{50}} = \frac{|\Delta d_{se,\Delta d_{50}}|}{|\Delta d_{se,\Delta h}| + |\Delta d_{se,\Delta S}| + |\Delta d_{se,\Delta d_{50}}|} \cdot 100 \quad (28c)$$

In order to compare the output values of P_h, P_S, and $P_{d_{50}}$, all the computations were based on a constant value

of c, i.e., $\Delta h = c \cdot h$; $\Delta S = c \cdot S$; $\Delta d_{50} = c \cdot d_{50}$. In this condition, P_h, P_S, and $P_{d_{50}}$ are independent from c; hence, combining Equations (8) and (28) the following formulae are obtained:

$$P_h = \frac{|\varepsilon_h|}{|\varepsilon_h| + |\varepsilon_S| + |\varepsilon_{d_{50}}|} \cdot 100 \quad (29a)$$

$$P_S = \frac{|\varepsilon_S|}{|\varepsilon_h| + |\varepsilon_S| + |\varepsilon_{d_{50}}|} \cdot 100 \quad (29b)$$

$$P_{d_{50}} = \frac{|\varepsilon_{d_{50}}|}{|\varepsilon_h| + |\varepsilon_S| + |\varepsilon_{d_{50}}|} \cdot 100 \quad (29c)$$

Equations (29a)–(29c) can be applied to the FL formula, according to the specific sensitivities given by Equations (13),

(18) and (22). The following values were obtained: $P_h = 70\%$, $P_S = 14\%$ and $P_{d_{50}} = 16\%$. In other words, the FL formula is more significantly sensitive to h rather than to S or d_{50}. Equations (29a)–(29c) can also be applied to the HC formula, looking at values shown in Table 3; it appears clear that P_h and then P_S represent the highest variation aliquots.

For the JF formula in clear-water conditions, P_h, P_S and $P_{d_{50}}$ were equal to 72, 0 and 28%, respectively; however, in live-bed conditions, the behaviour of this formula is a function of flow intensity, as presented in Figure 6. In fact, the JF formula in all conditions is significantly sensitive to h.

In the ML formula, each variation aliquot is constant for $b/d_{50} > 25$. Table 2 also shows that for $b/d_{50} > 25$ and $b/h < 0.7$, all the three influencing parameters have the same contribution in the total variation, whereas in the other conditions, the approach flow depth generates the highest variation aliquot with respect to the other influencing parameters for $b/h > 0.7$. The values of P_h, P_S, and $P_{d_{50}}$ are slightly dependent on b/d_{50} for $b/d_{50} \leq 25$. In this condition, the highest variation aliquot is $P_{d_{50}}$ for $b/h < 0.7$ (Figure 5(b)), whereas the highest variation aliquot is P_h for $b/h > 0.7$ (Figures 5(c)–5(f)).

For the BR formula in clear-water conditions, P_h and $P_S = P_{d_{50}}$ are functions of flow intensity and h/b (Figure 7). For $h/b < 3$, P_h increases with U/U_c, whereas for $h/b \geq 3$ P_h is almost constant as U/U_c varies. For $U/U_c > 0.5$, P_h increases as h/b decreases (for $U/U_c \leq 0.5$, the BR formula predicts $d_{se} = 0$; see Table 1). Note that in live-bed conditions the BR formula is only sensitive to h, i.e. $P_h = 100\%$ and $P_S = P_{d_{50}} = 0$.
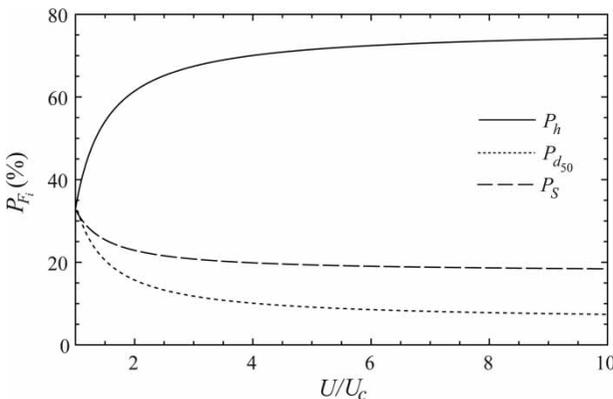
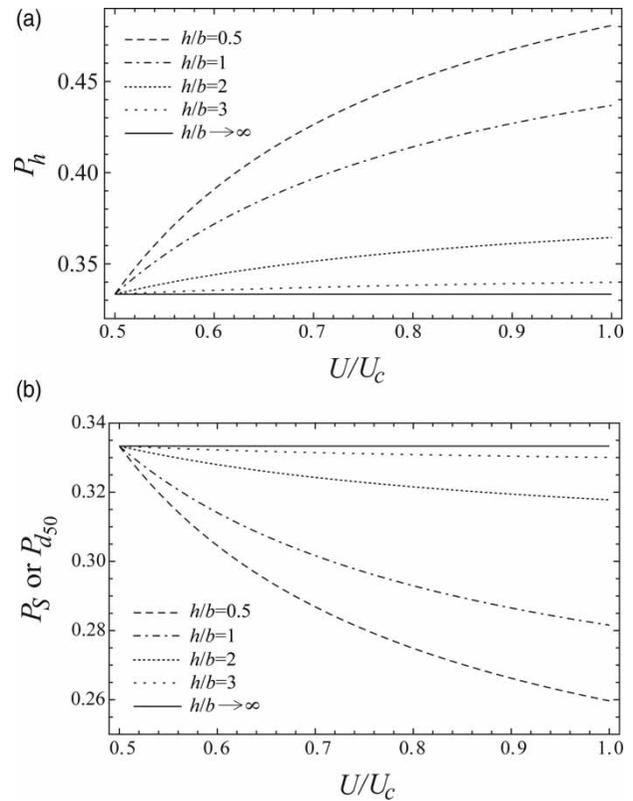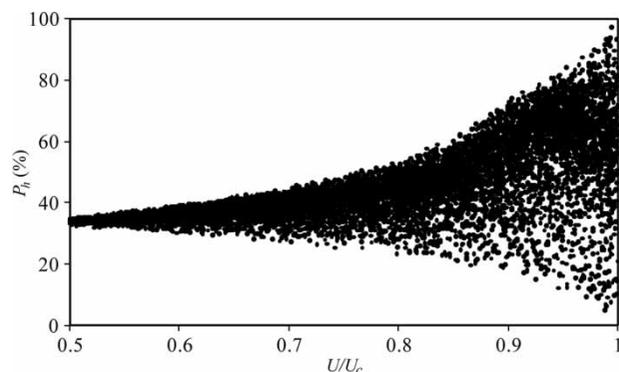**Figure 6** │ Percentage of variation aliquot of the JF formula in live-bed conditions.

Figure 7 │ Percentage of variation aliquot of the BR formula with respect to (a) h, and (b) S and d_{50} in clear-water conditions.

The behaviours of P_h, P_S and $P_{d_{50}}$ in the SH formula are different from those in the other formulae. In order to consider the behaviour variation aliquot of this formula, 10,000 triplet input data (h, b, d_{50}) were synthetically generated for clear-water conditions in the following ranges of values, which are typical of most natural watercourses: $0.5 \leq h \leq 10$ m, $0.5 \leq b \leq 5$ m and $2 \leq d_{50} \leq 64$ mm. Afterwards, for each triplet a value for S was randomly selected in order to ensure that live-bed scour conditions were obtained ($0.5 < U/U_c < 1$). For each dataset, values of P_h, P_S, and $P_{d_{50}}$ were computed. In this formula, ε_h, ε_S and $\varepsilon_{d_{50}}$ are functions of U/U_c, h/b and b/d_{50}. As mentioned before, dependency of ε_h and $\varepsilon_{d_{50}}$ on low values of h/b and b/d_{50} increases as flow intensity increases. For example, Figure 8 shows the calculated values of P_h based on synthetically generated data for this formula. This figure indicates an increasing scattering of data points on the right side of the graph for higher values of U/U_c where dependency of P_h to h/b and b/d_{50} increases.

Figure 8 | Percentage of variation aliquot of the SH formula in clear-water conditions with respect to h.

Therefore, in each case, percentage of variation aliquot of the SH formula should be directly calculated based on Equations (14), (19), (23) and (29).

CONCLUSIONS

Sensitivity analysis of six pier scour formulae was performed assuming that flow depth, riverbed slope and median sediment size are measured independently and mean approach flow velocity, critical flow velocity for the inception of sediment motion and maximum scour depth are calculated by Manning–Strickler equations, Neill formula and selected pier scour equations, respectively. Thus, if the approach flow velocity/flow depth/critical velocity for sediment motion are calculated with other methods, e.g., a site specific stage-discharge curve or a hydraulic/bed-morphodynamic model, a particular sensitivity analysis is needed.

The results clarify that some formulae in some conditions are very sensitive to input data, so that a preliminary sensitivity analysis is recommended to designers before using the predictions of selected empirical formulae, also if the input variables are affected by little uncertainty. In fact, even if such formulae might estimate the maximum scour depth with an acceptable approximation in specific conditions, small uncertainty due to measurement error in the input variables may produce high error in the output prediction. This also can be assumed as a reason that the formulae are more accurate in laboratory (with negligible uncertainty) than in field conditions (with higher level of uncertainty).

The outline of the present study is given by several equations derived for a pier scour case. As indicated by Yanmaz (2001), such equations cannot be easily quantified, owing to variations of factors related to local hydraulic, topographical and sedimentological characteristics, etc., which prevent a precise estimation of the local scour hole. Nevertheless, sensitivity analysis of pier scour depth formulae can be proficiently used as a useful tool in the application of selected formulae, identifying the conditions where a formula is considerably sensitive to input parameters.

Summarising, the main results of this study are as follows.

Sensitivity of three formulae, i.e. the SH and BR formulae in clear-water conditions and the JF formula in live-bed conditions ($Fr–Fr_c \geq 0.2$), depends especially on flow intensity. Sensitivity of these formulae to influence parameters decreases as flow intensity increases. For some formulae, the sensitivity values are just a function of flow intensity value, whereas for some others also h, b, d_{50} and S exert a non-negligible influence.

The BR and ML equations are independent of the approach flow velocity, the critical velocity of sediment motion and, consequently, the flow intensity under live-bed conditions. Thus, higher sensitivity is expected in clear-water than in live-bed conditions. The correction factor of wide pier in shallow water, i.e. K_w, recommended for the HC formula has greater exponents for clear-water applications; hence, employing this correction factor leads to higher sensitivity of the HC formula in clear-water conditions.

By neglecting the variation in pier width, in most formulae the higher variation aliquot is related to the approach flow depth. Actually, $d_{se} = f(U, h,…)$ and, if U is also estimated with h (e.g., by means of the Manning equation), the estimated d_{se} may be significantly sensitive to h.

Among the selected formulae, the HC and FL formulae showed lower sensitivity to h, d_{50} and S, having specific sensitivity less than 1 in all conditions, i.e. a certain error in a given h, d_{50} and S produces a lesser error in the maximum scour depth estimation. In fact, in these two formulae, scour depth is mostly a function of pier width b and, as mentioned before, pier width is generally employed in formulae as a deterministic parameter. Therefore, errors in estimation or measurement of influencing parameters have less influence on results of these two formulae.

ACKNOWLEDGEMENT

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