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A One-Dimensional Numerical Model for Temperature Studies in Lakes

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Most hydro-electric schemes change the seasonal variation of river discharge. As a result of this the temperature in lakes downstream of a hydro-electric plant will be changed. The numerical model described in this paper has been developed to calculate the change in the vertical temperature distribution. The model has been applied to two Norwegian lakes.

Introduction

In all lakes the temperature field is three-dimensional and varying with time. It is however generally agreed that the temperature distribution averaged over a period of one week or more can be described in one space dimension, the vertical direction. This is due to the fact that

a) the horizontal diffusion coefficient is about 10^4 greater than the vertical diffusion coefficient,
b) the acceleration of gravity will, if acting as the only force, give horizontal isotherms.

It is well known that wind acting on the surface will result in isotherms bending down in the direction of the wind. If the wind suddenly changes direction or becomes weaker, the water masses will start oscillating and we will have internal waves.

It is assumed that these events can be neglected in the description of a one-year cycle since they have a much smaller time scale.

In deep lakes the heat exchange with the bottom can be neglected and the heat exchange with the surroundings can be described by the quantities shown in Fig. 1.

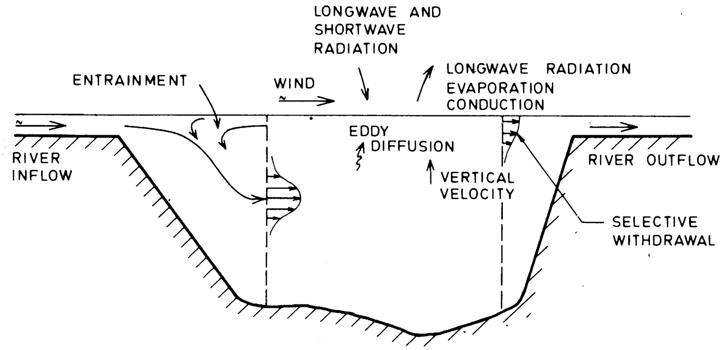

Fig.1. Quantities involved in the numerical analysis.

The Numerical Model

The mathematical analysis is an extension of the methods presented by Sundaram et.al. (1971) and Huber et.al. (1972).

The mathematical statement of the problem may be expressed as

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{1}{A}\frac{\partial}{\partial z}\left(A f \frac{\partial T}{\partial z}\right) - \frac{q_i}{A}(T - T_i) \tag{1}$$

where T is temperature, t is time, w is vertical velocity, A is the horizontal cross-section at any depth, f is the eddy diffusion coefficient, q_i represents horizontal inflow to the lake per unit of vertical height, T_i is the temperature of the incoming water and z is the vertical coordinate.

The boundary conditions are:

$$-\rho c f\frac{\partial T}{\partial z} = HN \quad \text{at} \quad z = 0 \tag{2}$$

$$-\rho c f\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = z_{max} \tag{3}$$

where HN is the net heat exchange with the atmosphere, c is the specific heat of water, ρ is the density of water and z_{max} is the maximum depth used in the calculation.

In periods when the lake is ice covered boundary condition Eq. (2) is replaced by

$$-\rho c f \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \qquad (4)$$

In the next sections we will present some of the quantities involved in the model.

Vertical Eddy Diffusion

Over the thermocline-region the diffusion coefficient is expressed by the equation.

$$f(z) \equiv f(0)\left(1 + \alpha \frac{\partial \rho / \partial z}{(W/z)^2}\right)^{-1} \qquad (5)$$

where $f(0)$ is the diffusion coefficient at $z=0$, W is the wind speed and α is a constant.

Below the thermocline the diffusion coefficient in the model is constant and equal to the calculated value at the thermocline.

In the model the thermocline is taken at a location below the depth of maximum density-gradient where

$$\frac{\partial \rho}{\partial z} = \beta \left(\frac{\partial \rho}{\partial z}\right)_{max} \qquad (6)$$

A combination of α and β where $\alpha = 10\ kg^{-1}\ m^4\ s^2$ and $\beta = 0.95$ seems to give good results for the two lakes tested. However, these numbers are expected to be some function of the surrounding topography and environmental variations from one year to another.

The vertical diffusion coefficient at the surface is, for lack of better knowledge, chosen to be constant over one year. This quantity is seldom measured in Norwegian lakes. It is probably a function of wind speed and direction, the thickness of the epilimnion and the size and configuration of the lake. The vertical diffusion coefficient at the surface is on the order of 0.5 - 5.0 cm^2/s. (For reference, see Hutchinson (1957) and Burtsaert et. al. (1969).)

Vertical Velocity

It is assumed that the vertical velocity is due to rivers entering the lake and flowing out of it. A sketch of the inlet and the outlet regions is shown in Fig. 2.

In the inlet region the river will mix with the surface water of the lake. There is no way of computing the rate of mixing in the inlet-region. It is supposed to be a function of sediment configuration, river velocity, the density difference between the river and the surface water and the density stratification of the lake. For lack of better ways to calculate these quantities Huber et. al. (1972) recommend a

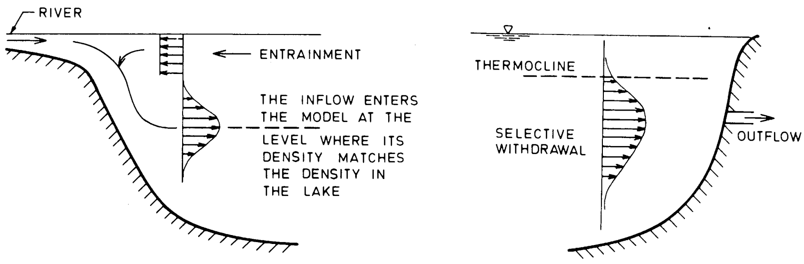

Fig.2. The inlet and outlet regions. The outflow depth may be arbitrarily chosen in the model.

mixing-volume of 50% of that of the river. This may be regarded as a very rough estimate. The results from parameter studies show that changing this quantity from 50% to 200% does not greatly influence the temperature distribution if the riverflow is small.

The outlet region is treated as a selective withdrawal problem. The thickness δ of the outflowing layer is calculated from the equation

$$\delta = 4.8 \left(\frac{q^2}{g\varepsilon}\right)^{\frac{1}{4}} \tag{7}$$

where q is the outflowing volume-flux per unit-width of the lake, g is the acceleration of gravity and ε is the normalized density gradient (Ryan et. al. 1971).

The calculations of the inlet and the outlet regions give horizontal fluxes to and from the model-area. The vertical velocities are then calculated from continuity.

Thermal Convection

When the surface layer is warmed from 0°C to 4°C in spring and cooled down to 4°C in autumn, we will have thermal convection in this region of the lake. This is due to the fact that pure water at 1 atm. pressure has its maximum density at 4°C.

The result of this process is that the upper layer becomes more or less completely mixed.

In the numerical model the process is simulated by mixing the upper layer downwards until a stable density statification is re-established whenever a negative density gradient is calculated.

The heat content is conserved in the model, but the release of potential energy when denser water is falling downwards is not been taken into account. It is likely that this energy will contribute to the vertical eddy diffusion also below the thermocline.

The Heat Exchange with the Atmosphere

According to Jirka et. al. (1975) the net heat transfer *HN* through a water surface is composed of net incident solar radiation *HSN,* net incident atmospheric radiation *HAR,* long wave radiation from the water surface *HW,* evaporation heat flux *HE* and conduction heat flux *HC.*

$$HN \equiv HSN + HAR - HW - HE - HC \qquad (8)$$

There exist several formulas for each of these quantities. For further information is referred to Jirca et. al (1975), Idso (1974) and Paily et. al. (1974).

The Numerical Method

The non-linear parabolic equation (1) is solved by the use of a predictor-corrector-method, see Remson et. al. (1971).

Time-steps of 2.5 days and 1 day have been used with success.

Case I – Jonsvannet

The model was first tested against data from Jonsvannet near Trondheim. The data were taken from Holtan (1961).

The area-depth-function is shown in Fig. 3. The river inflow and outflow were regarded small compared with other heatsources and hence neglected. The heat exchange with the atmosphere was calculated on the basis of mean monthly values from Trondheim.

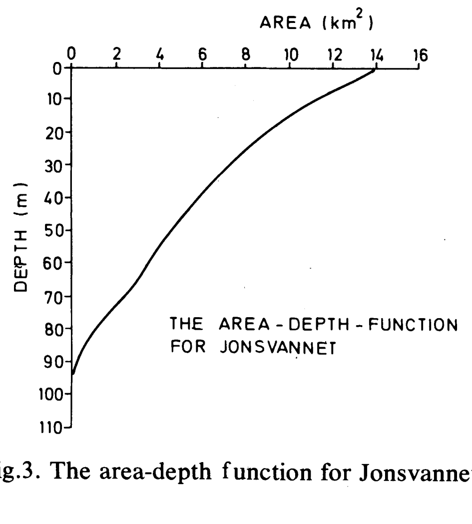

Fig.3. The area-depth function for Jonsvannet.

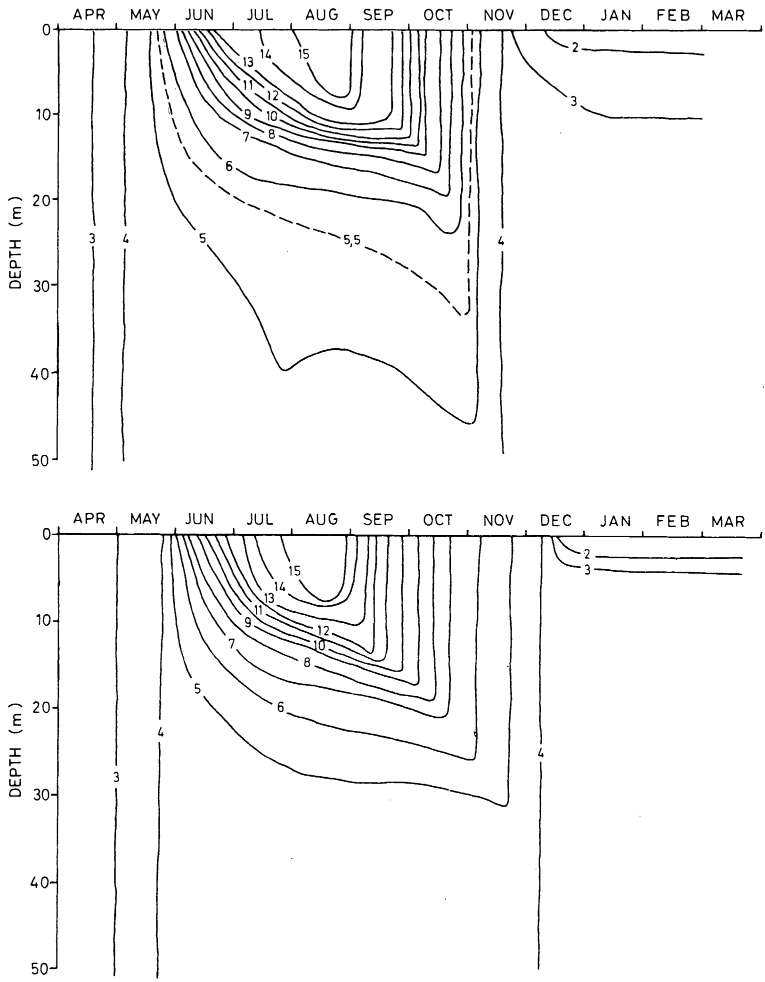

Fig.4. Measured (above)and calculated (below) isotherms.

Measured and calculated isopleths for temperature are presented in Fig. 4. In the calculation the diffusion coefficient in the surface layer was estimated to be 1.0 cm²/s for situations with no ice. When the lake was icecovered the diffusion coefficient was reduced by a factor of 10^3. The numbers were chosen in order to obtain a good fit between the measured and calculated temperature field.

As seen from the figures the model delays the time for spring and autumn turn-over by half a month. The maximum temperature and the time for this temperature was however correctly calculated. The depth of the epilimnion-layer was also correctly calculated throughout the summer and early autumn.

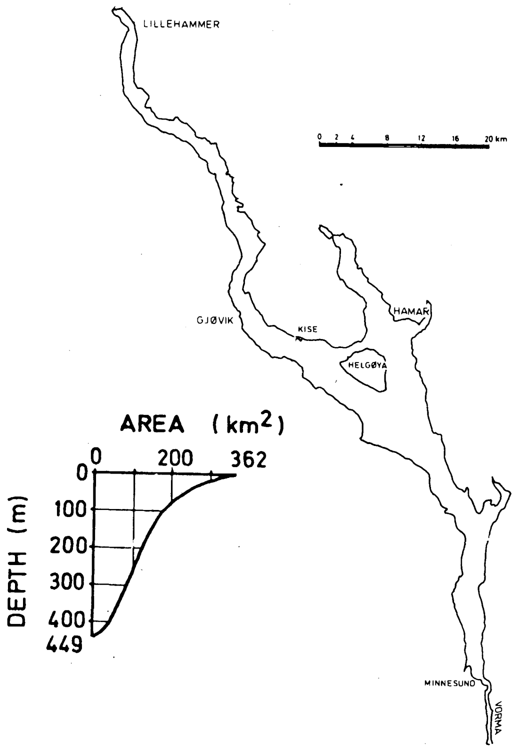

Fig.5.

Case II – Mjøsa

The configuration of the lake and the area-depth-function are shown in Fig. 5. The main river Lågen is entering the lake at Lillehammer far north and the outflowing river Vorma starts at Minnesund far south. The volume-flux of Lågen at Lillehammer and the river temperature for 1974 are shown in Fig. 6 as monthly average values. The water-flow in Vorma deviated somewhat from Lågen, but showed some of the same characteristics. For this reason it was believed that the error introduced by assuming the same water flow in Vorma as in Lågen was small. This approximation was necessary because the model is at present not capable of storing water.

From thermisor-chain recordings it was clear that the temperature conditions in Mjøsa north of Gjøvik differed systematically from the temperature conditions south of Gjøvik. Mjøsa south of Gjøvik showed an almost horizontally homogeneous temperature distribution throughout the year when filtering out the short-period fluctuations. It was therefore decided to run the model for the area south of Gjøvik only.

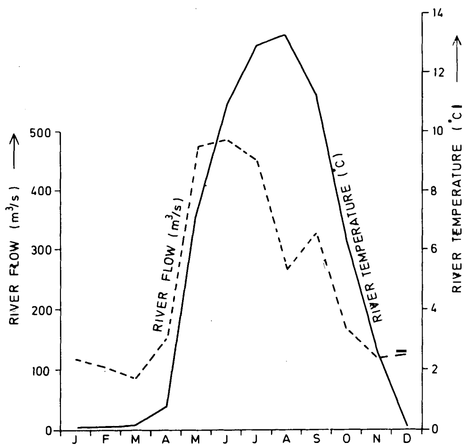

Fig.6. River-flow and river temperature for Lågen, 1974.

It is however obvious that there is heat-exchange between the basin south of Gjøvik and the basin north. The heat-exchange between these two basins was calculated from heat-budget considerations for the northern part. The result was:

Jan - March:	heat transported from the north to the southern part,
April - Aug:	heat transported from the southern part,
Sep - Dec:	heat transported to the southern part.

This heat transport turned out to be of the same order of magnitude as that caused by the river.

The explanation for this heat transport seems to be: In Dec - March the ice-free southern part of the lake is cooled while the ice-covered northern part is not. Therefore, heat is circulating to the southern part. The heat transport the rest of the year can be explained by the entrainment induced when Lågen enters Mjøsa.

Fig. 7 shows isotherms for September 1974 at Minnesund. The temperature at the surface is varying greatly with time due to internal waves. It is obvious that this will affect the temperature field of the lake since colder water than the surface water is leaving the lake for some periods of time. This is treated in the model by assuming that some part of the outflow leaves the lake below the thermocline. The principle is shown in Fig. 8.

The measured and calculated isopleths for temperature are shown in Fig. 9. Based on a comparison between the measured and calculated temperature field the diffusion coefficient is estimated to be 1.5 cm^2/s throughout the year. The heat exchange with the atmosphere is based on monthly averages from a weather station located at Kise.

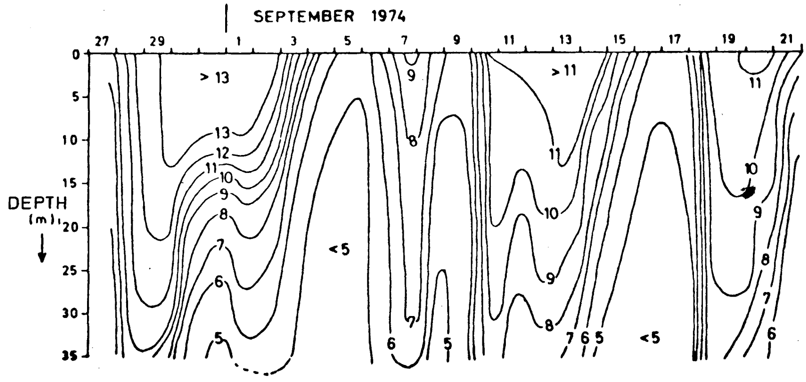

Fig.7. Isotherms at Minnesund, September 1974.

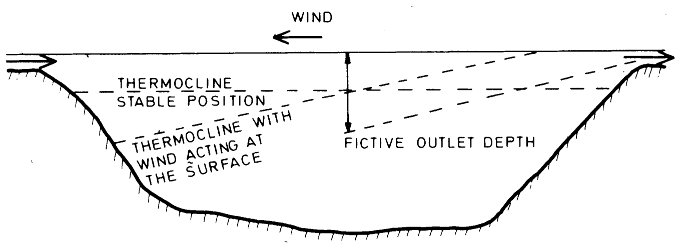

Fig.8. The principle for treating internal waves in the numerical model.

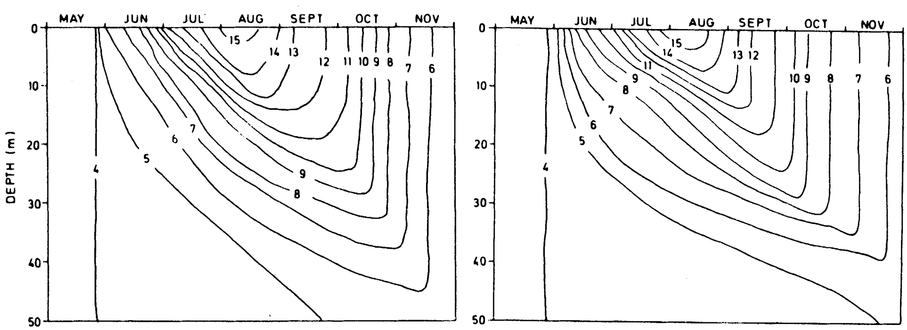

Fig.9. Measured (left) and calculated (right) isotherms, 1974.

Conclusions

The results so far indicate that the numerical model includes the main mechanisms for seasonal temperature variations in lakes. There are however some uncertainties in establishing the correct values for the constants entering the equation for the diffusion coefficients and in predicting the mixing processes at the river inflow. It is believed that these questions may be answered by further testing.

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