Crystalline Fouling Studies

N. Epstein. In his recent paper, R. B. Ritter presents the following three empirical equations for the fouling of aqueous calcium sulfate solutions undergoing sensible heating

\[ \theta = \frac{0.15}{k_t (\Delta C/C_s)} \times \text{hr} \quad (4) \]

\[ F_s = (3.8 \times 10^{-5}) k_t (\Delta C/C_s)^2, \text{(ft}^2\text{)} (\text{°F})/\text{Btu} \quad (5) \]

with standard deviations of ± 64 and ± 42 percent, respectively, and

\[ F_r = (1.3 \times 10^{-3}) \theta^{-0.6}, \text{(ft}^2\text{)} (\text{°F})/\text{Btu} \quad (8) \]

with an unstated standard deviation. (The units of each of the variables in equations (4), (5), and (8) are given in the Nomenclature.)

If we multiply equation (4) by equation (5), the result is

\[ F_r = (5.7 \times 10^{-3}) \theta^{-0.6}, \text{(ft}^2\text{)} (\text{°F})/\text{Btu} \quad (i) \]

It is clear to start with that equation (i), with a power of -1 on the \( \theta \), contradicts equation (8), where the power on \( \theta \) is -0.6. However, this apparent inconsistency may arise from the large standard deviations associated with equations (4) and (5), from which (i) was derived. More troubling is the very large discrepancy in the magnitudes of the coefficients in equations (8) and (i). The values of \( F_r \) which result from these two equations, over the reported measured induction time range of 0.15–50 hrs, are shown in the following table

<table>
<thead>
<tr>
<th>( \theta ) hours</th>
<th>( F_r \times 10^3 ) equation (8)</th>
<th>( F_r \times 10^3 ) equation (i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.046</td>
<td>0.038</td>
</tr>
<tr>
<td>1</td>
<td>1.3</td>
<td>0.0057</td>
</tr>
<tr>
<td>50</td>
<td>0.124</td>
<td>0.000114</td>
</tr>
</tbody>
</table>

The measured values of \( F_r \times 10^3 \) ranged from 0 to 3, with \( F_r \) in (\text{ft}\text{}^2\text{)} (\text{°F})/\text{Btu}, which is consistent with the results shown by equation (8) but is at least two orders of magnitude greater than those from equation (i). Evidently the values of the empirical constants in equation (4) and/or (5) are in error.

To determine the probable source of the error, \( F_r \) was first evaluated by equation (5) over the reported range of \( k_t = 16-140 \text{ lb/(ft}^2\text{)} \text{(hr)} \) and \( \Delta C/C_s = 0.09-1.70 \). The resulting possible range of \( F_r \) is then \( (3.8 \times 10^{-5})(16)(0.09)^2 \) or \( 4.92 \times 10^{-3} \) (\text{ft}^2\text{)} (\text{°F})/\text{Btu} at a minimum and \( (3.8 \times 10^{-5})(140)(1.70)^2 \) = \( 15.4 \times 10^{-3} \) (\text{ft}^2\text{)} (\text{°F})/\text{Btu} at a maximum. Since the measured values of fouling rate, \( F_r \), actually ranged from very small to \( 3 \times 10^{-3} \) (\text{ft}^2\text{)} (\text{°F})/\text{Btu}, it is likely that equation (5), which encompasses this range, is not in error. Equation (4) was then subjected to similar scrutiny, by evaluating \( \theta \) via this equation over the same ranges of \( k_t \) and \( \Delta C/C_s \). The resulting possible range of \( \theta \) is then \( 0.15/(140)(1.70)^2 \) or \( 3.71 \times 10^{-4} \) hr at a minimum and \( 0.15/(16)(0.09)^2 \) or 1.16 hrs at a maximum. Since the measured values of the induction period, \( \theta \), actually ranged from 0.15 to 50 hrs, it seems very likely that the constant, 0.15, in equation (4) is low by some two orders of magnitude.

Nomenclature

\( \Delta C/C_s \) = supersaturation at surface, dimensionless

\( F_r \) = fouling rate, (\text{ft}^2\text{)} (\text{°F})/\text{Btu}

\( k_t \) = mass transfer coefficient, \text{lb/(ft}^2\text{)} \text{(hr)}

\( \theta \) = induction period, \text{hr}

Author's Closure

Professor Epstein is correct in pointing out an error in the constant of equation (4). The value of the constant, \( A \), should be 55 when \( k_t \) is expressed as \text{lb/ft}^2\text{hr}, or 0.075 for \( k_t \) in S.I. units, \text{kg/m}^2\text{s}. The product of equations (4) and (5), expressed as equation (i) in Epstein's discussion, then becomes

\[ F_r = (2.1 \times 10^{-1}) \theta^{-1}, \text{(ft}^2\text{)} (\text{°F})/\text{Btu} \quad (i) \]

This equation and equation (8) give identical results at a midrange induction period, \( \theta \), of 3.3 hrs. The standard deviation of equation (8) was ± 85 percent.

Professor Epstein's careful analysis is certainly appreciated. We thank him for uncovering this error.