

ERRATUM | FEBRUARY 01 2003

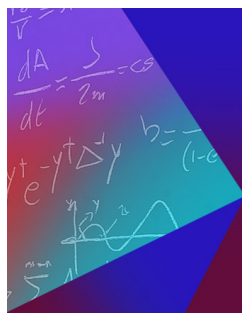
**Erratum: Pseudo-Hermiticity for a class of
nondiagonalizable Hamiltonians [J. Math. Phys. 43, 6343
(2002)]** **FREE**

Ali Mostafazadeh



J. Math. Phys. 44, 943 (2003)

<https://doi.org/10.1063/1.1540714>



Journal of Mathematical Physics

**Young Researcher Award:
Recognizing the Outstanding Work
of Early Career Researchers**

[Learn More!](#)

Erratum: Pseudo-Hermiticity for a class of nondiagonalizable Hamiltonians [J. Math. Phys. 43, 6343 (2002)]

Ali Mostafazadeh^{a)}

Department of Mathematics, Koç University,
Rumelifeneri Yolu, 34450 Sariyer, Istanbul, Turkey

[DOI: 10.1063/1.1540714]

Recently, the authors of Ref. 1 used the framework provided in Ref. 2 to re-examine the consequences of pseudo-Hermiticity for the class of block-diagonalizable Hamiltonians introduced in Ref. 2. In doing so, they discovered that Theorem 2 of Ref. 2 did not hold, as they could find a counter-example. This theorem must be replaced with the following.

Theorem 2: Let H be as in Theorem 1 of Ref. 2. Then H is pseudo-Hermitian if and only if it is Hermitian with respect to an inner product $\langle\langle\cdot,\cdot\rangle\rangle$ that supports a positive-semidefinite basis³ including the eigenvectors of H . In particular, for every eigenvector ψ of H , $\langle\langle\psi|\psi\rangle\rangle\geq 0$; if the corresponding eigenvalue is real and nondefective (algebraic and geometric multiplicities are equal), $\langle\langle\psi|\psi\rangle\rangle>0$; otherwise $\langle\langle\psi|\psi\rangle\rangle=0$.

Proof: As shown in Ref. 2, pseudo-Hermiticity of H implies that H is Hermitian with respect to the inner product $\langle\langle\cdot,\cdot\rangle\rangle_\eta$ with η given by Eq. (15) of Ref. 2 and $\sigma_{v_0,a}=1$. It is not difficult to check that indeed the basis vectors $|\psi_n,a,i\rangle$, constructed in Ref. 2, have the property that $\langle\langle\psi_n,a,i|\psi_n,a,i\rangle\rangle\geq 0$, and that $\langle\langle\psi_n,a,i|\psi_n,a,i\rangle\rangle>0$ only for the cases that $p_{n,a}=1$ and $E_n\in\mathbb{R}$, i.e., $|\psi_n,a,i=1\rangle$ is an eigenvector of H with a real eigenvalue. Furthermore, by construction, this basis includes all the eigenvectors of H . The proof of the converse is the same as the one given in Ref. 2.

It is important to note that having a positive-semidefinite basis does not imply that the inner product $\langle\langle\cdot,\cdot\rangle\rangle_\eta$ is positive-semidefinite (unless the Hamiltonian is diagonalizable and has a real spectrum in which case both the basis and the inner product $\langle\langle\cdot,\cdot\rangle\rangle_\eta$ are positive definite.⁴) If the Hamiltonian has defective or complex(-conjugate pair(s) of) eigenvalues, there will always be at least two null vectors with negative³ linear combinations. Unlike positive vectors, linear combinations of nonnegative vectors need not be nonnegative.

¹G. Sclarici and L. Solombrino, "On the pseudo-Hermitian nondiagonalizable Hamiltonians," arXiv:quant-ph/0211161.

²A. Mostafazadeh, J. Math. Phys. **43**, 6343 (2002).

³A vector ϕ is respectively said to be positive, null (zero), negative, if $\langle\langle\phi|\phi\rangle\rangle>0$, $\langle\langle\phi|\phi\rangle\rangle=0$, $\langle\langle\phi|\phi\rangle\rangle<0$. It is said to be nonnegative if $\langle\langle\phi|\phi\rangle\rangle\geq 0$. A basis is called positive-semidefinite if it consists of nonnegative vectors. See, for example, J. Bogner, *Indefinite Inner Product Spaces* (Springer, Berlin, 1974).

⁴A. Mostafazadeh, J. Math. Phys. **43**, 2814 (2002).

^{a)}Electronic mail: amostafazadeh@ku.edu.tr