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Erratum: Pseudo-Hermiticity for a class of nondiagonalizable Hamiltonians [J. Math. Phys. 43, 6343 (2002)]

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Recently, the authors of Ref. 1 used the framework provided in Ref. 2 to re-examine the consequences of pseudo-Hermiticity for the class of block-diagonalizable Hamiltonians introduced in Ref. 2. In doing so, they discovered that Theorem 2 of Ref. 2 did not hold, as they could find a counter-example. This theorem must be replaced with the following.

Theorem 2: Let H be as in Theorem 1 of Ref. 2. Then H is pseudo-Hermitian if and only if it is Hermitian with respect to an inner product $\langle\langle,\rangle\rangle$ that supports a positive-semidefinite basis[3] including the eigenvectors of H. In particular, for every eigenvector ψ of H, $\langle\langle\psi|\psi\rangle\rangle \geqslant 0$; if the corresponding eigenvalue is real and nondefective (algebraic and geometric multiplicities are equal), $\langle\langle\psi|\psi\rangle\rangle > 0$; otherwise $\langle\langle\psi|\psi\rangle\rangle = 0$.

Proof: As shown in Ref. 2, pseudo-Hermiticity of H implies that H is Hermitian with respect to the inner product $\langle\langle,\rangle\rangle_\eta$ with η given by Eq. (15) of Ref. 2 and $\sigma_{\nu_0,a}=1$. It is not difficult to check that indeed the basis vectors $|\psi_n,a,i\rangle$, constructed in Ref. 2, have the property that $\langle\langle\psi_n,a,i|\psi_n,a,i\rangle\rangle \geqslant 0$, and that $\langle\langle\psi_n,a,i|\psi_n,a,i\rangle\rangle > 0$ only for the cases that $p_{n,a}=1$ and $E_n \in \mathbb{R}$, i.e., $|\psi_n,a,i=1\rangle$ is an eigenvector of H with a real eigenvalue. Furthermore, by construction, this basis includes all the eigenvectors of H. The proof of the converse is the same as the one given in Ref. 2.

It is important to note that having a positive-semidefinite basis does not imply that the inner product $\langle\langle,\rangle\rangle_\eta$ is positive-semidefinite (unless the Hamiltonian is diagonalizable and has a real spectrum in which case both the basis and the inner product $\langle\langle,\rangle\rangle_\eta$ are positive definite.[4]) If the Hamiltonian has defective or complex(-conjugate pair(s) of) eigenvalues, there will always be at least two null vectors with negative[3] linear combinations. Unlike positive vectors, linear combinations of nonnegative vectors need not be nonnegative.

[1] G. Scolarici and L. Solombrino, ''On the pseudo-Hermitian nondiagonalizable Hamiltonians,'' arXiv:quant-ph/0211161.
[2] A. Mostafazadeh, J. Math. Phys. **43**, 6343 (2002).
[3] A vector ϕ is respectively said to be positive, null (zero), negative, if $\langle\langle\phi|\phi\rangle\rangle > 0$, $\langle\langle\phi|\phi\rangle\rangle = 0$, $\langle\langle\phi|\phi\rangle\rangle < 0$. It is said to be nonnegative if $\langle\langle\phi|\phi\rangle\rangle \geqslant 0$. A basis is called positive-semidefinite if it consists of nonnegative vectors. See, for example, J. Bognar, *Indefinite Inner Product Spaces* (Springer, Berlin, 1974).
[4] A. Mostafazadeh, J. Math. Phys. **43**, 2814 (2002).

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